Trade and Domestic Production Networks[*]

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November 26, 2017

Abstract

We use administrative data from Belgium with information on domestic firm-to-firm sales and foreign trade transactions to study how international trade affects firm efficiency and real wages. The data allow us to construct the buyer-supplier network of the Belgian economy. We document that most firms that do not directly import or export still have large indirect exposure to foreign trade, and that a firm's output is affected by idiosyncratic shocks to its buyers and suppliers. These empirical findings motivate and guide the development of a model with domestic production networks and international trade. We obtain new sufficient statistics results for the effects of trade in a model with fixed network structure, and we develop a tractable model of endogenous domestic production networks. Comparing our results to those we obtain using existing approaches highlights the importance of data on and modeling of domestic production networks in studies of international trade.

1 Introduction

Over the past few decades, the focus of research on international trade has shifted from countries and industries towards firms. This shift is in no small part due to the increased availability of firm-level transaction data on trade. One important insight from this data is that few firms directly import or export goods (Bernard, Jensen, Redding, and Schott, 2007). However, the concentration of imports and exports does not necessarily imply that few firms benefit from foreign trade. Even if firms themselves do not import or export, they may still buy from or sell to domestic firms that trade internationally. Capturing this channel, however, is challenging since domestic firm-to-firm transactions are rarely observed. In the absence of such data, quantification of the effects of foreign trade on all firms requires strong assumptions, such as a common intermediate good (Eaton and Kortum, 2002, and Blaum, Lelarge, and Peters, 2016) or the same import shares across importing firms within broad industries (Caliendo and Parro, 2015, and Costinot and Rodríguez-Clare, 2014).

The goal of this paper is to combine data on domestic firm-to-firm sales with information on foreign trade transactions to study how international trade affects real wages and efficiency of all firms, including those that do not directly export or import. Our analysis employs a panel dataset with detailed information on Belgian firms for the years 2002-2014. This dataset is based on several data sources that we have linked through identifiers. Annual accounts provide data on input factors and output, custom records and intra-EU declarations give information on exports and imports, and a value-added tax (VAT) registry provides information on domestic firm-to-firm transactions. Using this data, we empirically examine several new dimensions of firms in international trade before developing and estimating a model of trade and domestic production networks.

In Section 2, we describe the data, construct the domestic production (buyer-supplier) network of the Belgian economy, and provide two new empirical findings. The first is that most firms are exposed to foreign trade through their production network. While only 15% of firms import directly, 97% of firms obtain foreign inputs either directly or indirectly through domestic suppliers which use foreign inputs in their production process. Indeed, most firms are heavily dependent on foreign inputs, but only a small number of firms show that dependence through the direct imports observed in firm-level transaction data on trade. For example, in a majority of firms, at least 40% of input costs are spent on goods that are imported directly or indirectly.

The second empirical finding is that foreign trade shocks seem to propagate across firms within production networks. Following Hummels, Jørgensen, Munch, and Xiang (2014), we measure trade shocks as changes in world export supply and world import demand of country-product combinations in which the firm had a previous trade relationship. They argue that these shocks are plausibly exogenous and that their impact varies markedly across firms,

because the firms — even within the same sector — do not have all inputs in common. Using our data, we find that positive export shocks to the firm's buyers and positive import shocks to the firm's suppliers both tend to increase the firm's output, even after controlling for direct shocks to the firm itself and for shocks to the set of potential buyers and suppliers of the firm.

Taken together, these two empirical findings highlight that information about domestic firm-to-firm transactions is key to understand the extent to which firms rely on foreign input and to analyze the propagation of trade shocks. Motivated and guided by this evidence, we develop and estimate a model of domestic production networks and international trade. In our model, firms combine imports, inputs produced by other domestic firms, and labor to produce differentiated products with a constant elasticity of substitution production function. Firms are finite and monopolistically competitive.

In Section 3, we assume a fixed network structure (i.e., the buyer-supplier relationships do not change in response to trade shocks) and quantify how international trade affects firms' production costs and consumer prices. The cost reduction for an individual firm due to international trade depends on two quantities only: the share of input costs that is spent on goods that are imported directly or indirectly and the elasticity of substitution in the production function. We apply this sufficient statistics formula to our data, and find that international trade is important in reducing the cost of production for plausible values of the elasticity of substitution. For example, with an elasticity of substitution in the production function of 2, we calculate that shutting down international trade would increase the cost of the majority of Belgian firms by at least 70%. To compute the welfare gains from trade, we combine information on firms' sales to domestic households with an assumption about the elasticity of substitution in the utility function. Our baseline results imply that the consumer price index in Belgium would be 77% higher in the absence of international trade.

While assuming a fixed network structure is convenient to take the model to the data, it does not allow us to capture how buyer-supplier relationships may change in response to trade shocks. In Section 4, we therefore develop a model of trade with endogenous network formation. In particular, we let firms optimally choose their set of suppliers (i.e. the firm's sourcing strategy) subject to a buyer-supplier-specific fixed cost for adding a supplier. Allowing for endogenous network formation is challenging for two reasons. First, firms face a large discrete choice problem of which suppliers to include in their sourcing strategy. Second, firms' sourcing strategies are interdependent, creating a large fixed point problem: firms take into account the expected sourcing strategies of others in order to determine their own optimal sourcing strategy, all the while knowing that other firms are thinking in the same way.

Building on Jia (2008) and Antras, Fort, and Tintelnot (2017), we overcome the first

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challenge by using lattice theory to solve firms' large combinatorial discrete choice problems. To address the second challenge we consider the formation of an acyclic network, postulating an ordering of firms and restricting the eligible set of suppliers to firms that appear prior to the buyer.[1] While restrictive, this assumption allows us also to solve a model of firm trade with endogenous formation of domestic buyer-supplier relationships. We use method of simulated moments to estimate the model, and then perform counterfactuals to draw inference about the impact of shutting down international trade with and without endogenous network formation. Our findings suggest that allowing for endogenous formation of buyer-supplier relationships tend to attenuate the effects of banning trade on firms' cost. Under endogenous network formation, we find a price index increase around 15% lower than under a fixed network.

Our paper contributes to a growing literature on the economy-wide effects of *foreign sourcing*.[2] Many studies use aggregate data only, relying on the assumption that firms import intensities are equalized — which is at odds with the data. Using firm-level data on trade transactions, Blaum et al. (2016) show that accounting for heterogeneity in import exposure significantly affects the measurement of the gains from international trade. Their model assumes that firms can import directly and purchase a common intermediate good. Taking advantage of data on domestic firm-to-firm transactions, we relax the assumption of a common intermediate good, and derive a parsimonious sufficient statistics formula for a model with a fixed production network. We also go beyond the fixed network structure, solving a model of endogenous network formation with a finite number of firms and fixed costs for adding suppliers. This contribution builds on the global sourcing model of Antras et al. (2017). While they distinguish between final good and intermediate good sectors, we consider a more general input-output structure between firms. In addition, our model captures not only the firms' decisions with respect to foreign sourcing but also their choices of domestic sourcing strategies.[3]

Our paper also relates to a literature on the formation and consequences of *domestic production networks*.[4] Bernard, Moxnes, and Saito (2016b) adapt the model of Antras et al. (2017) to search for domestic suppliers in different locations, where each location has a continuum of intermediate-good-producing firms. They find significant improvements in firm

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[1]See Spiegler (2016) for a recent contribution in economics studying belief formation in a directed acyclic network.

[2]See for example Antràs and Helpman (2004), Antràs and Helpman (2008), Rodríguez-Clare (2010), Garetto (2013), Halpern, Koren, and Szeidl (2015), Gopinath and Neiman (2014), Amiti and Konings (2007), Goldberg, Khandelwal, Pavcnik, and Topalova (2010), and De Loecker, Goldberg, Khandelwal, and Pavcnik (2016).

[3]Our work is also related to the analysis in Caliendo and Parro (2015) and Ossa (2015). They find the gains from trade to be larger when taking sectoral input-output linkages into account.

[4]A growing body of work studies how firms meet international trading partners. See for example Chaney (2014), Chaney (2016), Morales, Sheu, and Zahler (2015), and Eaton, Kinkins, Tybout, and Xu (2016).

performance from a reduction in internal search costs in Japan. Furusawa, Inui, Ito, and Tang (2017) develop a variant of the global sourcing model of Antras et al. (2017), and use Japanese buyer-supplier link data to test the model's predictions. Oberfield (2017) develops a theory in which the network structure of production forms endogenously among firms that each purchase a single input. Lim (2015) develops a dynamic model of network formation in which each firm has a continuum of domestic suppliers. With a continuum of suppliers and buyers, the sales from one firm to another are negligibly small and a link between two particular firms has no effect on aggregate outcomes. In contrast to these papers, we develop a model of endogenous network formation with a finite set of suppliers, and incorporate both firm exporting and importing decisions. While our theory assumes simple solutions for the pricing game between firms, Kikkawa, Magerman, and Dhyne (2017) explore the segmentation of markets for different buyers, with supplier firms having heterogeneous bargaining power in the supplier-buyer relationships.

Finally, our paper relates to a literature that analyzes how production networks matter for *aggregate effects and transmission of idiosyncratic firm shocks*. Gabaix (2011) provides conditions under which granular shocks can affect aggregate fluctuations. Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012) study the transmission of shocks along sectoral input-output networks. Magerman, De Bruyne, Dhyne, and Van Hove (2016) test both channels with the Belgium domestic firm-to-firm data. Barrot and Sauvagnat (2016), Boehm, Flaaen, and Pandalai-Nayar (2015), and Carvalho, Nirei, Saito, and Tahbaz-Salehi (2016) use natural disasters to study the propagation of shocks in production networks. Carvalho and Voigtländer (2015) analyze the adoption of inputs by innovators and the evolution of the domestic production network. Hulten (1978) provides conditions under which the underlying network structure is irrelevant for quantifying the propagation of shocks — up to a first-order approximation — as long as firms' initial size and the magnitudes of the idiosyncratic shocks are observed.[5] In recent work, Baqaee and Farhi (2017) illustrate that the second-order effects of shock propagation arising from networks can be large.[6] Our paper extends the analysis of shock propagation to foreign trade shocks, while allowing buyer-supplier relationships to change in response to these shocks.

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[5]As shown by Kikkawa et al. (2017), to apply the Hulten theorem to a small open economy setting one would need very strong conditions that are immediately violated in our data: in particular, all firms would need have a constant ratio of export sales to sales to domestic final consumers.

[6]Other recent contributions to determining the effects of networks include Baqaee (2014), Carvalho and Grassi (2017), as well as in the context of financial frictions, Bigio and La'o (2016) and Liu (2016). Atalay, Hortacsu, Roberts, and Syverson (2011) characterize the buyer-supplier network of the US economy.

2 Trade and production networks: Data and evidence

This section describes the data, documents firms' direct and indirect exposure to foreign trade, and shows how the output of a firm is affected by trade shocks to its buyers and suppliers.

2.1 Data sources and sample selection

Our analysis draws on three administrative data sources from Belgium, accessible only at the National Bank of Belgium, for the years 2002-2014. These data sources can be linked through unique identifiers, assigned and recorded by the government for the purpose of collecting value-added taxes (VAT). Below we briefly describe our data and sample selection, while additional details are given in Appendix C.

The first data source is the Business-to-Business (B2B) transactions database (see also Dhyne, Magerman, and Rubinova (2015)). By law, all Belgian firms are required to file the annual sales to each buyer (provided the annual sales to a given buyer exceeds €250). Thus, the B2B dataset allows us to measure accurately the identity of the firms' suppliers and buyers. The second data source is the annual accounts filed by Belgian firms. These data contain detailed information from the firms balance sheets on output (such as revenues) and inputs (such as capital, labor, intermediates) as well as 4-digit (NACE) industry codes and geographical identifiers at the zip code level. In addition, the annual accounts include information about ownership shares in other enterprises. The third set of data source is the Belgian customs records and the intra-EU trade declarations. These data contain information about international trade transactions in each year and for every firm. Both imports and exports are disaggregated by product and origin or destination.

One challenge with using the Belgian data is that the information is recorded at the level of the VAT-identifier. The problem is that a given firm may have several VAT-identifiers (for accounting or tax reasons).[7] While organizational choices and transactions across units within a firm are of interest, our paper is centered on trade between firms. Thus, if a firm has multiple VAT-identifiers, we aggregate all data up to the firm level using information from the balance sheets about ownership structure. Details of the aggregation are outlined in Appendix C.1. In 2012, for example, the aggregation converts 896,000 unique VAT-identifiers into 860,000 unique firms. Of these firms, 842,000 had a single VAT-identifier. However, the 18,000 firms with multiple VAT-identifiers are important, accounting for around 60% of the total output in the dataset.

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[7]Existing papers tend to abstract from this issue, analyzing the data at the level of the VAT-identifier. See e.g. Amiti, Itskhoki, and Konings (2014), Magerman et al. (2016) and Bernard, Blanchard, Van Beveren, and Vandenbussche (2016a)).

After constructing a firm-level dataset, we impose a few sample restrictions. We exclude firms in the government or financial sector. In addition, we restrict the sample to firms with positive labor costs and employment, tangible assets of more than €100, and positive total assets in at least one year during our sample period. These criteria are similar to the ones used by De Loecker, Fuss, and Van Biesebroeck (2014). Applying these criteria reduces the number of firms significantly. In, 2012, for example, only 139,605 firms satisfy the above criteria. The large reduction in sample size is mostly driven by the exclusion of local firms without employees (self-employment) from the sample (687,700 firms in 2012). Lastly, we drop foreign firms with no local production activity in Belgium from the sample. These account for a sizable fraction of imports and exports, but have no domestic production activity in Belgium.

Table 1 illustrates that our selected estimation sample of firms provides relatively good coverage of aggregate value added, gross output, exports and imports. However, total sales in our sample is larger than what are reported in the national statistics. The reason is that the output of trade intermediaries in the national statistics is measured by their value added instead of their total sales. We refer to Appendix C.2 for the same statistics for all Belgian firms.

Table 1: Coverage of selected sample

| Year | GDP (Excl. Gov. & Fin.) | Output | Imports | Exports | Selected sample | | | | |
					Count	V.A.	Sales	Imports	Exports
2002	149	411	210	229	122,460	123	586	179	189
2007	192	546	300	314	136,370	157	757	280	269
2012	212	626	342	347	139,605	170	829	296	295

Notes: All numbers except for Count are denominated in billion Euro in current prices. Belgian GDP and output are for all sectors excluding public and financial sector. See Appendix C.2 for the same statistics for the total economy. Data for Belgian GDP, output, imports and exports are from Eurostat.

2.2 Direct and indirect exposure to foreign trade

The Belgian data allow us to construct the buyer-supplier relationships of the Belgian economy, and therefore document firms' direct and indirect exposure to foreign trade.

