Firm-productivity and cross border merger

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Abstract: We show that the market concentration effect plays an important role in determining the relation between firm-productivity and cross border merger. In contrast to the existing theoretical literature but in line with some empirical evidence, we show that a low-productive firm may prefer cross border merger whereas a high-productive firm may either prefer greenfield foreign direct investment or export.

Key words: Cross border merger; Export; Foreign direct investment; Productivity

JEL Classifications: F21; F23; G34; L23; D24

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1. Introduction

Following the trade and investment liberalization wave in the world economy, the number of firms involved in international trade and foreign investment activities has dramatically increased. Cross-border mergers and greenfield foreign direct investments are two important ways through which many multinational firms serve foreign markets. Additionally, over the last two decades cross-border mergers, have become more popular compared to greenfield FDI. The share of cross-border mergers in total foreign direct investment (FDI) flows has increased and almost caught up with the share of greenfield FDIs (UNCTAD 2008).¹

Following the recent trend in the international trade literature, the literature on trade and investment has started to uncover the effects of firm-productivity on cross-border mergers. In a recent paper, Nocke and Yeaple (2007: here-after NY) examines the relationship between firm-productivity and foreign market entry decisions in a heterogeneous firm framework. In their paper, NY consider that the efficiency motive is the main driver of merger activities, and show that the most productive firms engage in cross-border mergers, while relatively less efficient firms choose greenfield FDI, and the least efficient firms export or only serve the domestic market.²

The present paper follows NY’s line of research and contributes to the literature by incorporating the industry concentration effect of cross-border mergers. Focusing on a monopolistically competitive environment, NY ignore any strategic motives for a merger. In this

¹ As a result of the global economic crisis of 2008-09, both total FDI flow, and the level of cross-border mergers decreased dramatically, but, recently they have tended to recover from the crisis episodes (UNCTAD 2013).
² This result is valid for Sector M in NY, where the firms differ in terms of their mobile capabilities, which is the main interest of this paper.
paper, we introduce the market-concentration effect of cross-border mergers by developing a two-country oligopoly model of product differentiation, where the firms compete in the product market.

The main finding of our paper is that, in the presence of a higher market-concentration under merger, a relatively low productive firm may prefer cross-border merger while a relatively high productive firm may prefer either greenfield FDI or export. Our results are more likely to hold if the product differentiation is low, so that the associated competition effect is large and the gain from product-market concentration is high. Thus, in contrast to NY, our results suggest that a strict productivity ranking in terms of cross border merger may not occur. While NY is more applicable for conglomerate mergers, our paper is more appropriate for mergers between the competitors in a particular industry. Our paper also complements NY by showing that their result may hold even if the firms are engaged in strategic competition.

Empirical evidence on this issue is very limited. Some recent empirical studies provide mixed results regarding the relationship between entry mode decision and firm productivity, which is consistent with our predictions. Trax (2011), for instance, finds that the most efficient UK firms choose cross-border merger over greenfield FDI in high intangibles industries, while she cannot find such evidence in the low intangibles sector. In contrary, Raff et al. (2012) show that the most productive Japanese firms prefer greenfield FDI over cross-border merger. Looking at the US firms, Nocke and Yeaple (2008) also find that the higher productive firms prefer greenfield FDI over cross border merger. show that Our results, showing that the relationship between productivity and merger is non-monotonic, can offer an explanation for the mixed empirical results.
The remainder of the paper is organized as follows. In section 2, we provide a review of recent theoretical and empirical literature. We describe the model and discuss the results in sections 3 and 4. Section 5 considers some extensions of our analysis. We provide concluding remarks and a future research agenda in Section 6. Many mathematical details are relegated to Appendices.

2. Related Literature

Two recent developments in the literature on FDI are worth mentioning. Until recently, the literature has focused on the determinants of an internationalization decision of a firm by treating firms within an industry as homogeneous. However, the emerging empirical literature shows that firms selling abroad are rare among all producers in both developing and developed countries, and their decisions to participate in international markets are not random (Bernard and Jensen, 1999). Furthermore, Bernard and Jensen (1999) show that firms engaging in international trade are different in terms of their size, productivity and capital intensity than those who operate domestically only.

Following these new findings, the seminal paper by Melitz (2003) and Bernard et al. (2003) provide theoretical analysis for the link between firm-level productivity and the export decisions of firms. The main finding of Melitz (2003) is that more productive firms exports while the less productive firms operate only domestically. Head and Ries (2003) and Helpman et al. (2004) contribute to this literature by explicitly considering two ways to serve the foreign market – export and FDI – and show that only the most productive firms within an industry undertake FDIs, while the less efficient firms serve the foreign markets via export. Head and Ries (2003) also show that less productive firms may undertake FDI in the presences of wage difference

3 See Markusen (2002) and Antràs and Yeaple (2014) for a survey.
between countries. Mukherjee and Marjit (2009) show that the theoretical results of Helpman et al. (2004) may not hold in the presence of wage bargaining between the labour union and management. Mukherjee (2010) shows that the particular preference function considered in Helpman et al. (2004) may be important for their result and their result may not hold under a different consumer preference function. Mrazova and Neary (2013) confirm the findings of Helpman et al. (2004) only if a firm’s variable cost of production and the transportation cost it faces are complementary. Moreover, they show that it may not be the case when they consider different preference structures, where firms cannot be sorted on the basis of their productivity in a way that Helpman et al. (2004) proposed.

The second important development in the FDI literature concerns the composition of FDI. Although cross-border merger is considered to be an important type of FDI (UNCTAD, 2008), the above-mentioned literature did not pay attention to this aspect. Some recent papers, such as Ferrett (2005), Bjorvatn (2004), Mattoo et al. (2004), and Neary (2009), explicitly model the two different components of FDI to examine the internationalization decisions of the firm. Although these papers provide several important insights, unlike our paper, they did not analyze the effects of firm-productivity on cross border mergers.

As already mentioned, the paper by NY, considered the relation between firm-productivity and cross border merger. Since this paper is most closely related to our paper, it is worth discussing that paper in a greater detail. NY consider firm heterogeneity along with the heterogeneous nature of FDI, where firms which want to sell abroad have the following options: export, greenfield FDI, and cross border merger. In their paper, firm heterogeneity arises from the fact that firms have different types of capability. They consider two sectors: sectors M and N. Firms are heterogeneous in terms of their internationally mobile capabilities in sector M, while in
sector N, the source of heterogeneity is the immobile capabilities that firms own. NY suggest that the motivation of a firm engaging in a cross border merger is closely linked to a firm’s heterogeneous capabilities. A target or an acquiring firm’s main motivation for undertaking cross border merger is to exploit complementarities when combining the firm-specific capabilities. When considering sector M, where firms differ in terms of their internationally mobile capabilities only, they show that the more productive firms prefer cross border merger over greenfield FDI. However, they show that firms that engage in cross border merger are the least efficient in industry N, where firms differ in terms of internationally non-mobile capabilities. Unlike our paper, NY consider that the efficiency motive is the only motive for a merger, and ignore any strategic motive, which is our focus.

There is another paper by Nocke and Yeaple (2008) where they show that the high productive firms prefer greenfield FDI compared to cross border merger. However, their reason is different from ours.

They consider a model of “vertical FDI” with complementary “headquarter input” and “production input”. Merger in their paper means acquisition of a higher productive production unit by a firm with a lower productive production unit. Hence, merger in their paper allows the acquirer to enjoy a cost reduction by taking over a higher productive production unit and because the target production union is efficient than the acquirer, the acquirer’s benefit from cost reduction is independent of its productivity. Since the acquirer profit increases with its productivity, it is then immediate that the acquirer’s gain from merger decreases with its productivity, a higher productive acquirer prefers greenfield FDI than cross border merger. It is worth noting that if the acquirer has more efficient production unit than the target production

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4 While the production technology of a firm is considered as an example of mobile capabilities, local marketing experience or supply networks are associated with immobile capabilities.
unit, merger does not occur in their paper. Further, if the wage rates are the same in both countries, thus eliminating the benefit from greenfield FDI in their paper, cross border merger is always the preferable strategy.

In contrast to Nocke and Yeaple (2008), we consider “horizontal FDI” with no fragmentation of production. Second, unlike them, merger in our paper does not create any cost synergy. Thirdly, unlike them, the acquirer (whose productivity is the point of focus) is always more productive compared to the target firm and merger can be profitable in our paper even if the wages are the same in both countries. While the reason for our paper is due to the market concentration effect following a merger, the benefit from cost synergy is the driving force for their result.

3. The Model

Consider an economy with two countries: home (H) and foreign (F), and two rival firms, each already settled in one of these countries. Assume that Firm F is located in the foreign country, while Firm H is located in the home country. We assume that these firms produce differentiated products and compete in the home-country market. Firm F can serve country H in the following ways:
Export (X): Firm F serves country H through export by incurring a positive per-unit transportation cost t.
Greenfield FDI (G): Under greenfield FDI, Firm F sets up a plant in country H by incurring a setup cost, G, and serves country H from that plant.

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\footnote{We show in Section 4.5 that our qualitative results also hold when we consider that there are N number of firms located in the home country, where N>2.}
Cross border Mergers (M): Under cross border merger, Firms F and H merge, the merged firm produces with the best available technology and the merger activity involves a fixed cost, $K$. This cost may occur due to organizational, managerial or technological factors (see, e.g., Hart and Tirole, 1990).

Although Firm F has another option, i.e. not serving the home-country market, we will assume the parameters values in a way so that this option will not occur in equilibrium. We will do this because considering the option of not serving the home-country by Firm F will not add much new insights to our analysis.

As in NY, we consider that labour is the only factor of production, and the wage rates in both countries are the same and equal to $w$. The same wage rate in our paper eliminates the reason for FDI found in Nocke and Yeaple (2008). Firms differ in terms of their labour productivities, and the technology of Firms F and H are $q_F = \frac{L_F}{\lambda}$, and $q_H = \frac{L_H}{\lambda}$, respectively, where $\lambda$ is the inverse of labour productivity. Lower $\lambda$ implies higher labour productivity, and this implies that Firm F is more productive than Firm H.\(^6\) We finally assume that Firm F can transfer its technological advantage to the merged entity so that it can produce at the same marginal cost it faces in its home country. This assumption makes our results comparable to the results of NY for Sector M in their paper, where firms are heterogeneous in terms of their internationally mobile capabilities.

Following Bowley (1924), we assume that the representative consumer in country H has the following utility function:\(^7\)

$$U = \left(q_F + q_H\right) - \left(\frac{q_F^2}{2} + \frac{q_H^2}{2}\right) - \gamma q_F q_H + m$$

\(^6\) Hence we obtain labour demand for Firm F and H: $L_F = \lambda q_F$ and $L_H = q_H$

\(^7\) The Bowley type of market demand function is commonly used in the industrial organization literature.
where \( m \) stands for the numeraire commodity, and the parameter \( \gamma \in [0,1] \) is the degree of product differentiation. The resulting inverse demand functions for goods \( q_F \) and \( q_H \) are:
\[
\begin{align*}
p_F &= 1 - q_F - \gamma q_H \\
p_H &= 1 - q_H - \gamma q_F
\end{align*}
\]
where \( p_F \) and \( p_H \) are prices, \( q_F \) and \( q_H \) are outputs. The products are perfect substitutes if \( \gamma = 1 \), while the goods are isolated for \( \gamma = 0 \).

We consider the following two-stage game. At stage 1, Firm F determines whether to export or to undertake greenfield FDI or to merge with Firm H. At stage 2, the firms compete in country H if Firm F either exports or undertakes greenfield FDI. There is no competition if Firms F and H merge, and the merged firm becomes a monopolist with two differentiated products. The profits are realized according to Firm F’s production decision. We solve the game through backward induction.

We will consider Cournot competition in the next section and will consider Bertrand competition in Section 4.5.1, if Firm F either exports or undertakes greenfield FDI. Although the duopoly market structure helps us to show our results in the easiest way, we show later on that our qualitative results hold even if there are \( n \) number of host-country firms.