We define firm j's *total foreign input share* as the sum of firm j's *direct foreign input share*, s_{Fj}, and the direct foreign input share of firm j's suppliers, suppliers' suppliers, and so forth, each weighted by the firm-pair-specific input shares ($s_{ij}, s_{ki}, ...$):

$$s_{Fj}^{Total} = s_{Fj} + \sum_{i \in Z_j^D} s_{ij} \underbrace{\left[s_{Fi} + \sum_{k \in Z_i^D} s_{ki} \left(s_{Fk} + \cdots \right) \right]}_{s_{Fi}^{Total}}, \tag{1}$$

where Z_j^D denotes the set of domestic suppliers of firm j, and the denominator of the input shares is the sum of labor costs, purchases from other firms, and imports. Note that the definition of the total foreign input share is recursive: a firm's total foreign input share is sum of its direct foreign input share and the share of its inputs from other firms multiplied by those firms' total foreign input shares. While many firm-level datasets contain information about the direct foreign input share s_{Fj}, our data also offer information about firm-pair-specific input shares, s_{ij}. As a result, we are able to calculate the total foreign input share for every firm. We note that there is one inherent assumption in our definition of the total foreign input share: When a firm sells its output to multiple firms or final consumers, the foreign input share in the costs of producing these goods is assumed to be the same for all buyers (i.e., independent of the identity of the buyer). This assumption is consistent with the model we develop in Sections 3 and 4, where each firm produces a single product.

In Figure 1a, we display a histogram of the total and direct foreign input shares of the Belgian firms. While only 15% of firms import directly, 97% of firms obtain foreign inputs either directly or indirectly through domestic suppliers which use foreign inputs in their production process. Indeed, most firms are heavily dependent on foreign inputs, but only a small number of firms show that dependence through the direct foreign input shares observed in firm-level transaction data on trade. In the median firm, for example, the total foreign input share is 41%. By comparison, the total foreign input shares are 21% and 60% at at the 20th and 80th percentile. We present direct and total foreign input shares by sector in Appendix D.1. Even in the service sector, in which firms have a very low share of direct foreign inputs, the median firm's total foreign input share is as large as 28%.

Figure 1b performs a similar exercise, but now looks at *total export shares* and *direct export shares*. We calculate the total export share of firm j, r_{jF}^{Total}, as the sum of the share of revenue of firm j coming from directly export goods, r_{jF}, and the share of revenue coming from goods sold to other domestic firms, multiplied by those firms' total export shares:

$$r_{jF}^{Total} = r_{jF} + \sum_{i \in W_j} r_{ji} r_{iF}^{Total}, \tag{2}$$

where W_j denotes the set of domestic buyers of firm j, and the denominator of the export shares is the total revenue of the firm. Direct export is even more concentrated than direct import, both on the intensive and extensive margin. While only 10% of firms export directly,
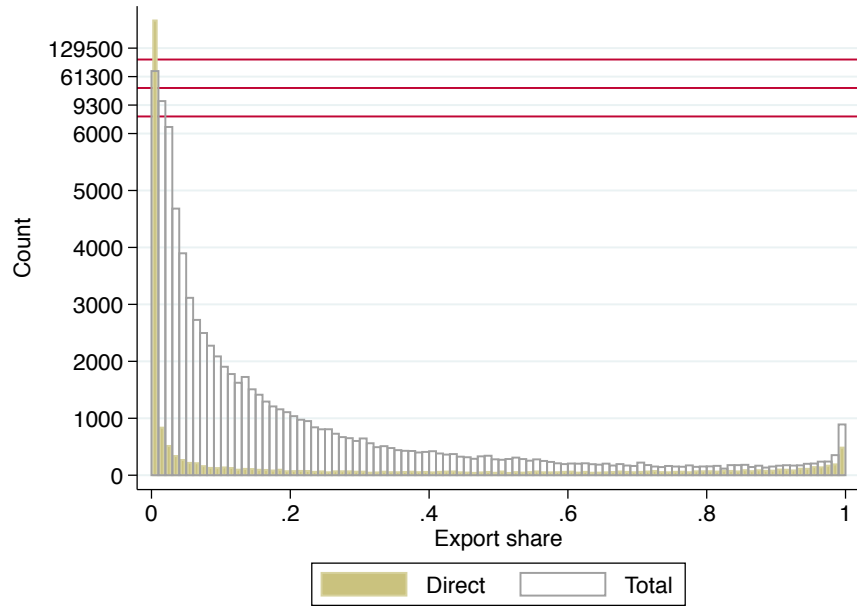
Figure 1: Histogram of direct and indirect linkages to foreign trade

(a) Direct and total foreign input share
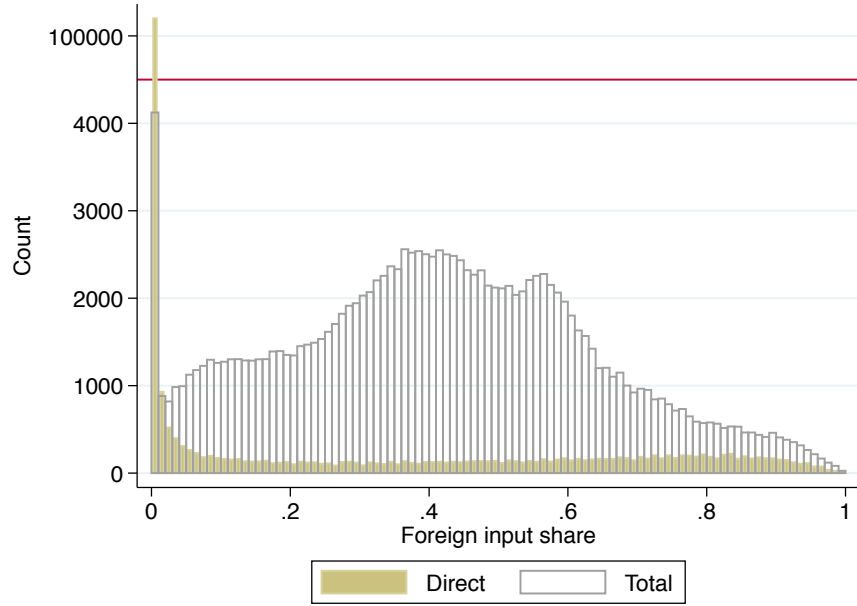

(b) Direct and total export share

Notes: Total foreign input share of firm i, s_{Fi}^{Total} is calculated by solving $s_{Fi}^{Total} = s_{Fi} + \sum_{j \in Z_i} s_{ji} s_{Fj}^{Total}$ where s_{Fi} is i's direct foreign input share, and s_{ji} is j's share among i's inputs. Total export share firm i, r_{iF}^{Total} is calculated by solving $r_{iF}^{Total} = r_{iF} + \sum_{j \in W_i} r_{ij} r_{jF}^{Total}$ where r_{iF} is i's share of exports in its revenue, and r_{ij} is share of i's revenue that arises from sales to firm j. The figures are based on the analysis of 139,605 private sector firms in Belgium in 2012. The horizontal lines represent scale breaks on the vertical axis.

Figure 2: Size premium of direct and indirect linkages to foreign trade

(a) Foreign inputs
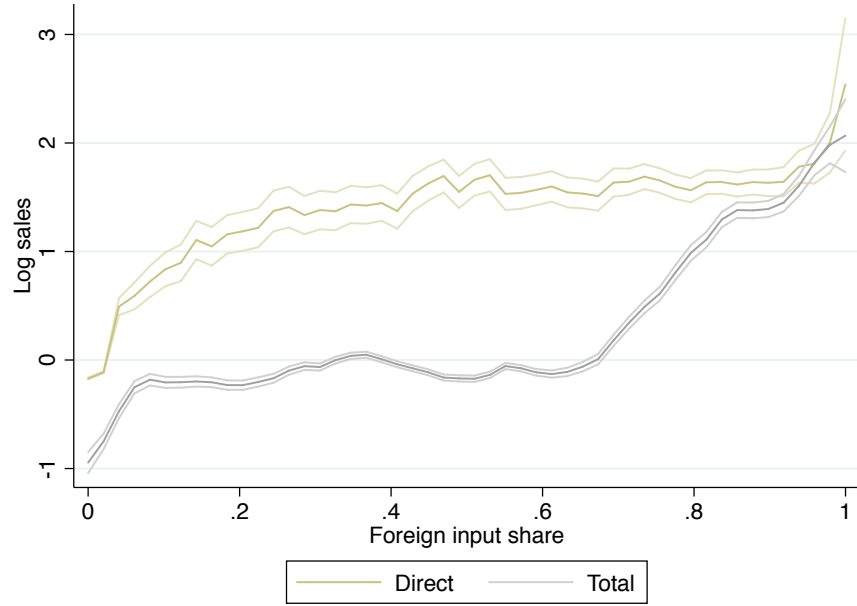
(b) Exports
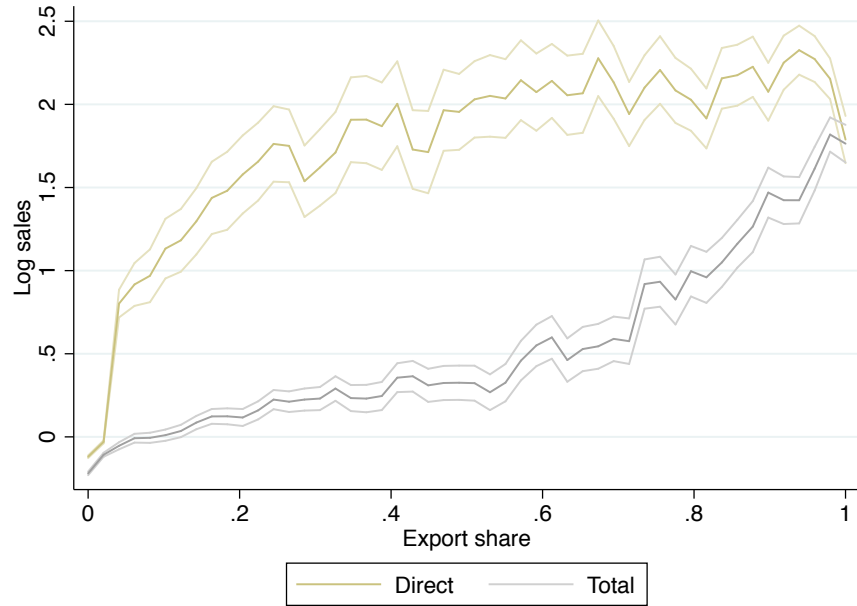
Notes: The two figures display the smoothed values with 95% confidence intervals of kernel-weighted local polynomial regression estimates of the relationship between firms' sales and their levels of participation in foreign trade. We use the Epanechnikov kernel function with kernel bandwidth of 0.01, pilot bandwidth of 0.02, degree of polynomial smooth at 0, and smooth obtained at 50 points. Log sales are demeaned with 4-digit industry fixed effects.

82% of firms export either directly or indirectly by selling to domestic buyers which subsequently trade internationally. In terms of trade volume, however, export remains relatively concentrated even after taking the indirect export into account. The total export share is only 2% in the median firm, whereas it is 19% at the 80th percentile. In contrast, Figure 1a showed that most firms are heavily dependent on foreign inputs. This difference is partly driven by the service sector. While many firms in this sector (e.g., restaurants) rely on foreign inputs — often obtained indirectly through domestic suppliers — relatively few export directly or sell to domestic firms that are exporting directly or indirectly (see Appendix D.2 for direct and total foreign input shares by sector).[8]

Across a wide range of countries and industries, firms that directly export or import have been shown to be larger than other firms. A natural question is whether the positive association between firm size and international trade also carries over to indirect export or import. Figure 2 investigates this, calculating the average size of firms by direct and total foreign input shares as well as by direct and total export shares. We demean the log of firm sales using the firm's four-digit industry average, so that a firm with log sales of zero is the size of an average firm in its industry. Figure 2 illustrates that average firm size is increasing in both direct and total foreign input shares. However, firms that import directly tend to be much larger than firms that buy foreign inputs through domestic firms. Indeed, firms with less than 60% in total foreign input shares are, on average, of similar size as the average firm in their industry. A similar pattern is evident for size and export. Firms with very high total export shares tend to be large. However, over most of the total export share distribution, there is only a weak relationship between firm size and total export share. Taken together, the results in Figure 2 suggest that firms do not have to be large to rely heavily on foreign inputs or to have most of their sales going ultimately to a foreign country.

2.3   Trade shocks and the production network

The analysis in Section 2.2 showed that most firms are exposed to foreign trade through their production network. This finding raises the questions of whether, and to what extent, trade shocks propagate across firms within production networks. To investigate these questions, we build on the work by Hummels et al. (2014), who construct measures of trade shocks from changes in world export supply and world import demand of country-product combinations in which the firm had a previous trade relationship. They argue these shocks are plausibly exogenous, and show that their impact varies markedly across firms, because the firms do not have all inputs in common. Hummels et al. (2014) use the trade shocks to estimate wage

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[8]Another possible explanation is that it is difficult to measure all forms of export in the service sector. For example, when a foreigner eats at a Belgian restaurant, it is technically an export transaction. However, such transactions are not recorded in our data.

effects of offshoring and exporting in Denmark. We apply the same identification strategy to the Belgian setting with the goal of examining whether trade shocks to the firm's actual buyers or suppliers have stronger impact on its output than trade shock to potential suppliers and customers.

To make the identification strategy precise, consider the following regression model in first-differences:

$$
\begin{aligned}
\Delta \log Y_{it} =& \beta_X^C \Delta \log X_{it}^C + \beta_M^S \Delta \log M_{it}^S \\
& + \beta_X^{PC} \Delta \log X_{it}^{PC} + \beta_M^{PS} \Delta \log M_{it}^{PS} \\
& + \beta_X \Delta \log X_{it} + \beta_M \Delta \log M_{it} + \varphi_t + \epsilon_{it}.
\end{aligned} \tag{3}
$$

where Y_{it} denotes the total sales of firm i in year t, and Δ denotes the change in the variable from year $t-1$ to t. In addition to calender time fixed effects φ_t, we include three sets of explanatory variables. The first is the measures of import shocks to firm i's suppliers $\Delta \log M_{it}^S$ and export shocks to firm i's' buyers $\Delta \log X_{it}^C$. Our goal is to consistently estimate the coefficients on these variables, β_X^C and β_M^S. However, there are several threats to identification. One is that $\Delta \log M_{it}^S$ and $\Delta \log X_{it}^C$ are likely to correlate with trade shocks to i's potential suppliers and buyers. We therefore include measures of import shocks to firm i's potential suppliers, $\Delta \log M_{it}^{PS}$, and export shocks to firm i's' potential buyers, $\Delta \log X_{it}^{PC}$. Another concern is that $\Delta \log M_{it}^S$ and $\Delta \log X_{it}^C$ could be correlated with trade shocks that affect the firm i directly (through it's direct import demand or supply). To address this concern, we control for export shocks, $\Delta \log X_{it}$, and import shocks, $\Delta \log M_{it}$, to firm i itself.

To take the regression model to the data, we need to construct the various measures of trade shocks. To this end, we follow the shift-share approach in Hummels et al. (2014). To construct an export shock for firm i, $\Delta \log X_{it}$, we use information about the firm's product-country-level exports in year $t-1$ (the share variable capturing firm-specific exposure), and the aggregate shift in world import demand for each country and product:

$$
\Delta \log X_{it} = \log \underbrace{\sum_{k,c} r_{ic,t-1}^{k,X} \mathrm{WID}_{k,c,t}}_{X_{it,t-1}} - \log \underbrace{\sum_{k,c} r_{ic,t-1}^{k,X} \mathrm{WID}_{k,c,t-1}}_{X_{it-1,t-1}}.
$$

The term $r_{ic,t-1}^{k,X}$ is the share of exports of firm i at year $t-1$ that falls on product k sold to country c, and $\mathrm{WID}_{k,c,t}$ is the world import demand (excluding imports from Belgium) of country c for product k.[9] We measure the export shock for firms' buyers in a similar way. For firm i's buyers, we construct the weighted average of their export demand shocks,

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[9]We use NACE 4 digit level to classify products k.