4. Equilibrium Analysis

4.1. Export vs. greenfield FDI

If Firm F chooses to export, it determines output by maximizing the following expression:
\[
\max_{q_F} (1 - q_F - \gamma q_H - \lambda w - t)q_F.
\]
Expression (1) shows that Firm F can sell \( q_F \) units of product with a transportation cost of \( tq_F \) while the production cost is equal to \( \lambda w q_f \).
Firm H determines its output to maximize the following expression:

$$\max_{q_H} (1 - q_H - \gamma q_F - w)q_H.$$  \hspace{1cm} (2)

Maximizing (1) and (2), we get the equilibrium outputs of Firms F and H respectively:

$$q_F^X = \frac{(2 - \gamma - w(2\lambda - \gamma) - 2t)}{(4 - \gamma^2)^2}, \quad q_H^X = \frac{(2 - \gamma - w(2\lambda - \gamma) + yt)}{(4 - \gamma^2)^2}.$$  

Let $\pi_F^X (\pi_H^X)$ represent Firm F’s (Firm H’s) equilibrium profit if Firm F chooses to export. The equilibrium profits of Firms F and H are:

$$\pi_F^X = \frac{(2 - \gamma - w(2\lambda - \gamma) - 2t)^2}{(4 - \gamma^2)^2}, \quad \pi_H^X = \frac{(2 - \gamma - w(2\lambda - \gamma) + yt)^2}{(4 - \gamma^2)^2}. $$  \hspace{1cm} (3)

(4)

We assume that $q_F^X$ and $q_H^X$ are positive, i.e. $t < \frac{(2 - \gamma - 2\lambda w + wy)}{2} \equiv \bar{t}$.

If Firm F undertakes greenfield FDI, it maximizes the following profit function:

$$\max_{q_F} (1 - q_F - \gamma q_H - \lambda w)q_F - G$$  \hspace{1cm} (5)

while the profit function for Firm H is:

$$\max_{q_H} (1 - q_H - \gamma q_F - w)q_H.$$  \hspace{1cm} (6)

Maximizing (5) and (6), we obtain the equilibrium outputs of Firms F and H respectively:

$$q_F^G = \frac{(2 - \gamma - w(2\lambda - \gamma))}{(4 - \gamma^2)}, \quad q_H^G = \frac{(2 - \gamma - w(2\lambda - \gamma))}{(4 - \gamma^2)}. $$

Let $\pi_F^G (\pi_H^G)$ represent Firm F’s (Firm H’s) equilibrium profit if Firm F undertakes greenfield FDI. The equilibrium profits of Firms F and H are:

$$\pi_F^G = \frac{(2 - \gamma - w(2\lambda - \gamma)^2}{(4 - \gamma^2)^2} - G$$  \hspace{1cm} (7)

$$\pi_H^G = \frac{(2 - \gamma - w(2\lambda - \gamma)^2}{(4 - \gamma^2)^2}.$$  \hspace{1cm} (8)

We assume that $q_F^G, q_H^G$ are positive, i.e. $w < \frac{(2 - \gamma)}{(2 - \lambda \gamma)} \equiv \bar{w}$. 
Firm F prefers greenfield FDI compared to export if \( \pi^G_F > \pi^X_F \) or:

\[
G < -\frac{4t(-2+t+2\lambda w+\gamma-\omega)}{(-4+\gamma^2)^2} \equiv G_1 \tag{9}
\]

where \( G_1 > 0 \). Firm F prefers export over greenfield FDI for \( G > G_1 \). Differentiating \( G_1 \) with respect to \( \lambda \) we find that \( \frac{\partial G_1}{\partial \lambda} = -\frac{8\omega t}{(-4+\gamma^2)^2} < 0 \).

The following result is immediate from the above discussion.

**Proposition 1:** If greenfield FDI is the alternative to export, Firm F’s incentive for greenfield FDI increases with its productivity, i.e., \( \frac{\partial G_1}{\partial \lambda} < 0 \).

The reason for the above result is that if the productivity of Firm F increases, it produces more, and faces a higher trade cost in total. Since this cost can be avoided by greenfield FDI, Firm F’s incentive for greenfield FDI increases. This result is in line with Helpman, Melitz and Yeaple (2004).

### 4.2. Export vs. cross border merger

From equation (9) we know that Firm F prefers export over greenfield FDI if \( G > G_1 \). Now we will see Firm F’s preference for cross-border merger when export is the alternative to merger.

We know from (3) and (4) that if Firm F exports, the profits of Firms F and H are

\[
\pi^X_F = \frac{(2-\gamma-w(2\lambda-\gamma)-2t)^2}{(4-\gamma^2)^2}, \quad \text{and} \quad \pi^X_H = \frac{(2-\gamma-w(2-\lambda \gamma)+\omega)^2}{(4-\gamma^2)^2}.
\]

On the other hand, if Firms F and H merge, the merged entity maximizes the following profit function:

\[
m_{q_F, q_H} \max (1 - q_F - \gamma q_H - \lambda w) q_F + (1 - q_H - \gamma q_F - \lambda w) q_H - K. \tag{10}
\]
The merged firm produces differentiated products in county H. Merger allows Firm F to avoid the trade cost but the merged firm incurs a cost of merger, K. Since Firm F is more efficient than Firm H, the merged firm uses the technology of Firm F and \( \lambda \) stands for the productivity of the merged firm.

If Firms F and H merge, the equilibrium outputs and the profit of merged firm are respectively:

\[
q_F^M = q_H^M = \frac{(1 - \lambda w)}{2(1 + \gamma)}
\]

\[
\pi_F^M + \pi_H^M = \frac{(1 - \lambda w)^2}{2(1 + \gamma)} - K. \tag{11}
\]

Merger between firms F and H occurs if the profit of merged firm, \( \pi_F^M + \pi_H^M \), exceeds the total profits of Firms F and H under export by Firm F, i.e. if:

\[
\pi_F^M + \pi_H^M > \pi_F^X + \pi_H^X.
\]

Since how the profit of the merged firm getting divided is not important for our purpose, we focus on the total profits after and before merger.

It follows from (3), (4) and (11), that Firm F prefers cross-border merger compared to export if:

\[
K < \frac{(1 - \lambda w)^2}{2(1 + \gamma)} - \left( \frac{(2 - \gamma - w(2\lambda - \gamma) - 2w)^2}{(4 - \gamma^2)^2} + \frac{(2 - \gamma - w(2\lambda + \gamma + 2\gamma) + \gamma t)^2}{(4 - \gamma^2)^2} \right) \equiv K_1. \tag{12}
\]

The above condition shows the maximum gain from merger compared to export, thus a higher value of \( K_1 \) suggests that the firms have a higher incentive to pursue a cross-border merger.

Now we show how the incentive for merger changes with respect to productivity. Equation (12) shows that \( K_1 \) consists of the profit of the merged entity and the profits of Firm F and H when Firm F chooses to export. All these profits in equation (12) depend on productivity term \( \lambda \). Therefore, the relationship between \( K_1 \) and \( \lambda \) depends on how these three profit terms
react to the productivity change. If the difference between the profit of the merged firm and the combined profit of Firms F and H under export by Firm F increases (decreases) in $\lambda$, we can conclude that a lower $\lambda$ increases (decreases) Firm F’s merger incentive.

Before examining this, we define:

$$t^* = \frac{8w\gamma(1+\gamma) - (-2 + \gamma)^2 (2+ \gamma (y+2)) + \lambda w (8 + \gamma (-8 + \gamma (-10 + \gamma (-2 + \gamma))))}{2(1+\gamma)(4 + \gamma^2)}$$

where $t^* > 0$ if $w > \frac{(\gamma-2)^2 (2+\gamma (y+\gamma))}{8\gamma(1+\gamma) + \lambda (8+\gamma (-8 + \gamma (-10 + (\gamma-2)\gamma)))} \equiv w^*$ and $w^* > 0$.

**Proposition 2:** If Firm F prefers export over greenfield FDI, i.e. $G > G_1$, a higher productivity of Firm F increases (decreases) the incentive for cross-border merger if $t \in (t^*, \bar{t})$ or if $(t \in (0, t^*))$.

**Proof:** By using condition (12) we can show that $\frac{\partial K_1}{\partial \lambda} < (>)0$ for $t > (<) t^*$, where $t^* < \bar{t}$. ■

The above result is in contrast to Nocke and Yeaple (2007) for $t < t^*$, showing that the incentive for a cross border merger is higher for a higher productive firm. If the trade cost is high, i.e. $t > t^*$, Firm H is a near monopoly and in this situation, a lower productivity of firm F reduces the industry profit under export. Hence, to keep the benefit of a near monopoly, the firms incentive for merger increases with a lower productivity of Firm F. On the other hand, if the trade cost is low, i.e. $t < t^*$, competition is fierce under export and a lower productivity of Firm F increases the industry profit under export by increasing production efficiency. A lower productivity of Firm F increases profit of the merged firm also. Since the total output is higher under export compared to merger, the effect of a lower productivity of Firm F is stronger under
export compared to merger, thus reducing the incentive for cross border merger following a higher productivity of Firm F.

In order to see the effect of the level of product differentiation on our findings, let us now consider how \( t^* \) changes with respect to the level of \( \gamma \).

**Proposition 3:** A lower level of product differentiation, i.e. higher \( \gamma \), increases the range for \( t \) over which a lower productivity of Firm F reduces the incentive for merger, i.e. \( \frac{\partial t^*}{\partial \gamma} > 0 \).

**Proof:** Differentiating \( t^* \) with respect to \( \gamma \), we find that:

\[
\frac{\partial t^*}{\partial \gamma} = \frac{(-4+\gamma^2)(-8+\gamma(8+\gamma(16+\gamma(2+\gamma))))+w(-8(1+\gamma)^2+\lambda(16+\gamma(24+\gamma(2+\gamma))))}{2(1+\gamma)^2(4+\gamma^2)^2} > 0.
\]

As product differentiation reduces, it increases competition and the output effect under export becomes more important compared to merger, thus increasing the possibility of a lower incentive for merger following a higher productivity of Firm F.

**4.3 Greenfield FDI vs. cross border merger**

Now we consider the case where \( G < G_1 \), i.e. Firm F prefers greenfield FDI compared to export.

We know from equations (7), and (8) that if Firm F undertakes greenfield FDI, the profits of firms F and H are 

\[
\pi_F^G = \frac{(2-\gamma-w(2-\lambda)\gamma)}{(4-\gamma)^2} - G, \quad \text{and} \quad \pi_H^G = \frac{(2-\gamma-w(2-\lambda)\gamma)}{(4-\gamma)^2}.
\]

If Firms F and H merge, we know from (11) that the profit of the merged entity is

\[
\pi_{F+H}^M = \frac{(1-\lambda)w^2}{2(1+\gamma)} - K.
\]

If greenfield FDI is the alternative to merger, a merger between firms F and H is profitable if:

\[
\pi_{F+H}^M > \pi_F^G + \pi_H^G.
\]
Hence, the cross-border merger occurs if:

\[
K < \frac{(1-\lambda w)^2}{2(1+\gamma)} - \frac{(2-\gamma-w(2-\lambda))}{(4-\gamma)^2} - G \equiv K_2.
\]  

(13)

The value of $K_2$ shows the maximum gain of Firm F from merger compared to greenfield FDI. A higher value for $K_2$ suggests that Firm F has a higher incentive for cross-border merger.

Before further analysis we define:

\[
w' = \frac{(y-2)^2(2+y(2+y))}{8y(1+y)+y(8+y(-10+y(-2+y)))} > 0.
\]

**Proposition 4:** If greenfield FDI is the alternative to merger, i.e. $G < G_1$, Firm F’s incentive for undertaking cross-border merger increases (reduces) with a higher productivity of Firm F, i.e. with a lower $\lambda$, if $w \in (0, w')$ or if $(w \in (w', \bar{w}))$.

**Proof:** The proof follows from rearranging the derivative of $K_2$ with respect to $\lambda$. Straightforward calculation shows that $\frac{\partial K_2}{\partial \lambda} < (>)0$ for $w < (>)w'$. The rest of the proof follows immediately. $^8\blacksquare$

If the wage is high, i.e. $w > w'$ Firm F is a near monopoly and a lower productivity of Firm F strengthen Firm F’s near monopoly position and reduces its incentive for cross border merger. However, if the wage is low, i.e. $w < w'$, competition is fierce and the firm’s incentive for monopolizing the market increases the incentive for cross border merger following a lower productivity of Firm F.

Now we show how the range of $w$ over which a higher productivity of Firm F reduces the incentive for a cross border merger varies with product differentiation.

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$^8$ Please see Appendix 3 Section C.1 for the proof of $\bar{w} > w'$.
Proposition 5: A lower level of product differentiation, i.e. higher \( \gamma \), increases \( (w-w')' \), i.e. increases the range of \( w \) over which a higher productivity of Firm F reduces the incentive for a cross border merger.

Proof: See Appendix 3 Section C.2 for the proof. ■

If wage is higher, a higher competition due to a lower product differentiation increases the possibility of a near monopoly by Firm F. Hence, a lower product differentiation reduces the possibility of a cross border merger following a lower productivity of Firm F.

5. Robustness and extensions

5.1. Bertrand Competition

It is well known that the results under horizontal merger may depend on the type of product market competition, viz., Cournot or Bertrand competition, (Salant et al., 1983 and Deneckere and Davidson, 1995). The purpose of this subsection is to see whether our results under Cournot competition shown above hold under Bertrand competition.