$\Delta \log X_{it}^C$, using i's output share to each buyer in the previous year as the weights:

$$\Delta \log X_{it}^C = \log \sum_j r_{ij,t-1} X_{jt,t-1} - \log \sum_j r_{ij,t-1} X_{jt-1,t-1}.$$

Finally, we measure the trade shocks to the firms' potential buyers, $\Delta \log X_{it}^{PC}$. The potential buyers of firm i include both the buyers of i's goods and other firms in the same (4-digit) sector as the actual buyers. We weight sectors for each firm according to the share of the firm's revenue that is sold to firms' from that sector, $r_{iu,t-1}$.[10] We then construct an export shock for each sector as a weighted aggregate of export shocks to all firms of that sector.[11] We combine these terms to construct an export shock to the potential buyers of firm i:

$$\Delta \log X_{it}^{PC} = \log \sum_u r_{iu,t-1} X_{ut,t-1}^{-i} - \log \sum_u r_{iu,t-1} X_{ut-1,t-1}^{-i}.$$

To construct the import shock variables to the firm itself, its suppliers, and its potential suppliers, we use a similar procedure. These variables use information about changes in aggregate export supply in foreign countries and the past sourcing of firms from these countries. We describe their construction in Appendix E.1.

Table 2 shows the estimation results from the regression model with changes in the firm's total sales as the dependent variable (see Table 14 in Appendix E.2 for results with changes in domestic sales and domestic inputs as dependent variables). The results in column 1 suggest that firms that experience positive trade shocks tend to increase their sales. The estimates in column 2 suggest that trade shocks to potential buyers and suppliers also affects the firm's sales. However, as shown in column 3, shocks to the firms' actual buyers and suppliers matter more for the firm's sales than shocks to the potential buyers and suppliers.[12] In terms

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[10]Let sectors, at the NACE 4-digit level, be denoted with u. $r_{iu,t} = \sum_{j \in W_{it}^u} \frac{\text{Sales}_{ijt}}{\text{TotalSales}_{it}}$, where W_{it}^u denotes the set of customers of i producing sector u goods at time t. We fix all weights at the previous year $t-1$.

[11]For the weights that firm i assigns to each firm within a sector, we use the firms' sales to domestic final demand as corresponding weights. We have experimented with different weights and obtained similar results. These weights vary at the firm i - sector u level, as we exclude firm i's own exports and imports if firm i is producing sector u good.

$$X_{ut,t-1}^{-i} = \sum_{j \in U_{t-1}, j \neq i} \frac{V_{jHt-1}}{\sum_{k \in U_{t-1}, k \neq i} V_{kHt-1}} X_{jt,t-1}$$

$$X_{ut-1,t-1}^{-i} = \sum_{j \in U_{t-1}, j \neq i} \frac{V_{jHt-1}}{\sum_{k \in U_{t-1}, k \neq i} V_{kHt-1}} X_{jt-1,t-1},$$

where U_t is the set of firms producing sector u good at t, and V_{iHt} is firm i's sales to domestic final demand at t.

[12]Note that the actual buyers and suppliers are included in the set of potential buyers and suppliers. Thus, the coefficients β_X^C and β_M^S should be interpreted as the additional effect of a trade shocks to the

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Table 2: Reduced form results

	Δ ln Total Sales	Δ ln Total Sales	Δ ln Total Sales
$\Delta \ln X_{it}$	0.106***	0.103***	0.089***
	(0.008)	(0.008)	(0.009)
$\Delta \ln M_{it}$	0.183***	0.178***	0.156***
	(0.010)	(0.010)	(0.012)
$\Delta \ln X_{it}^{PC}$		0.027***	0.025***
		(0.002)	(0.003)
$\Delta \ln M_{it}^{PS}$		0.040***	0.039***
		(0.004)	(0.005)
$\Delta \ln X_{it}^{C}$			0.122***
			(0.013)
$\Delta \ln M_{it}^{S}$			0.041***
			(0.018)
N	87100	87100	87100

Notes: Standard errors are clustered at the firm level. All variables are in terms of yearly log differences for the period 2002-2012. All specifications include year fixed effects. We truncate outliers of each variables at the top and bottom 1% level.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

of magnitudes, the estimates suggest that an 10 percent exogenous increase in the foreign demand of goods for firm i's buyers leads to a 1.2 percent increase in the sales of firm i. The pass through of shocks to firm i's suppliers is smaller. A 10 percent exogenous increase in the foreign supply of goods to firm i's suppliers leads to a .4 percent increase in the sales of firm i.

Taken together, the results in Table 2 suggest that sectoral input-output tables are not sufficient to analyze the propagation of trade shocks. The output of a firm is significantly affected by idiosyncratic shocks to its buyers and suppliers. This finding is consistent with Carvalho et al. (2016), who show that the disruption caused by a Japanese earthquake in 2011 propagated through upstream and downstream supply chains. Motivated by this evidence, we proceed by developing a model of international trade and domestic production networks.

firm's actual buyers and suppliers as compared to the effect of trade shocks to the firm's potential buyers and suppliers

3 A model of trade with fixed production networks

We now develop a model of trade and domestic production networks, and use it to quantify how international trade affects firms' production costs and consumer prices. While this section assumes a fixed network structure — which is convenient to take the model to the data — we allow, in Section 4, the buyer-supplier relationships to change in response to trade shocks.

3.1 Model

We describe a small open economy called Belgium. Before describing the model, we briefly discuss the notation. Since there exist many bilateral directed flows in our model, we will often have two subscripts. In such cases, the first subscript denotes the origin of the good and the second subscript denotes the destination of the good.

3.1.1 Preferences and Demand

Each consumer supplies one unit of labor inelastically. Consumers are assumed to have identical, homothetic CES preferences over consumption goods:

$$U = \left(\sum_{k \in \Omega} (\beta_{kH} q_{kH})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \tag{4}$$

where Ω denotes the set of available products in the small open economy, k denotes a product, and H denotes domestic final demand from households. Since all consumers have the same, homothethic CES preferences for consumption, we can write the aggregate final consumer demand (in quantities) for product k, given price p_{kH}, as:

$$q_{kH} = \beta_{kH}^{\sigma-1} \frac{p_{kH}^{-\sigma}}{P^{1-\sigma}} E, \tag{5}$$

where E denotes the aggregate expenditure in Belgium and P denotes the domestic consumer price index:

$$P = \left(\sum_{j \in \Omega} \beta_{jH}^{\sigma-1} p_{jH}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}. \tag{6}$$

We assume that final goods are substitutes and therefore $\sigma > 1$.

Demand from abroad for product k takes a similar functional form:

$$q_{kF} = \beta_{kF}^{\sigma-1} \frac{p_{kF}^{-\sigma}}{P_F^{1-\sigma}} E_F, \tag{7}$$

where β_{kF} is a product-k-specific foreign demand shifter, p_{kF} is the price of product k abroad, and P_F and E_F denote the foreign price index and expenditure, respectively .

3.1.2 Production and market structure

Firms produce single products. We will use i, j, k to index firms or products. The products are differentiated across firms. Firms sell the same product to final consumers and to other firms as an intermediate input, though not all firms sell to other firms, and not each pair of firms has a buyer-supplier relationship. Note that we allow Belgian firms to sell directly to foreign consumers, while all foreign products reach Belgian consumers indirectly through the importing of inputs by Belgian firms.[13]

We treat every firm as infinitesimal when selling to final consumers. Hence, when selling to domestic or foreign final consumers, we assume the market structure is monopolistic competition. When selling to other firms, the assumption of infinitesimal size is no longer reasonable, however, since most firms just have a few selected suppliers. We assume that in the Nash bargaining between buyer and supplier, the buyer has the full bargaining power. Given the assumptions on technology described below, this will imply that the supplier sells at marginal cost to the buyer firm. We note that in this section with exogenous networks, our main propositions and quantitative results are unchanged if firms charge positive and possibly heterogeneous mark-ups to customer firms as long as these are fixed. Kikkawa et al. (2017) analyze a network economy with variable firm-to-firm mark-ups. In our paper, the arguably strong assumption of the bargaining power in firm-to-firm transactions being on the buyer's side will be critical for modeling the network formation game in a tractable manner.[14]

This section assumes a fixed network structure: we take as given the set of firms, Z_j, from which each firm j is eligible to purchase inputs. For importing firms, Z_j contains also foreign, F, as an eligible supplier. Sometimes we will refer to the set of domestic suppliers of firm j, which we denote by Z_j^D.

Firms use a CES input bundle of workers and domestic and foreign inputs with elasticity of substitution $\rho > 1$ in the production function. We assume that $\sigma > \rho$, implying that consumers are more price-elastic than firms in their purchase of goods. Given the CES production function, we can write the cost function of firm j as:

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[13]The assumption that foreign goods reach Belgian consumers only through Belgian firms is reasonable because in the data nearly all imports are carried out by firms. We make the assumption that Belgian firms can reach foreign consumers directly to avoid modeling foreign firms in detail.

[14]Also in the industrial organization literature, corner solutions in the bargaining game are sometimes assumed to obtain tractable solutions for network formation problems. For example, when studying the determinants of the hospital networks offered by health plans, Ho (2009) assumes that hospitals make take-it-or-leave-it offers to all health plans in the market.

$$c_j(Z_j) = \frac{1}{\phi_j} \left(\sum_{k \in Z_j} \alpha_{kj}^{\rho-1} p_{kj}^{1-\rho} + \alpha_{\ell j}^{\rho-1} w_\ell^{1-\rho} \right)^{1/(1-\rho)}. \tag{8}$$

The first term in the cost function, ϕ_j, denotes the exogenous total factor productivity of firm j. Following Antras et al. (2017), we will call $\Theta_j(Z_j) = \sum_{k \in Z_j} \alpha_{kj}^{\rho-1} p_{kj}^{1-\rho} + \alpha_{\ell j}^{\rho-1} w_\ell^{1-\rho}$ the *sourcing capability* of firm j, and Z_j the *sourcing strategy* of firm j. The sourcing strategy may include both domestic and foreign sourcing. The price of labor is denoted by w_ℓ. The share of variable costs by firm j that is spent on intermediate inputs produced by firm $k \in Z_j$ is:

$$s_{kj} = \frac{p_{kj} q_{kj}}{c_j q_j} = \frac{\alpha_{kj}^{\rho-1} p_{kj}^{1-\rho}}{\Theta_j(Z_j)}, \tag{9}$$

where α_{kj} reflects how salient the good produced by firm k is as an input for firm j. Analogously, the share of variable costs by firm j that is spent on labor is:

$$s_{\ell j} = \frac{w_\ell \ell_j}{c_j q_j} = \frac{\alpha_{\ell j}^{\rho-1} w_\ell^{1-\rho}}{\Theta_j(Z_j)}, \tag{10}$$

while the *direct* foreign input share of firm j (assuming $F \in Z_j$) is:

$$s_{Fj} = \frac{p_{Fj} q_{Fj}}{c_j q_j} = \frac{\alpha_{Fj}^{\rho-1} p_{Fj}^{1-\rho}}{\Theta_j(Z_j)}. \tag{11}$$

Before deriving an expression for the total sales of a firm, we discuss the pricing problem of the firm. Due to CES preferences and monopolistic competition, firms charge a constant mark-up over marginal costs, $\mu = \frac{\sigma}{\sigma-1}$, when selling to final consumers at home or abroad. When selling to other firms, firms engage in Nash bargaining with the full bargaining power on the side of the buying firm. The buyer will make the supplier just indifferent between selling to the firm or not, and therefore firms sell at marginal costs to other firms.

In order to sell abroad, firms incur iceberg transport costs, τ. In this section, we take export participation, I_{jF}, as given ($I_{jF} = 1$ for all exporting firms and $I_{jF} = 0$ otherwise) and endogenize it in Section 4. Firms' total sales consist of the sum of domestic sales to final consumers, foreign sales to final consumers, and domestic sales to other firms. Let firm j's total sales be denoted by:

$$x_j = \underbrace{\beta_{jH}^{\sigma-1}\mu^{1-\sigma}\phi_j^{\sigma-1}\Theta_j(Z_j)^{(\sigma-1)/(\rho-1)}\frac{E}{P^{1-\sigma}}}_{\text{Domestic sales to final consumers}} + \underbrace{I_{jF}\beta_{jF}^{\sigma-1}\mu^{1-\sigma}\phi_j^{\sigma-1}\Theta_j(Z_j)^{(\sigma-1)/(\rho-1)}\tau^{1-\sigma}\frac{E_F}{P_F^{1-\sigma}}}_{\text{Exports}}$$

$$+ \underbrace{\sum_k I(j \in Z_k)\mu^{1-\rho}\phi_j^{\rho-1}\alpha_{jk}^{\rho-1}\Theta(j)\frac{x_k/\mu_k}{\Theta_k(Z_k)}}_{\text{Domestic sales to firms}}, \tag{12}$$

where μ_k denotes the average mark-up of firm k. Recall that the firm charges a constant mark-up to final consumers and a zero mark-up to other firms. Hence, μ_k depends on the distribution of firm k's sales.

Given that firms make their profits only on sales to final consumers, we can write the *variable* profits of firm j given a sourcing strategy, Z_j, and export participation, I_{jF}, as

$$\pi_j^{var}(Z_j, I_{jF}) = \frac{1}{\sigma}\beta_{jH}^{\sigma-1}\mu^{1-\sigma}\phi_j^{\sigma-1}\Theta_j(Z_j)^{(\sigma-1)/(\rho-1)}\frac{E}{P^{1-\sigma}}$$

$$+ I_{jF}\frac{1}{\sigma}\beta_{jF}^{\sigma-1}\mu^{1-\sigma}\phi_j^{\sigma-1}\Theta_j(Z_j)^{(\sigma-1)/(\rho-1)}\tau^{1-\sigma}\frac{E_F}{P_F^{1-\sigma}}. \tag{13}$$

3.1.3 Firms' dependence on foreign inputs

We now calculate the exposure of firms to foreign inputs, taking into account that the direct and total foreign input share can be substantially different. Let s_{Fj} denote the *total* foreign input share of firm j:

$$s_{Fj}^{Total} = s_{Fj} + \sum_{i \in Z_j^D} s_{ij}s_{Fi}^{Total}. \tag{14}$$

The definition of total foreign input share is intuitive in a model with single product firms in which each firm uses the same fraction of foreign inputs in the production sold to every buyer. Proposition 1 shows the link between the total foreign input shares and the cost reduction from international trade in our model.[15]. In the proposition, we also present results based on two alternative modeling assumptions that researchers often make when they do not have access to domestic firm-to-firm transaction information.