In order to solve the Bertrand game, first we obtain the direct demand functions by utilizing the inverse demand functions. They take the following form:

\[
q_F = \frac{(1-\gamma)P_F + \gamma P_H}{1-\gamma^2} \quad \text{and} \quad q_H = \frac{(1-\gamma)P_H + \gamma P_F}{1-\gamma^2}
\]

For our analysis under Bertrand competition, we illustrate the calculations for equilibrium outputs and profits in Appendix 3 Section A.
5.1.1. Equilibrium Analysis

Export vs. greenfield FDI

Consider that Firm F chooses between export and greenfield FDI in order to serve country H. If Firm F chooses to export, the equilibrium outputs $q_F^X$ and $q_H^X$ are positive if $t < 1 - \lambda w - \frac{(w-1)\gamma}{\gamma^2 - 2} \equiv \bar{t}$. However, if Firm F undertook greenfield FDI, the equilibrium outputs $q_F^G$ and $q_H^G$ are positive if $\frac{(\gamma - 1)(2 + \gamma)}{\gamma + \lambda (\gamma^2 - 2)} \equiv \bar{w}$. We assume that these conditions hold.

Firm F chooses to undertake greenfield FDI if and only if $\pi_F^G > \pi_F^X$ or:

$$G' < \left(\frac{t(\gamma - 1)(2 + \gamma) + 2(1 + w)\gamma + \gamma^2(-2 + t + 2\lambda w)}{(-4 + 4\gamma^2)(-1 + \gamma^2)}\right) \equiv G_2. \quad (14)$$

Export vs. cross border merger

From equation (14), we know that Firm F prefers export over greenfield FDI if $G' > G_2$. We find that cross border merger is profitable compared to export if:

$$K' < \frac{(1 - \lambda w)^2}{2(1 + \gamma)} \left(\frac{(-1 + t + \lambda w + \gamma - w\gamma - (-1 + t + \lambda w)\gamma^2)^2}{(-4 + 4\gamma^2)(-1 + \gamma^2)} + \frac{(-2 + \gamma(-1 + t - \gamma) + w(-2 + \gamma(\lambda + \gamma))^2}{(-4 + 4\gamma^2)(-1 + \gamma^2)}\right) \equiv K_3. \quad (15)$$

Let's define:

$$t^{**} = \frac{-8 + 8(1 + w)\gamma + 2\gamma^2 - 4(1 + w)\lambda + \lambda^2 + \lambda^4 + \lambda^5 - \lambda w(-8 + \gamma(16 + \gamma(2 + \gamma(-8 + \gamma + \gamma^2)))}{2(4 - 3\gamma + \gamma^3)}$$

where $t^{**} > 0$ if $w > \frac{(\gamma - 1)(\gamma + 2)^2(2 + \gamma(-2 + \gamma))}{4\gamma(-2 + \gamma)^2 + \lambda(-8 + \gamma(16 + \gamma(2 + \gamma(-8 + \gamma + \gamma^2)))} \equiv w^{**}$. 

16
Proposition 6: If export is the alternative to merger, \((G' > G_2)\), a higher productivity of Firm F increases (reduces) the incentive for cross-border merger, if \(t \in (t^{**}, \bar{t})\) or if \((t \in (0, t^{**})\).

Proof: By using condition (15) we can show that \(\frac{\partial K_3}{\partial \lambda} < (>) 0\) for \(t > (<) t^{**}\) where \(t^{**} > \bar{t}\). ■

Like the case under Cournot competition, this result is in contrast to NY for \(t < t^{**}\). The intuition is similar to that of Proposition 2.

Greenfield FDI vs. cross border merger

Now consider that \(G' < G_2\), i.e. greenfield FDI is preferable over export. We find that cross border merger is profitable compared to greenfield FDI by Firm F if:

\[
K'' < \frac{(1-\lambda w)^2}{2(1+\gamma)} + \left(\frac{(-2+\gamma+y^2-w(-2+y(\lambda+y))}{(-4+y^2)^2(-1+y^2)}\right) + \left(\frac{(2+y(-1+w-y)+y\lambda(-2+y^2)^2}{(-4+y^2)^2(-1+y^2)} - G'\right) \equiv K_4.
\]

(16)

Condition (16) tells us that Firm F prefers to do cross-border merger over greenfield FDI if and only if \(K'' < K_4\).

Let’s define:

\[
w'' = \frac{-8+8y+2y^2-4y^3+y^4+y^5}{-8\lambda-8y+16\lambda y+2\lambda y^2+4y^3-8\lambda y^3+3\lambda y^4+3\lambda y^5} > 0.
\]
Proposition 7: If greenfield FDI is the alternative to merger, i.e. \( G' < G_2 \), the incentive for cross border merger increases (reduces) with higher productivity of Firm F if \( w \in (0, w'') \) or if \( (w \in (w'', \tilde{w})) \).

Proof: By using condition (16) we can show that \( \frac{\partial K_i}{\partial \lambda} < (>)0 \) for \( w < (>)w'' \). ■

The intuition is similar to that of for Proposition 4.

5.2. N Firms in the country H

To convey our point in the simplest way, we assumed in section 2 that there is only one firm in country H. We show in this section that our findings of section 2 hold true when there are \( n \) symmetric firms in country H, where \( n>2 \).

If the products are homogeneous, the inverse demand functions for Firm F and for the ith home-country firm are respectively:

\[
p_F = 1 - q_F - \sum_{i=1}^{n} q_i
\]

\[
p_i = 1 - \sum_{i=1}^{n} q_i - q_F
\]

where \( p_F \) and \( p_i \) are prices, \( q_F, q_i \) are outputs.

We assume in this section that the firms compete like Cournot oligopolists. We illustrate the calculations for equilibrium outputs and profits in Appendix 3 Section B.

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9 The revised version of the paper will include the differentiated goods case.
**Equilibrium Analysis**

**Export vs. greenfield FDI**

First, consider Firm F’s choice between export and greenfield FDI.

If Firm F chooses to export, the equilibrium outputs $q^X_F$ and $q^X_H$ are positive if $t < \frac{1-\lambda w+nw-\lambda nw}{1+n} \equiv \bar{t}$. On the other hand, if Firm F undertakes greenfield FDI, the equilibrium outputs $q^G_F$ and $q^G_H$ are positive if $w < \frac{1}{2+\lambda} \equiv \bar{w}$. We assume that these conditions hold.

Firm F prefers greenfield FDI over export if:

$$G'' < \left(-\frac{(1+n)t(-2+t+n+t+2w(\lambda+w(1+\lambda)))}{(n+2)^2}\right) \equiv G_3. \quad (17)$$

If condition (17) is not satisfied, Firm F prefers export to greenfield FDI.

**Export vs. cross border merger**

We find that cross border merger is profitable compared to export by Firm F if:

$$K'' < \frac{(1-\lambda nw+w(n-1))^2}{(n+1)^2} - \left(\frac{(1-\lambda w(n+1)-t(n+1)+wn)^2}{(n+2)^2} + \frac{(1-2w+\lambda w+t)^2}{(n+2)^2}\right) \equiv K_5. \quad (18)$$

A cross-border merger between Firm F and the ith firm in country H is not preferable over export if the cost of merger, $K''$, is greater than $K_5$, showing the maximum gain from merger compared to export.

Let’s define:

$$t^{***} = \frac{-n(3+2n)+(2+n(9+5n)-\lambda(2+3n(2+n)))w}{(n+1)^2(2+n(2+n))}.$$

Moreover $t^{***} > 0$ if $w > \frac{n(3+2n)}{-2+2\lambda-9n+6n^2-5n^2+3n^2} \equiv w^{***}$. 


Proposition 8: If export is Firm F’s preferred choice as an alternative to merger, i.e. $G'' > G_3$.

Firm F’s higher productivity increases (reduces) the incentive for cross-border merger, if
$$t \in (t^**, \bar{t}) \ (t \in (0, t^**) ).$$

Proof: By using condition (18) we can show that $\frac{\partial K_6}{\partial \lambda} < (>) 0$ for $t > (<) t^{**}$, where $t^{**} < \bar{t}$. ■

This result is in contrast to Nocke and Yeaple (2007) for $t^{**} < \bar{t}$.

Greenfield FDI vs. cross border merger

We consider in this subsection that $G'' < G_3$, i.e. greenfield FDI is Firm F’s alternative to merger.

Firm F undertakes cross-border merger compared to greenfield FDI if:

$$K'' < \frac{(1-\lambda w + \omega(n-1))^2}{(n+1)^2} \left( \frac{(1-\lambda w(n+1)+\omega n)^2}{(n+2)^2} + \frac{(1-2w+\lambda \omega)^2}{(n+2)^2} - G'' \right) \equiv K_6.$$  \hspace{1cm} (19)

The value of $K_6$ shows the maximum gain from merger compared to greenfield FDI. A higher value for $K_6$ suggests that Firm F has a higher incentive for cross-border merger.

Let’s define:

$$w'' = \frac{-3n^2 - 2n^2}{-2 + 2\lambda - 9n + 6\lambda n + 5n^2 + 3\lambda n^2}.$$
Proposition 9: If greenfield FDI is Firm F’s alternative to merger, i.e. $G'' < G_3$, the incentive for undertaking cross-border merger increases (reduces) with its higher productivity if $w \in (0, w')$ (if $w \in (w'', \bar{w})$).

Proof: A straightforward calculation shows that $\frac{\partial K_6}{\partial \lambda} < (>) 0$ for $w < (>) w''$. The rest of the proof follows immediately. ■

Again, this result is in contrast to Nocke and Yeaple (2007) for $w > w''$.

6. Conclusion and Future Research Agenda

In this paper, we showed how the productivity of a foreign firm affects the incentive for a cross-border merger. In a two-country oligopolistic model with differentiated goods, we showed that the predictions of Nocke and Yeaple (2007), suggesting that the most productive firms prefer cross-border merger, may not hold true if the competition reducing effect of a merger is considered. We observed a non-monotonic relationship between productivity and a cross-border merger, and showed that our results hold under both Cournot and Bertrand competition.

In order to focus on the strategic motive for merger, we kept our model simple by assuming that there is only one foreign firm. However, it would be interesting to investigate a model where there are two or more asymmetric (in terms of productivity) foreign firms, to examine the strategy of different foreign firms, where competition among the foreign firms for greenfield FDI and cross-border merger creates further strategic effects. We leave this issue for future research.
Appendix I Section A

Bertrand Competition and the International Organization of Production

Equilibrium Analysis

Export vs Greenfield Investment

First, consider firm F’s choice between export and greenfield FDI. If Firm F chooses to export, the objective functions for Firms F and H are $\pi_F = (P_F - \lambda w - t)q_F$ and $\pi_H = (P_H - w)q_H$, respectively. The resulting equilibrium prices, and profits are:

$$P_F^E = \frac{-2-\gamma + 2\lambda w + \lambda w + 2t - \gamma^2}{-1-\gamma^2} , \text{ and}$$

$$\pi_F^E = \frac{(2(-1 + t + \lambda w + \gamma - w\gamma - (1 + t + \lambda w)^2 + 2(-1 + t + \lambda w)w - \gamma^2)^2}{(-4 + \gamma^2)(-1 + \gamma^2)^2}$$

$$(A1)$$

$$P_H^E = \frac{-2-\gamma + 2\lambda w + \lambda w + 2t - \gamma^2}{-1-\gamma^2} , \text{ and}$$

$$\pi_H^E = \frac{(2 + (-1 + t - \gamma)\gamma + w(-2 + \gamma(\lambda + \gamma))^2}{(-4 + \gamma^2)(-1 + \gamma^2)^2}$$

$$(A2)$$

If Firm F undertakes greenfield FDI, the objective functions for firms F and H are $\pi_F^G = (P_F - \lambda w)q_F - G'$ and $\pi_H^G = (P_H - w)q_H$, respectively. The resulting equilibrium prices and profits are:

$$P_F^G = \frac{-2-\gamma + 2\lambda w + \lambda w + \gamma^2}{-4-\gamma^2} , \text{ and}$$

$$\pi_F^G = \frac{(2 + (-1 + w - \gamma)\gamma + \lambda(2 + \gamma)^2}{(-4 + \gamma^2)^2(-1 + \gamma^2)^2} - G'$$

$$(A3)$$

$$P_H^G = \frac{-2-\gamma + 2\lambda w + \lambda w + \gamma^2}{-4-\gamma^2} , \text{ and}$$

$$\pi_H^G = \frac{(-2 + \gamma + \gamma^2 - w(-2 + \gamma(\lambda + \gamma)))^2}{(-4 + \gamma^2)^2(-1 + \gamma^2)^2}$$

$$(A4)$$
Firm F undertakes greenfield FDI if and only if \( \pi_F^G > \pi_F^X \) or:

\[
- \frac{(2+(-1+\gamma+\lambda+\gamma+\lambda+\gamma))\gamma^2}{(-1+\gamma)^2} - G' > - \frac{(2+(-1+\gamma+\lambda+\gamma+\lambda+\gamma))\gamma^2}{(-1+\gamma)^2} 
\]

which can be shown as follows:

\[
G' < \left( \frac{(t(-2+\gamma^2)(-2+(-1+\gamma+\lambda+\gamma+\lambda+\gamma))\gamma^2}{(-1+\gamma)^2} \right) \equiv G_2 \quad (A5)
\]

Export vs. cross border merger

From equation (A5) we know that Firm F prefers greenfield FDI if and only if \( G' < G_2 \), otherwise it prefers to export.