Proposition 1 (Cost increases from banning foreign inputs) *Assume $\rho > 1$.*

Given fixed linkages between firms, and leaving domestic nominal wages, w_ℓ, unchanged, the total cost increase from banning foreign inputs is:

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[15]Note that the assumption made in the proposition that nominal wages are unchanged is not that restrictive, since nominal wages can be normalized to any value under autarky

$$\hat{c}_j \,|_{total}^{p_{F.} \to \infty} = \left(1 - s_{Fj}^{Total}\right)^{1/(1-\rho)}. \qquad (15)$$

Ignoring linkages and indirect effects (i.e., assuming there is no pass-through of cost changes from domestic suppliers) and leaving domestic nominal wages, w_ℓ, unchanged, the direct cost increase from banning foreign inputs is:

$$\hat{c}_j \,|_{direct}^{p_{F.} \to \infty} = (1 - s_{Fj})^{1/(1-\rho)}. \qquad (16)$$

Finally, if one assumes an economy with roundabout production in which firms' outputs are aggregated to a composite intermediate input according to equation (4) and the composite intermediate input is the only firm-to-firm input in equation (8), the cost increase from banning foreign inputs, $\hat{c}_j \,|_{roundabout}^{p_{F.} \to \infty}$, is implicitly defined as:

$$\left(\hat{c}_j \,|_{roundabout}^{p_{F.} \to \infty}\right)^{1-\rho} = s_{\ell j} + s_{Dj} \left(\sum_k s_{kD} \left(\hat{c}_k \,|_{roundabout}^{p_{F.} \to \infty}\right)^{1-\sigma}\right)^{\frac{1-\rho}{1-\sigma}}, \qquad (17)$$

where s_{Dj} is the share of firm j's domestic intermediate good purchases and s_{kD} is the share of firm k in the intermediate good bundle (measured by firm k's share in total domestic sales).

The result on the firm-level cost increase from banning imports, $\hat{c}_j \,|_{total}^{p_{F.} \to \infty}$, reflects that a firm's cost will not only rise according to it's own direct foreign input share, but also according to its suppliers' foreign input shares, suppliers' suppliers' foreign input share, and so forth. This firm-level exposure is summarized in the total foreign input share expression. The elasticity of substitution in the production function, ρ, indicates how easy it is to switch to alternative inputs, including labor. Given the observed total foreign input share, a lower value of ρ leads to larger cost increases from banning foreign trade.

In contrast, the expression $\hat{c}_j \,|_{direct}^{p_{F.} \to \infty}$, yields only the direct effect of on firm-level cost from banning foreign trade (mirroring the results in earlier work by Arkolakis, Costinot, and Rodríguez-Clare (2012) and Blaum et al. (2016)). It can be rationalized from our model under a network in which importers sell all their output to final consumers, and hence the total foreign input share of domestic suppliers of other firms in equation (14) is zero. Since the observed production networks differ from this extreme case, using the formula from equation (16) leads to cost increases from banning foreign inputs that are too low compared to the full effect summarized in equation (15).

In the absence of firm-to-firm transaction data, it is possible to approximate the indirect effect by assuming a roundabout production structure (as for example in Blaum et al. (2016)).[16] Under the roundabout production assumption, every firm with the same inter-

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[16]Blaum et al. (2016) obtain a total cost reduction result from foreign inputs in which each firm buys

mediate input share will have the same indirect exposure to foreign goods. However, this assumption is at odds with our data, and could create bias in the calculation of the firm-level cost effects from trade.

3.1.4 Aggregation and equilibrium

We now describe the aggregation of our model, discuss how firm profits are redistributed to consumers and define the equilibrium. In the model with a fixed production network we abstract from fixed costs of linkage formation, and hence $\pi_j = \pi_j^{var}$.

We assume that the set of Belgian firms is fixed and that firm profits are distributed to workers in Belgium. We consider Belgium as a small-open economy and assume that there are no foreign asset holdings and that trade is balanced. Hence aggregate expenditure in Belgium is given by

$$E = w_\ell L + \sum_k \pi_k. \tag{18}$$

Balanced trade implies that aggregate exports are equal to aggregate imports:

$$\sum_j I_{jF}\beta_{jF}^{\sigma-1}\mu^{1-\sigma}\phi_j^{\sigma-1}\Theta_j(Z_j)^{(\sigma-1)/(\rho-1)}\tau^{1-\sigma}\frac{E_F}{P_F^{1-\sigma}} = \sum_j \frac{1}{\mu_j}s_{Fj}x_j. \tag{19}$$

Labor market clearing implies that labor income is equal to firms' labor costs:

$$w_\ell L = \sum_j \frac{1}{\mu_j}s_{\ell j}x_j. \tag{20}$$

We next define the equilibrium for the small open economy.

Definition 1 (Equilibrium given a fixed network structure) *Given foreign expenditure, E_F, foreign price index, P_F, and a set of prices by foreign suppliers, $\{p_{Fj}\}_j$, an equilibrium for the model with a fixed network structure and fixed export participation is a wage level, w_ℓ, price index for the consumer, P, and aggregate expenditure, E, such that equations (6), (8), (9), (10), (12), (13), (18), (19), and (20) hold.*

We establish uniqueness of the equilibrium in the closed economy in the following lemma.

Lemma 1 (Uniqueness of equilibrium under closed economy) *Define a $K \times K$ matrix A where where the (i,j) element is $\phi_j^{\rho-1}\alpha_{ij}^{\rho-1}$ if $i \in Z_j$ and 0 otherwise, and K denotes*

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the same CES bundle of intermediate inputs. The production function in their paper is Cobb-Douglas in intermediate inputs and own labor input. Our roundabout cost reduction result in equation (17) is similar to their result, but derived for a production function which is CES between labor input and intermediates.

the number of Belgium firms. Assume the matrix $\left(I - A'\right)$ is invertible, where I is the identity matrix. Then under a closed economy, for a given nominal wage, there exists a unique equilibrium defined in Definition 1.

Lemma (1) is useful because it implies that the counterfactual equilibrium without trade is unique. We next proceed to discuss the change in the aggregate price index arising from banning international trade.

Proposition 2 (Change in aggregate price index from banning international trade)
Let s_{iH} denote firm i's share in household demand in in the initial equilibrium prior to raising the barriers to trade.
Given a fixed set of firms, network structure and nominal wage, the price index change from banning international trade can be summarized as follows:

$$\hat{P} \mid_{total}^{p_{F.}\to\infty} = \left(\sum_i s_{iH} \left(\hat{c}_i \mid_{total}^{p_{F.}\to\infty}\right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}. \tag{21}$$

If the price of intermediate goods is assumed to be unchanged, the price index change can be expressed as

$$\hat{P} \mid_{direct}^{p_{F.}\to\infty} = \left(\sum_i s_{iH} \left(\hat{c}_i \mid_{direct}^{p_{F.}\to\infty}\right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}. \tag{22}$$

Finally, if one assumes roundabout production as defined in proposition 1, then the expression becomes

$$\hat{P} \mid_{roundabout}^{p_{F.}\to\infty} = \left(\sum_i s_{iH} \left(\hat{c}_i \mid_{roundabout}^{p_{F.}\to\infty}\right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}. \tag{23}$$

The change in the price index from banning international trade is a weighted aggregate of each firm's cost increase with the weight equal to the firm's share in domestic household demand, s_{iH}, in the initial economy with international trade. Propositions 1 and 2 imply that the change in the aggregate price index depends on the underlying network structure. To see this, consider a production network in which all the imports are made by firms that had no sales to other domestic firms or to the domestic final consumers. In that case, all the cost increases from banning foreign goods would accrue to firms for which the share in domestic household demand is zero, and therefore the price index effect would be zero as well. Suppose instead that importers of foreign goods had no sales to other domestic firms, but sold all their output to domestic final consumers. Then, the price index increase is given by equations (16) and (22). However, with positive sales to other domestic firms by the importers, the price index effect is given by equations (15) and (21).

A corollary of the results from propositions 1 and 2 is that two economies with the same elasticities of substitutions in production and in the utility functions, and the same levels of aggregate imports and exports, GDP, *and* gross production, can have different gains from trade. We illustrate this in a simple numerical example in Appendix B.[17]

The above results illustrates that knowing the underlying micro-structure of the economy is relevant for the quantitative analysis of the gains from trade. In the following subsection, we make use of the detailed information about domestic firm-to-firm transactions in our data when we calculate the welfare gains from trade for the Belgian economy.

3.2 Empirical results

In this section, we provide a quantitative analysis of how international trade affects firms' production costs and the consumer price index. As shown above, this analysis requires information on the observed firm-to-firm transactions, firm-level output, international trade flows, and labor input, in combination with estimates or assumptions for the elasticity of substitution in the production function, ρ, and the utility function, σ. Throughout the paper, the baseline specification assumes σ is equal to 4 and ρ is equal to 2.[18] We perform sensitivity analysis to examine how the results vary with the choice of these parameter values.

To assess the implications of banning foreign inputs, we compute the firm level cost increases by making use of proposition 1. We use the firm-to-firm network structure as observed in 2012. Proposition 1 tells us that the total shares of foreign inputs for each firm, s_{Fj}^{Total}, translates to the cost changes that firms face from banning foreign inputs. Figure 3 displays the cost increase of firms from banning international trade (in the red line). This figure also reports the cost increase under the assumptions of the direct effect (blue line) and roundabout economy (green line). In these two cases, it is not necessary to observe domestic firm-to-firm transactions to calculate the cost increases.

As evident from Figure 3, international trade matters much more for firms' production costs if we use our model with fixed domestic production networks than in the direct effect and roundabout economy. For the median firm (in the distribution of cost changes), the cost increase from banning foreign inputs is 70% in our model with domestic production networks, 41% in the model with the roundabout economy assumption, and zero when considering only the direct effect. As expected, the cost increases are the lowest under the

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[17]This example is related to the discussion in Melitz and Redding (2014) that the gains from trade can be arbitrarily large in a model with sequential production, as the number of stages of production increases. However, note that the ratio of gross production to GDP also rises in their example as the number of stages gets larger. Here we hold the level of gross production and GDP fixed, and illustrate that the gains from trade can still differ.

[18]Using data for the US, Antras et al. (2017) estimate $\sigma = 3.85$ and Oberfield and Raval (2014) estimate a level of σ between 3 and 5 among various manufacturing industries.

direct effect assumption. In the roundabout production economy, the non-importing firm's cost increase is bounded above by the price increase of the composite intermediate good. By comparison, when taking the actual production network into consideration, many firms have a cost increase above 65%, while the roundabout model suggest that very few importing firms have cost increases of that magnitude.

Figure 3: Distributions of $\hat{c}$ from banning imports
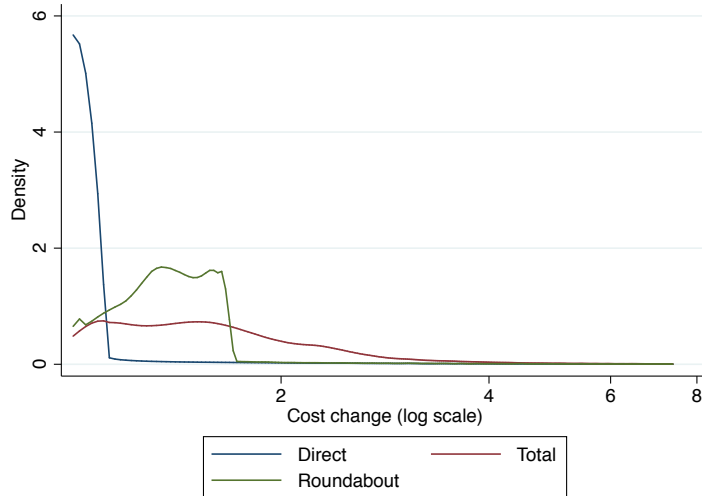
Notes: The parameters used are $\rho = 2, \sigma = 4$. See the appendix for derivations and plots for different parameter values.
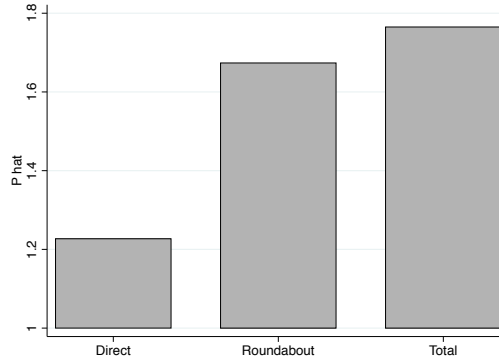
In Appendix I, we plot the same distributions for different parameter values of σ and ρ. Even for alternative parameters, the distribution of cost changes under the actual production network has a much thicker right tail than the distribution under the roundabout production assumption. While the distributions become closer to each other if $\sigma = \rho = 2$, even in that case, the 90th percentile of firm-level cost increases under roundabout is 152%, whereas it is 245% when the actual firm-to-firm transactions are used in the calculation (see Table 17 in the Appendix).[19]

In Figure 4, we make use of proposition 2 to analyze the implications for the aggregate consumer price index from banning foreign inputs. This figure reports the changes in aggregate price indices, $\hat{P}$, for the three different cases.[20] The increase in the price index is much larger when one takes into account the domestic production network than when one only

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[19]To read Table 17, recall that in the roundabout economy, σ is the parameter used in the aggregation to the intermediate input bundle. Note that altering σ does not affect the calculation of $\hat{c}_j \mid_{total}^{p_{F.} \to \infty}$, but it affects the calculation of $\hat{c}_j \mid_{roundabout}^{p_{F.} \to \infty}$.

[20]Note that s_{iH} is not observable in standard datasets. However, firm i's share of total domestic sales, s_{iD}, is usually observed. As shows in Appendix section A.1, s_{iD} and s_{iH} are identical in the roundabout case. We use s_{iD} when calculating the price index changes for the direct effect economy and the roundabout economy.

Figure 4: Change in aggregate price index, $\hat{P}$, from banning imports

takes into account firms' direct exposure to foreign inputs. While the price index increases by 23% if only the direct effect of banning imports is considered, it rises by as much as 77% if the full network structure of domestic production is taken into account. Compared to the model of roundabout production, we also find a larger change in the price index when using the information contained in the firm-to-firm transaction data. Under roundabout production, the domestic price index increases by 67%.

In Appendix I, we report the analogous numbers for $\hat{P}$ under different parameters of σ and ρ. As one would expect, the gains from trade get larger as σ and ρ get lower. We robustly find that the price index increase from banning international trade is larger in our model than under the roundabout economy assumption, as long as σ is sufficiently large relative to ρ. On the one hand, the assumption in the roundabout economy that each firm obtains the same exposure to foreign goods through its domestic intermediate input purchases leads to larger gains from trade in the roundabout economy, as it is harder for consumers to substitute away from firms that face higher costs. On the other hand, the common assumption in roundabout production economy models (Eaton and Kortum, 2002 and Blaum et al., 2016) is that aggregation of the intermediate input bundle occurs analogous to the aggregation for final consumption goods (equation (4)), using σ as the elasticity of substitution in that aggregation. That makes it easier in the intermediate input bundle to substitute from firms exposed to large imports to non-importing firms, leading to lower cost increases and lower gains from trade. The second effect dominates when σ is sufficiently large as compared to ρ, leading to larger gains from trade when using the actual firm-to-firm production network data.

Our analysis so far has taken the network structure of firms as fixed. This has allowed us to derive analytic solutions for the firm-level cost and aggregate price index changes in the absence of international trade that could then be calculated easily with the firm-to-firm

transaction and international trade data available to us. In the following section, we analyze how the network forms endogenously and the implications of endogenous networks for the quantification of the gains from trade.

4 A model of trade with endogenous production networks

This section develops a model of trade with endogenous network formation, allowing buyer-supplier relationships to change in response to trade shocks. The model builds on the theoretical framework presented in Section 3.1. We assume the same preferences, demand functions, production technology, and market structure as in the model with fixed production networks. What is new is that firms now optimally choose their set of suppliers (i.e. the firm's sourcing strategy) and decide whether to import and export. We first describe the model with endogenous network formation and discuss how it can be solved. Then we estimate the model, and use it to quantify how international trade affects firms' production costs and consumer prices with and without endogenous network formation.