Let’s assume that \( G' > G_2 \), in other words, export is Firm F’s preferred choice as an alternative to merger. We know from (A2) and (A3) that if Firm F exports, the equilibrium profits of Firms F and H are:

\[
\pi_F^X = \frac{(2+(-1+\gamma+\lambda+\gamma+\lambda+\gamma))\gamma^2}{(-1+\gamma)^2} , \quad \text{and} \quad \pi_H^X = \frac{(2+(-1+\gamma+\lambda+\gamma+\lambda+\gamma))\gamma^2}{(-1+\gamma)^2}
\]

On the other hand, if firm F and H merge, the merged firm maximize the following expression:

\[
\pi_{F+H}^M = (P_F - \lambda w)q_F + (P_H - \lambda w)q_H - K'
\]

The merged firm produces both the products, and the equilibrium prices and the profit of merged firm are:

\[
P_F^M = P_H^M = \frac{(1+\lambda w)}{2},
\]

and

\[
\pi_{F+H}^M = \frac{(-1+\lambda w)^2}{2(1+\gamma)} - K' \quad (A6)
\]

Cross-border merger is profitable compared to export by Firm F if:
\[
\frac{\pi_{M}}{\pi_{F+H}} > \pi_{F} + \pi_{H}
\]

where, \(\pi_{M}\) denotes the profit of the merged firm, while \(\pi_{F}\) and \(\pi_{H}\) denote the profit of Firms F and H under export by Firm F. Following (A1), (A2) and (A6), we get that cross-border merger if preferable to export by Firm F if and only if:

\[
\frac{(-1+\lambda w)^2}{2(1+\gamma)} - K' > \left( \frac{-2((1+t+\lambda w)+\gamma-w\gamma-(1+t+\lambda w)\gamma^2)^2}{(-4+\gamma^2)^2(1+\gamma^2)} + \frac{(-2+(1+t-\gamma)\gamma+w(-2+\gamma(1+\gamma)))^2}{(-4+\gamma^2)^2(1+\gamma^2)} \right)
\]

or

\[
K' < \frac{(-1+\lambda w)^2}{2(1+\gamma)} - \left( \frac{-2((1+t+\lambda w)+\gamma-w\gamma-(1+t+\lambda w)\gamma^2)^2}{(-4+\gamma^2)^2(1+\gamma^2)} + \frac{(-2+(1+t-\gamma)\gamma+w(-2+\gamma(1+\gamma)))^2}{(-4+\gamma^2)^2(1+\gamma^2)} \right) \equiv K_3
\]

**Greenfield FDI vs. cross border merger**

Now consider the case of \(G' < G_2\) i.e. greenfield FDI is Firm F’s preferred choice as an alternative to merger. We know from (A3) and (A4) that when Firm F undertakes greenfield FDI, the profits of firm F and H are:

\[
\pi^G_F = \frac{(2+(1+w-\gamma)\gamma+\lambda(-2+\gamma^2))}{(-4+\gamma^2)^2(1+\gamma^2)} - G', \text{ and } \pi^G_H = \frac{(-2+\gamma+\gamma^2-w(-2+\gamma(1+\gamma)))^2}{(-4+\gamma^2)^2(1+\gamma^2)}
\]

However, if Firms F and H merge, we know from (A6) that:

\[
\frac{\pi}{\pi_{F+H}} = \frac{(-1+\lambda w)^2}{2(1+\gamma)} - K''
\]

A cross-border merger between Firms F and H is profitable compared to greenfield FDI by Firm F if:
where, $\pi_{F+H}^M$ denotes the profit of the merged firm, while $\pi_F^G$ and $\pi_H^G$ denote the profits of Firms F and H under greenfield FDI by Firm F. Following (A3), (A4) and (A6), we get that cross-border merger is profitable than greenfield FDI by Firm F if and only if:

$$\frac{(-1+\lambda w)^2}{2(1+\kappa)} - M'' > \frac{(-2+(-1+w-\lambda)(-2+\gamma^2))^2}{(-4+2\gamma^2)(-1+\gamma^2)} - G' + \frac{(-2+\gamma^2-\lambda(-2+\gamma(\lambda+\gamma)))^2}{(-4+2\gamma^2)(-1+\gamma^2)}$$

or

$$K'' < \frac{(-1+\lambda w)^2}{2(1+\kappa)} - \frac{(-2+(-1+w-\lambda)(-2+\gamma^2))^2}{(-4+2\gamma^2)(-1+\gamma^2)} - G' + \frac{(-2+\gamma^2-\lambda(-2+\gamma(\lambda+\gamma)))^2}{(-4+2\gamma^2)(-1+\gamma^2)} \equiv K_4$$

Section B

N firms in country H

To show our results in the simplest way, we assumed in section 2 that there is only one firm in the home country. We show in this section that our results in section 2 hold true even if there are n symmetric firms in the home country, where n>2. If there are n home-country firms, the resulting inverse demand functions are:

$$p_F = 1 - q_F - \sum_{i=1}^{n} q_i$$

$$p_i = 1 - \sum_{i=1}^{n} q_i - q_F$$

where $p_F$, and $p_i$ are prices, $q_F$, $q_i$ are outputs.
Equilibrium Analysis

Export vs. greenfield FDI

First, consider the foreign firm’s choice between export and greenfield FDI. If Firm F exports, it determines output by maximizing the following expression:

$$\max_{q_F} (1 - q_F - \sum_{i=1}^{n} q_i - \lambda w - t)q_F$$

while profit maximization problem of ith firm in home country is:

$$\max_{q_i} (1 - \sum_{i=1}^{n} q_i - q_F - w)q_i$$

The resulting equilibrium outputs, and profits are;

$$q_F^X = (1-\lambda w(n+1)-(n+1)+wn) \quad \text{and}$$

$$\pi_F^X = \frac{(1-\lambda w(n+1)-(n+1)+wn)^2}{(n+2)^2}$$  \hspace{1cm} (A.7)

and output and profit of ith home-country firm are:

$$q_i^X = \frac{(1-2w-\lambda w+i)}{(2+n)} \quad \text{and}$$

$$\pi_i^X = \frac{(1-2w-\lambda w+i)^2}{(2+n)^2}$$  \hspace{1cm} (A.8)