4.1 Model

4.1.1 Determination of firm sourcing strategy, import and export participation

We assume that only buyers initiate linkages with other domestic firms. Forming linkages to suppliers is costly, and firm j incurs a random, firm-pair-specific, fixed cost f_{kj} to add supplier k. The realization of fixed costs is known to the firm at the time it selects suppliers. Firms in our model make profits due to positive mark-ups in sales to domestic and foreign final consumers. Since the buyer is assumed to have all the bargaining power, firms do not make profits from sales to other firms. Hence, variable profits are proportional to firm-level sales to final consumers.

Given a sourcing strategy, Z_j, and export participation choice, I_{jF}, the profits of firm j are equal to variable profits minus the fixed costs of domestic and foreign sourcing, $\sum_{k \in Z_j} f_{kj} w_\ell$, and the fixed costs of exporting, $I_{jF} f_{jF} w_\ell$:

$$\pi_j(Z_j, I_{jF}) = \pi_j^{var}(Z_j, I_{jF}) - \sum_{k \in Z_j} f_{kj} w_\ell - I_{jF} f_{jF} w_\ell. \tag{24}$$

We assume that firm j exogenously meets a set of eligible suppliers, $\mathbf{Z}_j$. Firm j then endogenously decides on the set of suppliers and whether or not to export:

$$\max_{Z_j, I_{jF}} \pi_j(Z_j, I_{jF}) \quad \text{s.t.} \quad Z_j \subseteq \mathbf{Z}_j \ , \ I_{jF} \in \{0,1\}. \tag{25}$$

Under the assumption made earlier that final demand is more elastic than the elasticity of substitution between inputs in the production function, $\sigma > \rho$, the sourcing decision is complementary across suppliers as well as complementary to the exporting decision. In other words, the marginal benefit of adding a supplier is increasing in the set of existing suppliers and it is higher if the firm exports. We can therefore follow the same approach as Antras et al. (2017), namely the adaption of the Jia (2008) algorithm, to solve the problem described in (25), given knowledge about the costs of the eligible suppliers in $\mathbf{Z}_j$.

To solve the model with endogenous network formation, we therefore need to determine the set of eligible suppliers, $\mathbf{Z}_j$. When solving the problem for every firm described in (25), a key issue that arises is that each firm needs to guess a set of costs for its eligible suppliers, where the costs themselves are equilibrium objects and depend on everyone's sourcing decision. Not only is it extremely challenging computationally to find a fixed point in the set of costs for all firms so that these costs are consistent with everyone's optimal sourcing decision, but also the uniqueness of such a fixed point is rather unlikely. On the one hand, if firms guess that suppliers have very high unit costs, this could result in the formation of very few linkages and lead to high unit costs overall. On the other hand, if firms guess that suppliers have very low unit costs, this could result in the formation of many linkages and lead to low unit costs overall. To get around these problems, we consider the formation of an acyclic network, postulating an ordering of firms and restricting the eligible set of suppliers to firms that appear prior to the buyer.

To be concrete, all firms can choose to import foreign inputs and to export their output abroad. However, the set of eligible domestic suppliers varies across firms. Specifically, we order firms in a sequence $S = \{1, 2, 3, ..., N\}$ that restricts the set of eligible suppliers, as illustrated in Figure 5. Because we assumed buyers has the full bargaining power in any firm-to-firm transactions, firms only need to know the choices of the firms prior in the sequence. Taken together, these assumptions make the network formation tractable, as we describe below.

Firm 1 is first in the sequence and can only hire labor inputs. To make its decision of how much labor to hire, firm 1 only needs to know the wage level, w_ℓ, and domestic market demand, $\frac{E}{P^{1-\sigma}}$. Firm 2 is second in the sequence and can hire both labor inputs as well as purchase the input produced by firm 1. To make these decisions, firm 2 needs to know the wage and market demand level as well as the cost of its eligible supplier (firm 1). Firm 3 can hire labor and purchase the inputs from firm 1, firm 2, or both. And so on. Given a guess for equilibrium wages, w_ℓ, and domestic market demand, $\frac{E}{P^{1-\sigma}}$, one can solve the problems of the firms sequentially.

Figure 5: Endogenous network formation – eligible connections
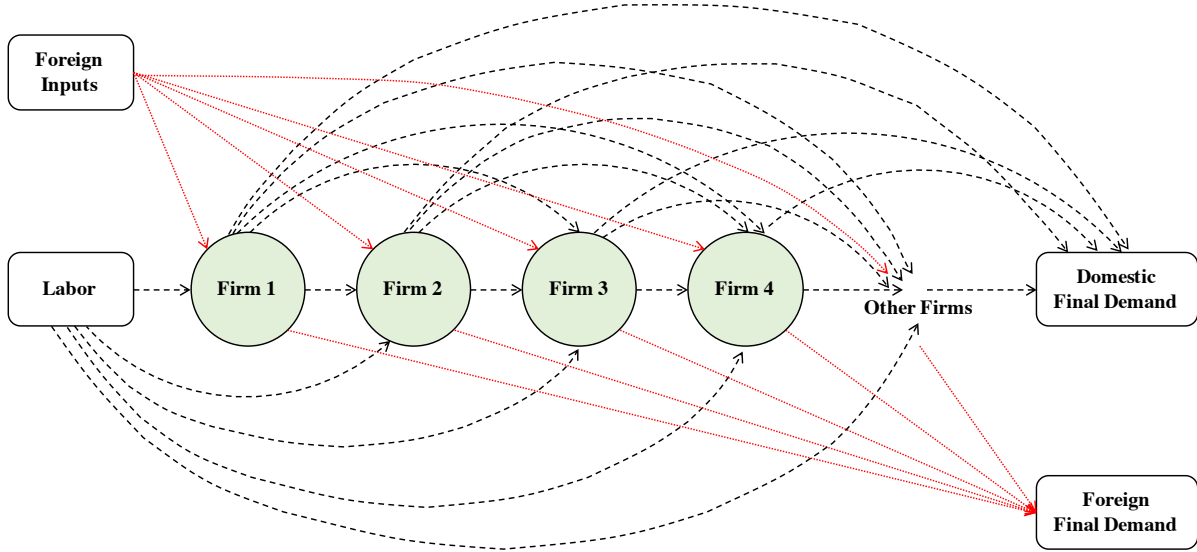
Although the above ordering of firms simplifies the problem, we are still limited in the number of possible sourcing strategies we can feasibly evaluate. We therefore restrict the set of eligible suppliers for firm j, $\mathbf{Z}_j$, to be a random subset from the set of firms prior to firm j in the sequence. The suppliers for firm j are then optimally chosen as the solution to the problem in (25). In practice, we choose the cardinality of $\mathbf{Z}_j$ to be at most 200, so the firm still chooses among 2^{200} possible supplier sets. By following Antras et al. (2017)'s adaption of Jia (2008)'s algorithm, we are then able to tractably solve the discrete choice problem.[21] The firm's order in the sequence of supplier choices and its set of eligible suppliers are becoming attributes of the firm and therefore primitives of the model.

Imposing a tie-breaking rule that in the case of indifference a supplier is included ensures a unique solution to the problem in (25). As a consequence, the network formation will also be unique given a set of wages and a guess for the price index. We can then alter wages, price index, and expenditure to achieve labor market clearing, trade balance, and a fixed point for the price index and expenditure. Importantly, we are searching here only for a fixed point in wages and price index (only 2 scalars) as opposed to searching for a large fixed point vector in every firm's costs and searching strategies. In other words, the ordering approach implies that even with a rich micro structure and firm-level heterogeneity, knowing only two equilibrium variables is sufficient to solve sequentially the firms' problems. We discuss the aggregation and the equilibrium with endogenous network structure more formally below.

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[21] We describe the computational algorithm to solve for the firm's problem in Appendix G. The procedure is very similar to the one described in Antras et al. (2017). Here, we also develop a greedy algorithm in the case the the differences in the lower and upper bounds for the optimal solution are too wide to evaluate the profits of all feasible combinations in between. As in Antras et al. (2017), we find that in about 99% of the cases the lower and upper bounds are perfectly overlapping (see Table 15 in the Appendix).

4.1.2 Aggregation and equilibrium

The model aggregation and equilibrium are broadly similar to the case with fixed networks. However, there are a few notable differences. The first is that firms incur fixed costs – paid in units of labor – to add a domestic buyer, import, or export. Therefore, the labor market clearing condition becomes:

$$w_\ell L = \sum_j \frac{\mu_j - 1}{\mu_j} s_{\ell j} x_j + w_\ell \sum_j \left(\sum_{k \in Z_j} f_{kj} + I_{jF} f_{jF} \right). \tag{26}$$

Additionally, a firm's profit function now subtracts the incurred fixed costs, and a firm's sourcing strategy and export participation are now endogenous choices. However, the trade balance condition remains unchanged. The following definition formally describes equilibrium with an endogenous network structure.

Definition 2 (Equilibrium with endogenous network structure) *Given foreign expenditure, E_F, foreign price index, P_F, and a set of prices by foreign suppliers, $\{p_{Fj}\}_j$, as well as set of eligible suppliers, $\mathbf{Z}_j$ that satisfies acyclicity of the network, an equilibrium for the model with endogenous network structure and endogenous export participation is a wage level, w_ℓ, price index for the consumer, P, and aggregate expenditure, E, as well as a set of sourcing strategies and export participation choices, such that the firm's optimization problem in (25), and equations (6), (8), (9), (10), (12), (13), (18), (19), and (26) hold.*

4.2 Assessing the assumptions about the shape of the network

Given the assumptions we invoked to solve the endogenous network formation, the resulting network will be acyclic. As shown in Figure 6, in an acyclic firm network, there exists at least one way to sort firms so that all directed edges face one direction. In contrast, in a cyclical network at least one edge will face the opposite direction. This feature of our network formation mechanism is admittedly restrictive. We now perform two checks to assess how well the Belgian data can be approximated by an acyclical network.

4.2.1 How cyclic is the production network?

Let $\nu(i)$ be an ordering of firms that maps firms $\{i, j, k, \cdots\} \in \Theta$ into numbers from $\{1, \cdots N\}$. To describe how cyclical the Belgian production network is, we want to find the optimal $\nu(k)$ that minimizes the following objective function:

$$\min_{\{\nu(k)\}} \sum_{i,j} \mathbf{1}\{i \in Z_j\} \mathbf{1}\{\nu(i) > \nu(j)\},$$

Figure 6: Examples of acyclic and cyclic networks

Directed acyclic network

Directed cyclic network

Directed acyclic network, sorted

Directed cyclic network, sorted
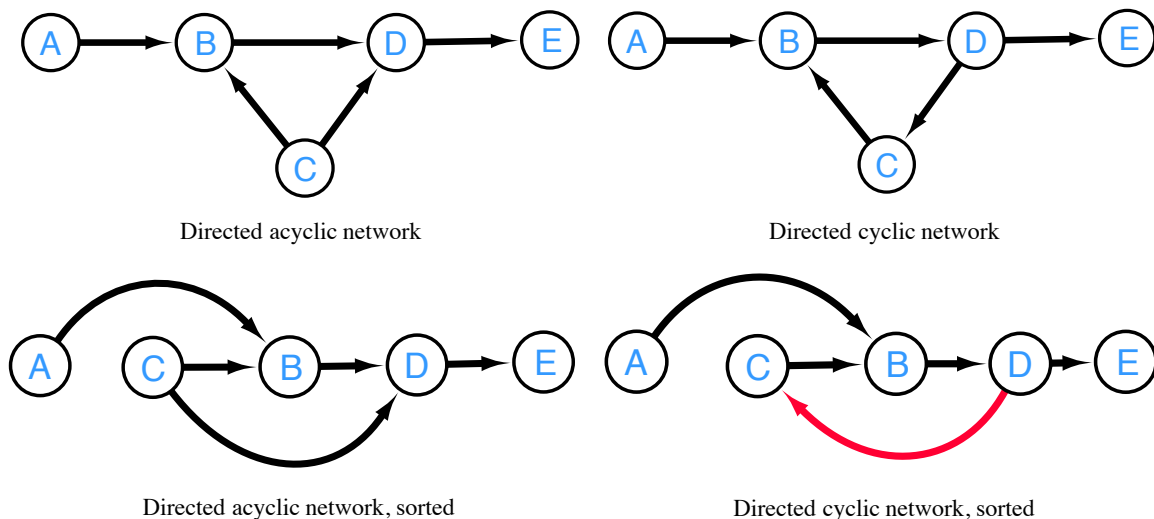
where Z_j is the supplier set of firm j. Solving this problem corresponds to minimizing the number of directed edges that are facing the direction opposite to that of the sorting order. In other words, we try to find an ordering that minimizes the number of arrows facing to the left in the cyclic network in Figure 6.

To solve this problem, which is also known as the feedback arc set problem, we adopt an algorithm proposed by Eades, Lin, and Smyth (1993). The details of the computational algorithm and implementation are presented in Appendix F. The algorithm offers a local minimum, showing that at most 17% of edges in the whole firm-to-firm network in 2012 violate acyclicity.[22] We also search for an ordering that minimizes the value of firm-to-firm sales in violation of acyclicity.[23] We find that no more than 22% of firm-to-firm sales are in violation of acyclicity. We will refer to the former as the unweighted ordering algorithm and the latter as the weighted ordering algorithm.

A natural question that arises is how different the structure of an economy with an acyclic network is in comparison to the economy observed in the data. One way to make this comparison is to calculate input-output tables with and without the firms in buyer-supplier relationships that violate acyclicity. We find that when calculating input-output tables with 72 sectors, the correlation between the input-output table coefficients from the full data and the data without links in violation of the ordering is 0.92 when using the the unweighted ordering algorithm output. The correlation is even higher, 0.97, when using the weighted

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[22]While there is no perfect reference point for this figure, we can compare it to the structure of the directed social network Twitter. Simpson, Srinivasan, and Thomo (2016) calculate that 23% of edges are in violation of acyclicity in the Twitter network in the year 2010.

[23]Specifically, we solve the following problem: $\min_{\{\nu(k)\}} \sum_{i,j} x_{ij} \mathbf{1} \{\nu(i) > \nu(j)\}$, where x_{ij} is the value of the sales from firm i to firm j.

ordering algorithm output.[24]

4.2.2 Gains from trade under fixed networks: cyclic versus acyclic production networks

Another way to assess the assumption of an acyclic network is to examine how the results based on the exogenous network model change if we exclude transactions that violate acyclicity. It is reassuring to find that estimated effect of international trade on consumer prices and firms' cost of production are very similar if we only uses firm-to-firm sales that are consistent with the acyclic network obtained by the ordering algorithm described in Section 4.2.1. Specifically, we keep the direct import share of each firm the same as in the data, set all transactions in violation of the ordering to zero, and adjust all other domestic firm-to-firm input shares such that share of each firm j's input purchases, $\sum_{i \in Z_j} s_{ij}$, is unchanged.[25] The results presented in Table 19 in the Appendix show that the gains from trade under an exogenous network are virtually identical if we only use the subset of transactions for which the domestic production network is acyclic.

4.3 Estimation of model parameters given endogenous network

When allowing for endogenous network formation, we are not able to analytically solve the model. Instead, we will structurally estimate the model and provide numerical results for the counterfactual analysis.

In the estimation of our model, we simulate 100,000 Belgian firms, which is close to the 139,605 firms in our sample in 2012. A firm is characterized by a core productivity level, a set of eligible suppliers that satisfies the ordering, a vector of fixed cost draws for all eligible suppliers, a vector of firm-pair-specific cost shifters, a foreign input cost shifter, fixed costs of importing and exporting, and a foreign demand shifter. We normalize firms' labor productivity shifters, $\alpha_{\ell j} = 1$, and firms' domestic final demand shifters, $\beta_{jD} = 1$.

As a first step of the estimation, we recover the productivity distribution of firms (scaled by some general equilibrium objects) from the identity

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[24]To construct the input-output table coefficients, we aggregate firm-to-firm transactions within the supplying and buying sector. We note that this procedure differs from the national account definition of an input-output table. First, the rows and columns of our aggregated tables are referring to the main sectors of the buyers and suppliers, but these firms can also have a significant share of their production in other sectors. Second, in national account tables, the contribution of the retail and wholesale sectors to the production of the other goods only refer to the trade margin of retailers and wholesalers. In our data, the retail and wholesale sectors are accounted based on their total sales and total input consumption and not on their trade margin.

[25]The only exception is when there is no other domestic supplier of a firm, in which case in the data with only acyclic transactions the domestic firm-to-firm input share is set to zero.

$$\frac{x_{iH}^{1/(\sigma-1)}}{s_{\ell i}^{1/(1-\rho)}} = \phi_i \frac{PE^{1/(\sigma-1)}}{\mu w_\ell}. \tag{27}$$

Observing all the terms on the left hand side enables us to estimate the distribution $\phi_i \frac{PE^{1/(\sigma-1)}}{\mu w_\ell}$. After visually inspecting the distribution, we assume it is log-normal, and estimate the scale parameter to be -2.12 and the dispersion parameter to be 1.37.

We next turn to the estimation of the parameters for the distribution of the firm-pair-specific shifter in the production function, α_{kj}, the foreign input cost shifter, α_{Fj}, the foreign demand shifter, β_{jF}, as well as the distribution of fixed cost parameters for domestic firm-to-firm purchases, $w_\ell f_{kj}$, fixed cost of importing, $w_\ell f_{Fj}$, and fixed cost of exporting $w_\ell f_{jF}$. We again impose log-normality of the distributions and estimate the scale and dispersion parameters of those distributions. We assume that α_{Fj}, α_{kj}, and β_{jF} are independent draws from three log-normal distributions which share a common dispersion parameter, $\Phi_{disp}^{\alpha,\beta}$, and have different scale parameters, $\Phi_{scale}^{\alpha_F}$, $\Phi_{scale}^{\alpha_{\mathrm{dom}}}$, $\Phi_{scale}^{\beta_F}$, respectively. Similarly, the fixed cost draws for domestic purchases from other firms, imports, and exports, are drawn independently from three log-normal distributions with scale parameters $\Phi_{scale}^{f_{\mathrm{dom}}}$, $\Phi_{scale}^{f_{\mathrm{imp}}}$, $\Phi_{scale}^{f_{\mathrm{exp}}}$, and a common dispersion parameter, Φ_{disp}^{f}. The parametric restrictions on the common dispersion parameters imply we need to estimate only 2 instead of 6 dispersion parameters. Overall, there are 8 parameters to be estimated.

We use method of simulated moments to estimate our parameters. We target three sets of moments to match. The first set of moments is helpful in estimating the parameters affecting domestic-firm-to firm purchases. The draws of the fixed costs govern the extensive margins of firm-to-firm trade. Thus, we use information about firms' numbers of suppliers to identify $\Phi_{scale}^{f_{\mathrm{dom}}}$ and Φ_{disp}^{f}. To do this, we match the model to have the same quartile distribution of number of suppliers. Following the procedure used by Eaton, Kortum, and Kramarz (2011), we include in the first vector of moments generated by the model, $\hat{m}_1(\Phi)$, the proportion of firms that has a number of suppliers equal to the first, second, third, and fourth quartile in the data. The draws of α_{kj} govern the distribution of both the intensive margin and the extensive margin of firm-to-firm transactions. To identify the parameters Φ_{scale}^{α} and $\Phi_{disp}^{\alpha,\beta}$, we target statistics on the labor share of firms. Again, we aim to match the fraction of firms in the data that have labor shares in the first, second, third, and fourth quartile of the actual labor share in the data. Relatedly, we also aim to match the distribution of the actual firm-to-firm input shares (conditional on observing trade between firms). Using the same procedure as above, we include the fraction of firms in the four quartile bins of that distribution (using as thresholds the quartiles observed in the data). This generates 12 elements in the vector $\hat{m}_1(\Phi)$.

The second set of moments is helpful in estimating the parameters affecting imports and

exports. We include in $\hat{m}_2(\Phi)$ the share of firms that import and export, respectively. We also include the fraction of firms falling into the bins of the first, second, third, and fourth quartile of imports in firm-level inputs in the data. Similarly, the fraction of firms that have the ratio of exports to firm-level domestic sales as in the quartiles in the data. There are 10 elements in the vector $\hat{m}_2(\Phi)$. Finally, as a third set of moments we include aggregate targets such as the ratio of aggregate exports to aggregate final demand, and the weighted aggregate of firm-level sales to households and foreign input shares, which correspond to the sufficient statistics for the price index increase under fixed networks. There are 3 elements in the vector $\hat{m}_3(\Phi)$.

We describe the difference between the moments in the data and in the simulated model by $\hat{y}(\Phi)$:

$$\hat{y}(\Phi) = m - \hat{m}(\Phi) = \begin{bmatrix} m_1 - \hat{m}_1(\Phi) \\ m_2 - \hat{m}_2(\Phi) \\ m_3 - \hat{m}_3(\Phi) \end{bmatrix},$$

and the following moment condition is assumed to hold at the true parameter value Φ_0:

$$E\left[\hat{y}(\Phi_0)\right] = 0. \tag{28}$$

The method of simulated moments selects the model parameters that minimize the following objective function:

$$\hat{\Phi} = \arg\min_{\Phi} \left[\hat{y}(\Phi)\right]^{\top} \mathbf{W} \left[\hat{y}(\Phi)\right], \tag{29}$$

where $\mathbf{W}$ is a weighting matrix.[26]

4.3.1 Estimation results

Table 3 shows the values of the estimated parameters.

Table 3: Estimated parameters

Preference and production				Fixed costs			
$\hat{\Phi}_{scale}^{\alpha_{\mathrm{dom}}}$	$\hat{\Phi}_{scale}^{\alpha_F}$	$\hat{\Phi}_{scale}^{\beta_F}$	$\hat{\Phi}_{disp}^{\alpha,\beta}$	$\hat{\Phi}_{scale}^{f_{\mathrm{dom}}}$	$\hat{\Phi}_{scale}^{f_{\mathrm{imp}}}$	$\hat{\Phi}_{scale}^{f_{\mathrm{exp}}}$	$\hat{\Phi}_{disp}^{f}$
-6.69	-2.36	-1.94	2.40	-2.64	8.01	9.47	8.65

The magnitude of the estimated parameters are difficult to interpret, since their scale is affected by the choice of normalizations for the foreign market size and the price of the foreign input. We therefore focus on the model fit given these parameter estimates.

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[26]We weight the moments equally, hence the weighting matrix is the identity matrix.

Table 4 shows how well we fit the moments that we target. Note that instead of showing the moments directly (i.e., the fraction of firms falling into each quartile bin), we show the values of the 25th, 50th, and 75th percentiles in both the data and model. The model does a pretty good job at fitting the targeted statistics of firm-to-firm transactions. Most statistics are very close between the model and the data. With respect to fitting firm-level statistics of trade, the model under-predicts the fraction of firms directly involved into exporting and importing (9.4% and 15.4% in the data and 6.7% and 7.9% in the model, respectively). However, the model does a decent job of fitting the aggregate importance of trade, illustrated by a similar ratio of aggregate exports to aggregate sales to domestic final demand in both model and data. The statistics summarizing the gains from trade under fixed networks are also similar in model and data.

We also examine how well the model fits moments that were not directly targeted in the estimation. Specifically, we have not targeted directly the association of size between buyers and suppliers that trade with each other. Consistent with the data, the model predicts a weak negative correlation between the number of suppliers of the buying firms (indegree buyer) and the number of buyer firms of suppliers (outdegree supplier). Similarly, the correlation between sales of the buying and selling firm is close to zero both in the data and in the model.

4.4 Counterfactual with endogenous network formation

Equipped with the parameter estimates of our model, we next turn to the quantitative analysis of the gains from trade for the Belgian economy under endogenous production networks. We repeat the same counterfactual experiment as under fixed networks, and ask how the economic outcomes in the Belgian economy would change if the barriers to trade were infinite. To shut down trade, we make the costs to import and export prohibitively large, so that no firm will engage in international trade. We keep the size of the domestic labor force and all other parameters at their estimated level. We then solve the problem of a closed economy and normalize the nominal wage to the same level it took in the open economy. To solve for the counterfactual equilibrium, we find a fixed point in the market demand, $\frac{E}{P^{1-\sigma}}$. After finding the fixed point in market demand, the labor market clears as well due to Walras' law.

We are interested in how international trade affects the firms' production costs, the structure of the domestic production network, and the aggregate price index. We start by discussing the change in the overall price index, and then look into the micro-changes that lead to these price index effects. In our simulated economy, the price index increase by 93 percent with a fixed network structure and 80 percent when we allow for endogenous network formation. Hence, we find that in our simulated economy with endogenous networks, the

Table 4: Model fit: targeted moments

	Data	Model
Number of suppliers 25th percentile	15	15
Number of suppliers 50th percentile	28	30
Number of suppliers 75th percentile	49	53
Share of labor costs 25th percentile	0.12	0.21
Share of labor costs 50th percentile	0.28	0.33
Share of labor costs 75th percentile	0.50	0.66
Firm-to-Firm share 25th percentile	0.0002	0.0001
Firm-to-Firm share 50th percentile	0.0015	0.0004
Firm-to-Firm share 75th percentile	0.0069	0.0035
Share of firms that export	0.094	0.067
Share of exports in total firm sales 25th percentile	0.008	0.001
Share of exports in total firm sales 50th percentile	0.120	0.020
Share of exports in total firm sales 75th percentile	0.649	0.185
Share of firms that import	0.154	0.079
Share of imports in firm inputs 25th percentile	0.008	0.012
Share of imports in firm inputs 50th percentile	0.239	0.066
Share of imports in firm inputs 75th percentile	0.653	0.317
Ratio of aggregate exports to aggregate sales to domestic final demand	1.02	0.87
$\hat{P}\mid_{total}^{p_{F.}\to\infty}= \left(\sum_i s_{iH}\left(1-s_{Fj}^{Total}\right)^{\frac{1-\sigma}{1-\rho}}\right)^{\frac{1}{1-\sigma}}$	1.77	1.93
$\hat{P}\mid_{direct}^{p_{F.}\to\infty}= \left(\sum_i s_{iH}\left(1-s_{Fj}\right)^{\frac{1-\sigma}{1-\rho}}\right)^{\frac{1}{1-\sigma}}$	1.21	1.13

Notes: Percentiles are calculated based on all firms in the sample; Share of labor costs refers to the fraction of labor costs in costs (labor costs + domestic purchases + imports) and the percentiles are calculated based on all firms the sample; Firm-to-Firm share refers to the fraction of costs a firm spends on one particular supplier and the percentiles are calculated for all firm-to-firm transactions; The percentiles for share of exports in total firm sales are calculated for all firms with positive export sales. The percentiles for share of imports in firm inputs are calculated for all firms with positive import purchases.

Table 5: Model fit: non-targeted moments

	Data	Model
Corr (Indegree Buyer, Outdegree Supplier)	-0.05	-0.06
Corr (Sales Buyer, Sales Supplier)	-0.02	0.01

price index rises around 15 percent less than under a fixed network.[27]

We next look at the distribution of firms' cost changes. Under fixed networks, we have seen that every firm's cost weakly increases when banning international trade. Interestingly,

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[27]Specifically, $\frac{80}{93} - 1 = -.14$.

this is no longer the case under endogenous networks. Firms with little direct or indirect engagement in foreign trade actually benefit from banning international trade. This is because they can expand relatively to the firms' that are engaged in international trade. Our simulated model suggests that in the absence of international trade, 24 percent of firms actually would have weakly *lower* costs than with international trade.

Compared to the counterfactual analysis with a fixed network structure, most firms have lower cost increases under endogenous network structure. We show the distribution of relative cost increases under endogenous and exogenous networks in Figure 7a. Note that a small fraction of firms does have higher cost increases under endogenous networks. This occurs if a firm not only stops importing and exporting but also drops domestic suppliers from its sourcing strategy. Figure 7b shows that firms that have relatively large cost increases under fixed networks tend to have even larger cost increases under endogenous networks.

Figure 7: Cost changes: Endogenous vs. exogenous network

(a) Density (b) Scatter
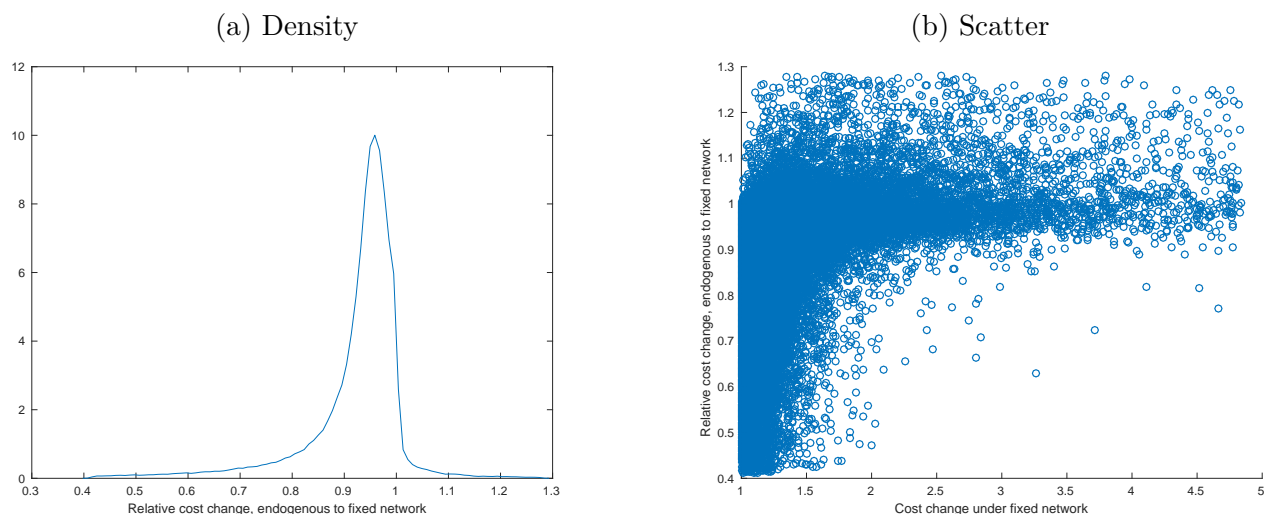
Notes: The left panel shows the density of the ratio of cost changes from banning international trade under endogenous and fixed networks. For most firms this ratio is smaller than one, implying lower cost increases under endogenous networks for those firms. The right panel is a scatter plot of the ratio of cost increases under endogenous and fixed networks (vertical axis) against the cost increases from banning international trade under fixed networks (horizontal axis). Firms that have high cost increases under fixed networks sometimes have even larger cost increases under endogenous networks. Observations below the 1st and above the 99th percentile of cost increases are excluded from the figure.