If firm F undertakes greenfield FDI, it maximizes the following profit function:

$$\max_{q_F} (1 - q_F - \sum_{i=1}^{n} q_i - \lambda w)q_F - G$$

while profit maximization problem of the ith home-country firm is:
\[ \max_{q_i} \left( 1 - \sum_{i=1}^{n} q_i - q_F - w \right) q_i \]

The resulting equilibrium outputs and profits are;

\[ q_F^G = \frac{(1-\lambda w(n+1)+wn)}{(n+2)} \text{, and} \]
\[ \pi_F^G = \frac{(1-\lambda w(n+1)+wn)^2}{(n+2)^2} - G \]

(A.9)

and output and profit of ith firm in home country are;

\[ q_i^G = \frac{(1-2w+\lambda w)}{(n+2)} \text{, and} \]
\[ \pi_i^G = \frac{(1-2w+\lambda w)^2}{(n+2)^2} \]

(A.10)

So, Firm F prefers greenfield FDI compared to export if \( \pi_F^G > \pi_F^X \)

\[ \frac{(1-\lambda w(n+1)+wn)^2}{(n+2)^2} - G'' > \frac{(1-\lambda w(n+1) - t(n+1) + wn)^2}{(n+2)^2} \]

which can be shown as follows;

\[ G'' < \left( -\frac{(1+n)(-2+2n+2\lambda n+\lambda w)}{(n+2)^2} \right) \equiv G_3 \]

(A.11)

**Export vs. cross border merger**

From (A.11) we know that Firm F prefers greenfield FDI if and only if \( G'' < G_3 \), otherwise prefers to export.

Let’s assume that \( G'' > G_3 \), export is Firm F’s available strategy as an alternative to merger. We know from (A7) and (A8) that when Firm F chooses to export, the profits of Firm F and the ith home-country firm are:
However, if Firm F and the ith home-country firm merge, the merged firm determines output to maximize the following profit function:

$$\max_{q_M} (1 - q_M - \sum_{j=1}^{n-1} q_j - \lambda w)q_M - M''$$

and each of the remaining home-country firm, say, firm j, determines output to maximize the following expression:

$$\max_{q_i} (1 - \sum_{j=1}^{n-1} q_j - q_M - w)q_i$$

If Firms F and the ith home-country firm merge, the equilibrium output and the profit of merged firm are:

$$q^*_M = \frac{1 - \lambda w + w(n-1)}{(n+1)}$$, and

$$\pi^*_M = \frac{(1 - \lambda w + w(n-1))^2}{(n+1)^2} - M''$$ (A.12)

A merger between Firms F and the ith home-country firm is preferable compared to export by Firm F if:

$$\pi^*_M > \pi^*_F + \pi^*_i$$

Following (A.7), (A.8) and (A.12) we get that merger occurs if and only if:
In this sub-section, we consider that \( G'' < G_3 \), i.e. greenfield FDI is Firm F’s available strategy as an alternative to merger.

We know from equations (A9) and (A10) that if Firm F undertakes greenfield FDI, the profits of firms Firm F and the \( i \)th home-country firm are:

\[
\pi_F^G = \frac{(1-\lambda_w(n+1)+wn)^2}{(n+2)^2} - G \quad \text{and} \quad \pi_i^G = \frac{(1-2\lambda_w+\lambda w)^2}{(n+2)^2}
\]

If Firm F and the \( i \)th firm merge, we know from (A3.12) that the profit of the merged firm is:

\[
\pi_M = \frac{(1-\lambda_w(n+1)+wn)^2}{(n+1)^2} - K''
\]

A merger between Firm F and the \( i \)th home-country firm is profitable compared to greenfield FDI by Firm F if:

\[
\pi_M^G > \pi_F^G + \pi_i^G
\]

or

\[
K'' < \frac{(1-\lambda w)^2}{2(1+\gamma)} - \left( \frac{(2-\gamma-w(2\lambda-\gamma))^2}{(4-\gamma^2)} + \frac{(2-\gamma-w(2-\lambda\gamma))^2}{(4-\gamma^2)} - G'' \right) \equiv K_6 \quad (A.14)
\]
Section C.1

Proof of $\bar{w} - w' > 0$

$$\bar{w} - w' = \left( (\gamma - 2) \left( \frac{1}{\lambda \gamma - 2} + \frac{4 + 2 \gamma - \gamma^3}{8 \gamma (1 + \gamma) + \lambda (8 + \gamma (-10 + (\gamma - 2) \gamma))} \right) \right)$$

$\bar{w} - w' > 0$ if $\lambda \epsilon [0, 1]$ and $\gamma \epsilon [0.732, 1]$ (see Figure 4.1 below)

![Figure 4.1: $(\bar{w} - w') > 0$](image)

Section C.2:

Proof of Proposition 5
\[
\frac{\partial (\hat{w} - w^*)}{\partial \gamma} = \left( \gamma - 2 \right) \left( \frac{\left( \frac{1}{\lambda \gamma - 2} + \frac{4+2\gamma^2}{8\gamma(1+\gamma) + \lambda(8+\gamma(-8+\gamma(-10+(\gamma-2)\gamma))})}{(1+\gamma)^2} + \frac{2(1+2\gamma)(-4-2\gamma+\gamma^2)(4+\lambda(-4-2\gamma+\gamma^2))}{(8\gamma(1+\gamma)+\lambda(8-8\gamma-10\gamma^2-2\gamma^3+\gamma^4))} \right) \\
+ \left( \gamma - 2 \right) \left( \frac{2-3\gamma^2}{8\gamma(1+\gamma)+\lambda(8+\gamma(-8+\gamma(-10+(\gamma-2)\gamma)))} \right)
\]

Plotting the above expression for \( \lambda \in (0, 1) \) and \( \gamma \in (0, 1) \) completes the proof.

![Figure 4.2: \( \frac{d(\hat{w} - w^*)}{dy} > 0 \) for \( \lambda \in (0, 1) \) and \( \gamma \in (0, 1) \)](image-url)
References


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