These heterogeneous cost changes are driven by changes in the domestic linkages between firms. We find that in the simulated economy, the number of domestic firm-to-firm linkages increases from 3.70 million to 4.16 million when trade is shut down, an increase of around 12 percent.[28] Underlying this net increase, there is churn in firm-to-firm linkages: Around

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[28]In the actual Belgian firm-to-firm data, there are around 6 million domestic firm-to-firm transactions, hence our simulated economy under-predicts the number of linkages between firms.

2 percent of the domestic linkages that exist under free trade are no longer active in the absence of trade.

5   Conclusion

In this paper, we used administrative data from Belgium with information on domestic firm-to-firm sales and foreign trade transactions to study how international trade affects firm efficiency and real wages. Our paper offered three sets of results. First, we documented that most firms that do not directly import or export still have large indirect exposure to foreign trade, and that a firm's output is affected by idiosyncratic shocks to its buyers and suppliers. These empirical insights motivated the development of a model with domestic production networks and trade.

Second, we derived new sufficient statistics results for how international trade affects firms' production costs. Assuming a fixed network structure, the cost reduction for an individual firm due to international trade depends only on the share of input costs that is spent on goods that are imported directly or indirectly and the elasticity of substitution in the production function. We applied this sufficient statistics formula to our data, and compared the results to those we obtain using existing approaches. This comparison highlights the importance of data on and modeling of domestic production networks in studies of international trade.

Lastly, we developed a novel framework for analyzing the endogenous formation of the production network. We make the model tractable by focusing on the formation of an acyclic rather than a cyclic production network. While restrictive, this allowed us to solve a model of firm trade with endogenous formation of domestic buyer-supplier relationships. Reassuringly, we found that the vast majority of buyer-supplier relationships in Belgium can be described by an acyclic production network. Moreover, both sectoral input-output tables and the gains from trade under a fixed network structure do not change materially if we restrict attention to transactions for which the domestic production network is acyclic. Our approach to endogenous network formation may prove useful in contexts other than trade where researchers are increasingly interested in the formation and consequences of domestic production networks.

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A Theoretical Results

A.1 Proof of Proposition 1

Proof.

Total cost increase

We have

$$
s_{Fj}^{Total} = s_{Fj} + \sum_i s_{ij} s_{Fi}^{Total}
$$

$$
= s_{Fj} + \sum_i s_{ij} \left[s_{Fi} + \sum_k s_{ki} \left(s_{Fk} + \cdots \right) \right]
$$

and

$$
c_j^{1-\rho} = \sum_k \alpha_{kj}^{\rho-1} \phi_j^{\rho-1} c_k^{1-\rho} + \alpha_{\ell j}^{\rho-1} \phi_j^{\rho-1} w_\ell^{1-\rho} + \alpha_{Fj}^{\rho-1} \phi_j^{\rho-1} p_{Fj}^{1-\rho}.
$$

Consider the indirect effects (i.e., assuming that suppliers' cost increases will translate into price increases for their customers). The unit cost after $p_{Fj} \to \infty$ for all j (assuming that the nominal wage, w_ℓ, does not change) is

$$
\tilde{c}_j^{1-\rho} = \sum_k \alpha_{kj}^{\rho-1} \phi_j^{\rho-1} \tilde{c}_k^{1-\rho} + \alpha_{\ell j}^{\rho-1} \phi_j^{\rho-1} w_\ell^{1-\rho},
$$

thus

$$\hat{c}_j^{1-\rho} \mid_{total}^{p_{F.}\to\infty} = \frac{\tilde{c}_j^{1-\rho}}{c_j^{1-\rho}}$$

$$= \frac{\sum_k \alpha_{kj}^{\rho-1} \phi_j^{\rho-1} \tilde{c}_k^{1-\rho} + \alpha_{\ell j}^{\rho-1} \phi_j^{\rho-1} w_\ell^{1-\rho}}{c_j^{1-\rho}}$$

$$= s_{\ell j} + \sum_k s_{kj}\hat{c}_k^{1-\rho} \mid_{total}^{p_{F.}\to\infty}$$

$$= s_{\ell j} + \sum_k s_{kj}\left[s_{\ell k} + \sum_i s_{ik}\left(s_{\ell i}+\cdots\right)\right]$$

$$= 1 - s_{Fj} - \sum_k s_{kj} + \sum_k s_{kj}\left[\left(1 - s_{Fk} - \sum_i s_{ik}\right) + \sum_i s_{ik}\left(\left(1 - s_{Fi} + \sum_l s_{li}\right) + \cdots\right)\right]$$

$$= 1 - \left(s_{Fj} + \sum_k s_{kj}\left[s_{Fk} + \sum_i s_{ik} - \sum_i s_{ik}\left(1 - s_{Fi} + \sum_l s_{li} + \cdots\right)\right]\right)$$

$$= 1 - \left(s_{Fj} + \sum_k s_{kj}\left[s_{Fk} + \sum_i s_{ik}\left(s_{Fi} + \cdots\right)\right]\right)$$

$$= 1 - s_{Fj}^{Total}.$$

Therefore, firms' change in unit costs upon $p_f \to \infty$ when considering the full network effects are as follows:

$$\hat{c}_j \mid_{total}^{p_{F.}\to\infty} = \left(1 - s_{Fj}^{Total}\right)^{\frac{1}{1-\rho}}.$$

Direct cost increase

If only considering the direct effect (i.e., assuming that suppliers' cost increases will not translate into price increases for their customers), with $\rho > 1$ and $p_{Fj} \to \infty$ for all j (i.e., autarky), the cost for firm j becomes

$$\bar{c}_j^{1-\rho} = \sum_k \alpha_{kj}^{\rho-1} \phi_j^{\rho-1} c_k^{1-\rho} + \alpha_{\ell j}^{\rho-1} \phi_j^{\rho-1} w_\ell^{1-\rho}.$$

Therefore,

$$\hat{c}_j^{1-\rho} \mid_{direct}^{p_{F.}\to\infty} = \frac{\bar{c}_j^{1-\rho}}{c_j^{1-\rho}} = 1 - s_{Fj}.$$

Re-arranging yields the change in unit cost when considering only the direct effect:

$$\hat{c}_j \mid_{direct}^{p_{F.}\to\infty} = (1 - s_{Fj})^{\frac{1}{1-\rho}} .$$

Cost increase under roundabout production

In this roundabout production economy, firm j produces its goods with a CES production technology, using domestic intermediate goods, foreign imports, and labor. The implied unit cost of firm j becomes

$$c_j = \phi_j^{-1} \left(\alpha_{Dj}^{\rho-1} P_D^{1-\rho} + \alpha_{Fj}^{\rho-1} p_{Fj}^{1-\rho} + \alpha_{\ell j}^{\rho-1} w_\ell^{1-\rho} \right)^{\frac{1}{1-\rho}} ,$$

where P_D is a price index of domestic intermediate goods. Associated input shares are $s_{Dj} = \frac{\phi_j^{\rho-1}\alpha_{Dj}^{\rho-1}P_D^{1-\rho}}{c_j^{1-\rho}}$, $s_{Fj} = \frac{\phi_j^{\rho-1}\alpha_{Fj}^{\rho-1}p_{Fj}^{1-\rho}}{c_j^{1-\rho}}$, and $s_{\ell j} = \frac{\phi_j^{\rho-1}\alpha_{\ell j}^{\rho-1}w_\ell^{1-\rho}}{c_j^{1-\rho}}$.

As in Blaum et al. (2016), we let domestic intermediate goods be produced via roundabout production, with CES substitution parameter σ. The price of an intermediate good is therefore equal to the CES price index,

$$P_D = \left(\sum_j \alpha_{jD}^{\sigma-1} p_{jD}^{1-\sigma} \right)^{\frac{1}{1-\sigma}} ,$$

where p_{jD} is the price that firm j charges in the aggregation process. Let $p_{jD} = c_j$. We can additionally define $s_{jD} = \frac{\alpha_{jD}^{\sigma-1} p_{jD}^{1-\sigma}}{P_D^{1-\sigma}}$, which is the firm j's contribution to the intermediate good. We use the firm's share of domestic sales:

$$s_{jD} = \frac{\text{B2B sales}_j + \text{Sales to HH}_j}{\sum_i (\text{B2B sales}_i + \text{Sales to HH}_i)} .$$

We assume $\alpha_{jD} = \beta_{jD}$, so that the two shares s_{jD} and s_{jH} are the same in the model.

Consider a change in c_j, upon $p_{Fj} \to \infty$ for all j.

$$c_j = \phi_j^{-1} \left(\alpha_{Dj}^{\rho-1} P_D^{1-\rho} + \alpha_{Fj}^{\rho-1} p_{Fj}^{1-\rho} + \alpha_{\ell j}^{\rho-1} w_\ell^{1-\rho} \right)^{\frac{1}{1-\rho}}$$

$$\tilde{c}_j = \phi_j^{-1} \left(\alpha_{Dj}^{\rho-1} \tilde{P}_D^{1-\rho} + \alpha_{\ell j}^{\rho-1} w_\ell^{1-\rho} \right)^{\frac{1}{1-\rho}}$$

$$\tilde{P}_D^{1-\sigma} = \sum_j \alpha_{jD}^{\sigma-1} \tilde{c}_j^{1-\sigma}$$

Combining these,

$$\tilde{c}_j^{1-\rho} = \phi_j^{\rho-1}\alpha_{Dj}^{\rho-1}\left(\sum_j \alpha_{jD}^{\sigma-1}\tilde{c}_j^{1-\sigma}\right)^{\frac{1-\rho}{1-\sigma}} + \phi_j^{\rho-1}\alpha_{\ell j}^{\rho-1}w_\ell^{1-\rho}.$$

Thus,

$$
\begin{aligned}
\hat{c}_j^{1-\rho}\,|_{roundabout}^{p_{F.}\to\infty} &= \frac{\phi_j^{\rho-1}\alpha_{Dj}^{\rho-1}\left(\sum_j \alpha_{jD}^{\sigma-1}\tilde{c}_j^{1-\sigma}\right)^{\frac{1-\rho}{1-\sigma}} + \phi_j^{\rho-1}\alpha_{\ell j}^{\rho-1}w_\ell^{1-\rho}}{c_j^{1-\rho}} \\
&= s_{\ell j} + s_{Dj}\frac{\left(\sum_j \alpha_{jD}^{\sigma-1}\tilde{c}_j^{1-\sigma}\right)^{\frac{1-\rho}{1-\sigma}}}{P_D^{1-\rho}} \\
&= s_{\ell j} + s_{Dj}\left(\frac{\sum_j \alpha_{jD}^{\sigma-1}c_j^{1-\sigma}\hat{c}_j^{1-\sigma}\,|_{roundabout}^{p_{F.}\to\infty}}{P_D^{1-\sigma}}\right)^{\frac{1-\rho}{1-\sigma}} \\
&= s_{\ell j} + s_{Dj}\left(\sum_j s_{jD}\hat{c}_j^{1-\sigma}\,|_{roundabout}^{p_{F.}\to\infty}\right)^{\frac{1-\rho}{1-\sigma}}.
\end{aligned}
$$

The solution to this system of equations $\hat{c}_j^{1-\rho}\,|_{roundabout}^{p_{F.}\to\infty}$ is the change in unit costs of each firm, upon autarky.

∎

A.2   Proof of Lemma 1

Using the result that firms sell to other firms are marginal cost and rearranging equation (8), we obtain

$$c_j^{1-\rho} = \sum_k \phi_j^{\rho-1}\alpha_{kj}^{\rho-1}c_k^{1-\rho} + \phi_j^{\rho-1}\alpha_{\ell j}^{\rho-1}w_\ell^{1-\rho}.$$

In matrix form, this equation becomes

$$\boldsymbol{c}^{1-\rho} = \left(I - A'\right)^{-1}\boldsymbol{\phi}^{\rho-1}\circ\boldsymbol{\alpha}_\ell^{\rho-1}\circ w_\ell^{1-\rho},$$

where the (i,j) element of A is $\phi_j^{\rho-1}\alpha_{ij}^{\rho-1}$ if $i \in Z_j$ and 0 otherwise. The assumption that the matrix $\left(I - A'\right)$ is invertible and the fact that under closed economy one can normalize wage w_ℓ, guarantee that there is a unique vector $\boldsymbol{c}$ that solves the equation above. With the cost vector $\boldsymbol{c}$ and constant mark-ups in sales to final consumers, one can compute a unique aggregate price index P according to equation (6). Given the cost vector and aggregate price index, one can then compute a unique aggregate expenditure E from equations (13)

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and (18).

A.3 Proof of Proposition 2

Denote post-shock equilibrium variable x with $\tilde{x}$. From equation (6), we have the expression for the price index after the shock,

$$\tilde{P} = \left(\sum_i \beta_i^{\sigma-1} \mu^{1-\sigma} \tilde{c}_i^{1-\sigma} \right)^{\frac{1}{1-\sigma}}.$$

Combining this expression with the pre-shock price index P, we have

$$\hat{P} = \frac{\tilde{P}}{P}$$

$$= \left(\frac{\sum_i \beta_i^{\sigma-1} \mu^{1-\sigma} \tilde{c}_i^{1-\sigma}}{P^{1-\sigma}} \right)^{\frac{1}{1-\sigma}}$$

$$= \left(\sum_i \frac{\beta_i^{\sigma-1} \mu^{1-\sigma} c_i^{1-\sigma}}{P^{1-\sigma}} \hat{c}_i^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$$

$$= \left(\sum_i s_{iH} \hat{c}_i^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$$

where s_{iH} denotes firm i's share in final consumption. This equation says that the change in the aggregate price index depends on each firm's change in cost and its share in final consumption before the shock.

B   Numerical example: gains from trade

Here we demonstrate that economies with identical sets of aggregate exports, aggregate imports, aggregate gross production and GDP but with different firm-to-firm network structures can potentially generate different gains from trade. We will work with a simple example comparing two economies, where in both economies aggregate exports and imports are €100, aggregate gross production is €300, and GDP is €100.

Both economies consist of two firms. Table 6 lays out the details of the two economies. In economy 1, both firms are identical, and the firm-to-firm network is symmetric in the sense that firms sell the same amount of goods to each other. In economy 2, firm 1 sells €100 to firm 2, while firm 2 does not sell any of its goods to firm 1. There is also asymmetry in sales to household and in exports, where firm 1 sells €10 to households and exports €40, while firm 2 sells €90 to households and exports €60. In the two economies, the aggregate values of imports, exports, gross production and value added are identical.

Table 6: Two economies

| | Economy 1 | | Economy 2 | |
	Firm 1	Firm 2	Firm 1	Firm 2
Imports	50	50	50	50
Exports	50	50	0	100
Gross production	150	150	150	150
Firm-to-firm sales	x_{12}	x_{21}	x_{12}	x_{21}
	50	50	100	0
Labor costs / Value added	50	50	100	0
Sales to households	50	50	50	50

Now let us compute the direct and total shares of foreign inputs for the two firms in two economies. Table 7 summarizes the foreign input shares. The direct shares of foreign inputs for both firms in both economies are 1/3, as they all import €50 while their total inputs are €150. We can also compute the total shares of foreign inputs by solving the system of equations, from equation (14). In economy 2, firm 2 has higher exposure to foreign inputs because not only does the firm import directly, it also relies heavily on goods from firm 1, which also imports directly.

Table 7: Direct and total shares of foreign inputs

| | Economy 1 | | Economy 2 | |
	Firm 1	Firm 2	Firm 1	Firm 2
s_{Fi}	1/3	1/3	1/3	1/3
s_{Fi}^{Total}	1/2	1/2	1/3	5/9

Finally, let us compute the change in aggregate price index from banning imports. Applying equations (15) and (21), we obtain the changes in price index for economy 1 and economy 2:

$$\hat{P}_1 = \left(\frac{1}{2} \left(1 - \frac{1}{2} \right)^{\frac{1-\sigma}{1-\rho}} + \frac{1}{2} \left(1 - \frac{1}{2} \right)^{\frac{1-\sigma}{1-\rho}} \right)^{\frac{1}{1-\sigma}}$$

$$\hat{P}_2 = \left(\frac{1}{2} \left(1 - \frac{1}{3} \right)^{\frac{1-\sigma}{1-\rho}} + \frac{1}{2} \left(1 - \frac{5}{9} \right)^{\frac{1-\sigma}{1-\rho}} \right)^{\frac{1}{1-\sigma}} .$$

If we apply $\rho = 2$ and $\sigma = 4$, then we obtain $\hat{P}_1 = 2$ and $\hat{P}_2 = 1.73$. Even though the aggregate statistics (GDP, gross production, export and import) are identical across the two economies, the gains from trade in economy 1 turns out to be larger. This is because of the asymmetry in firm-to-firm trade where it creates different s_{Fi}^{Total} for the two firms. While in economy 1 the two firms have identical total exposure to foreign inputs, in economy 2, firm 2 has larger exposure. Upon ban of foreign inputs, in economy 2, households will be able to substitute away toward firm 1's goods, resulting in a smaller increase in aggregate price index.

C Data Appendix

C.1 Grouping VAT-identifiers into firms

As mentioned in the main text, all our datasets are recorded at the VAT-identifier level. We utilize ownership filings in the Annual Accounts and information from the Balance of Payments survey in order to aggregate multiple VAT-identifiers into firms. In the ownership filings, each enterprise reports a list of all other enterprises of which it has an ownership share of at least 10% and the value of the share. In the Balance of Payments survey, Belgian enterprises with international financial linkages have to report their stock and flows of financial links. They have to report both the international participation they own and the foreign owners of financial participation in their capital if the participation represents at least 10% of the capital. The survey is designed to cover the population of Belgian enterprises involved in international financial transactions.

We group all VAT-identifiers into firms if they are linked with more than or equal to 50% of ownership. In addition, we group all VAT-identifiers into firms if they share the same foreign parent firm that holds more than or equal to 50% of their shares. We use a "fuzzy string matching" method to determine whether they share the same foreign parent firm, by obtaining similarity measures of all possible pairs of foreign firms' names. Lastly, in order to correct for misreportings, we also add links to the VAT-identifier pairs if the two were linked one year before and one year after. We define a firm as the group of VAT-identifiers that are directly and indirectly linked.

Given these groupings of VAT-identifiers, we then choose the "most representative" VAT-identifier for each firm. We use this "head VAT-identifier" as the identifier of the firm.[29] Then, in order to make the identifiers consistent over time, we make the following adjustment: We take firms whose head VAT-identifier was not an identifier of any firm in the previous year. For such firms, if there exists a VAT-identifier within the firm which was a head VAT-identifier in the previous year, then we switch the firm identifier to that former head VAT-identifier.[30]

Having determined the head VAT-identifier for each firm with multiple VAT-identifiers, we aggregate all the variables up to the firm level. For variables such as total sales and inputs, we adjust the aggregated variables with the amount of B2B trade that occurred

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[29]The criteria for determining the head VAT-identifier is as follows: (i) If there is only one VAT-identifier in the firm that filed all the full annual accounts, the VAT declarations, and the B2B filings, then this VAT-identifier is chosen as the head. (ii) If there are no such VAT-identifiers or multiple of them, then we choose the VAT-identifier that has the largest total assets reported. (iii) If there are no VAT-identifier that filed the annual accounts, then we choose the VAT-identifier that has the largest amount of total inputs, which is the sum of labor costs, B2B inputs, and imports.

[30]If there are multiple such VAT-identifier, then we choose the "most representative" VAT-identifier, using the same criteria as above.

Table 8: Number of VAT-identifier in firms with multiple VAT-identifiers

	Mean	10%	25%	50%	75%	90%	max
Num. VAT-identifier	3	2	2	2	3	4	372

within the firm, correcting for double counting. For other non-numeric variables such as firms' primary sector, we take the value of its head VAT-identifier.

The number of VAT-identifiers for firms with multiple VAT-identifiers are shown in Table 8.

C.2 Firm selection

Table 9 displays the same numbers for Table 1, with statistics for all Belgian firms added.

Table 9: Coverage of all Belgian firms and selected sample

Year	All Belgian Firms					Selected sample				
	Count	V.A.	Sales	Imports	Exports	Count	V.A.	Sales	Imports	Exports
2002	714,469	210	812	204	217	122,460	123	586	179	189
2007	782,006	274	1080	294	282	136,370	157	757	280	269
2012	860,373	300	1244	320	317	139,605	170	829	296	295

Notes: All numbers except for Count are denominated in billion Euro in current prices. Data for Belgian GDP, output, imports and exports are from Eurostat.

C.3 Reporting thresholds of the international trade dataset

There are different reporting thresholds for the international trade dataset, depending on if the trade occurred with an extra-EU country or within the EU. The dataset covers all extra-EU exports and imports by firms with values higher than €1,000 or with weights bigger than 1,000kg. Nevertheless, we also observe values less than €1,000 as more firms use electronic reporting procedures. For intra-EU trade prior to 2006, the dataset covers all exports and imports by firms whose combined imports from intra-EU countries that are more than €250,000 a year. For intra-EU trade from 2006 onward, the thresholds for exports and imports changed to €1,000,000 and €400,000, respectively. Import reporting thresholds became €700,000 per year in 2010. While these reporting threshold for intra-EU trade imply we miss some trade transaction, they are set to capture at least 93% of aggregate Belgian trade in the micro-data, hence our data still contains the overwhelming majority of the value of Belgian trade.

C.4 Mapping CN codes into NACE codes

Our international trade dataset records products in Combined Nomenclature (CN) codes, up to 8 digits. On the other hand, all other datasets that we use record the enterprise's primary sector in NACE Rev.2 code. To concord the two classifications, we convert the CN 8 digit codes into NACE Rev.2 codes. As the first 6 digits of CN codes are identical to the contemporary Harmonized System (HS) codes, we first convert those HS 6-digit codes to Classification of Products by Activity (CPA) codes. We then convert CPA codes to NACE codes, using the fact that CPA 2008 codes are identical to NACE Rev.2 codes up to 4 digits. This conversion allows us to convert more than 98% of all international trade recorded in our dataset, in terms of values (in 2012).

D Descriptive statistics

D.1 Direct and Total foreign input shares

In Figures 1a and 8 we present both the direct and total foreign input shares first for the entire sample of private sector firms in Belgium and then differentiated by major sector.

Figure 8: Histogram of direct and total foreign input share by firms' sector
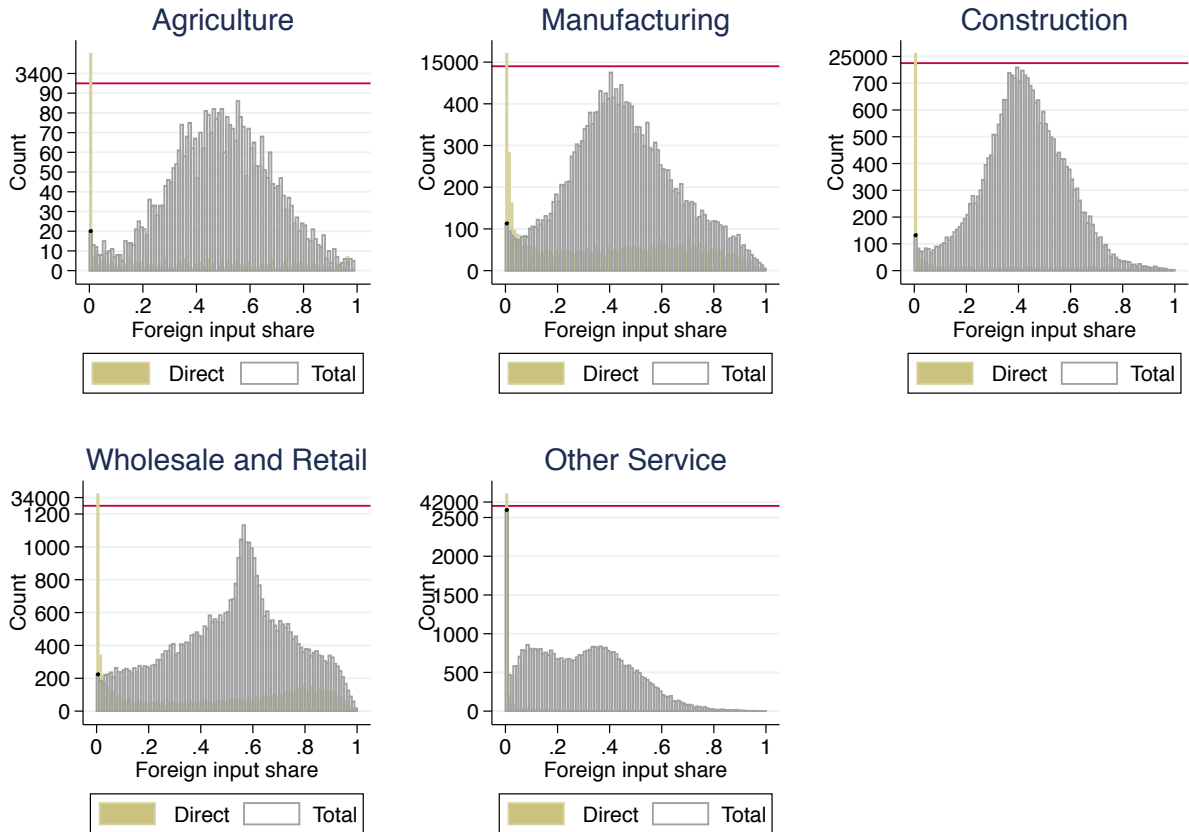
Notes: The black dot indicates the ending of the bar for the total foreign input share. Total foreign input share of firm i, s_{Fi}^{Total} is calculated by solving $s_{Fi}^{Total} = s_{Fi} + \sum_{j \in Z_i} s_{ji} s_{Fj}^{Total}$ where s_{Fi} is i's direct foreign input share, and s_{ji} is j's share among i's inputs. The figure is based on the analysis of 139,605 private sector firms in Belgium in 2012. The horizontal lines represent scale breaks on the vertical axis.

We summarize statistics on the distribution of the the direct and total foreign input share by firm's sector in Table 10.

D.2 Direct and Total export share

In Figures 1b and 9 we present both the direct and total export share for the entire sample of private sector firms in Belgium and then differentiated by major sector.

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Table 10: Distribution of direct and total foreign input share by firms' sector

Sector	Direct			Total		
	Mean	Weighted Mean	Median	Mean	Weighted Mean	Median
Agriculture	0.03	0.27	0	0.49	0.68	0.49
Construction	0.01	0.11	0	0.42	0.48	0.42
Manufacturing	0.11	0.59	0	0.45	0.75	0.44
Wholesale and Retail	0.10	0.42	0	0.52	0.75	0.55
Other Services	0.01	0.18	0	0.29	0.41	0.28
Total	0.05	0.42	0	0.42	0.67	0.41

Notes: The numbers for the weighted mean are calculated using total input purchases of firms as the weight.

Figure 9: Histogram of direct and total export share by firms' sector
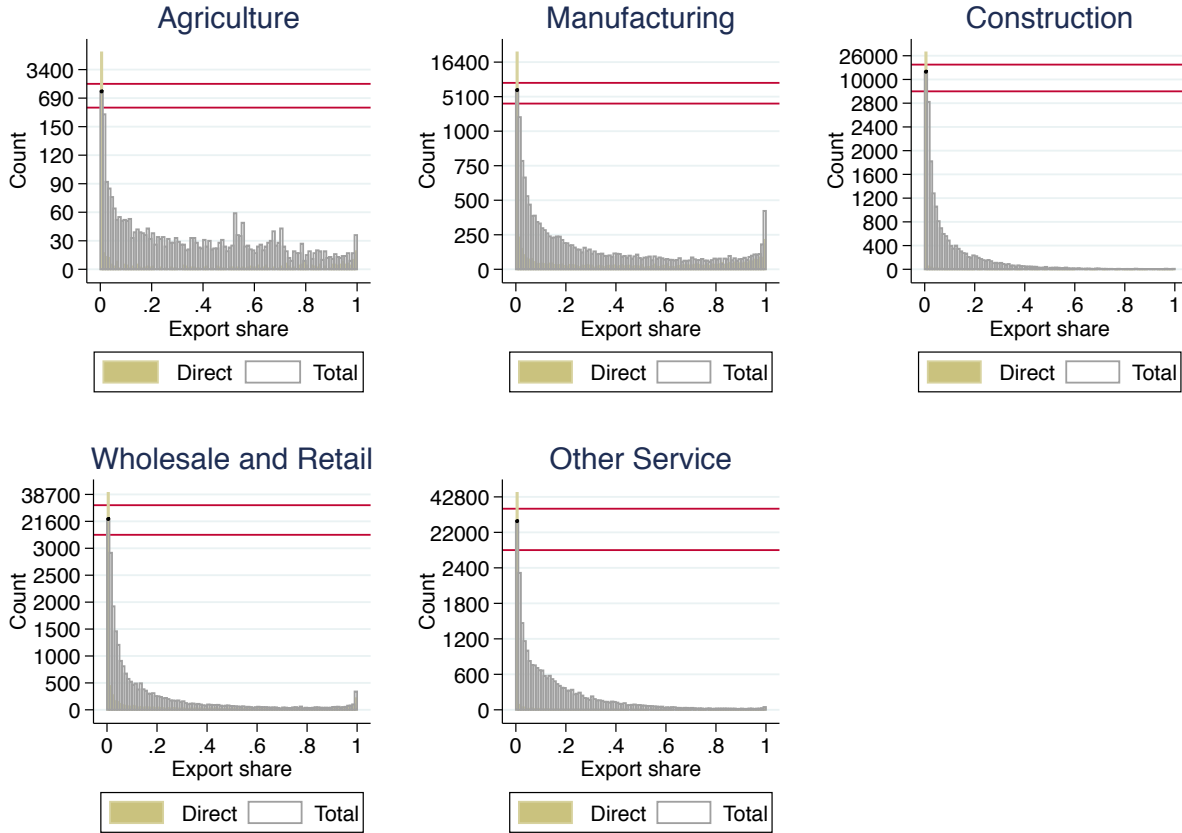
Notes: The black dot indicates the ending of the bar for the total export share. Total export share of firm i, s_{iF}^{Total} is calculated by solving $s_{iF}^{Total} = \tilde{s}_{iF} + \sum_{j \in W_i} \tilde{s}_{ij} s_{jF}^{Total}$ where $\tilde{s}_{iF}$ is i's share of exports out of its output, and $\tilde{s}_{ij}$ is share of i's output that went to firm j. W_i is the set of customers of i. The figure is based on the analysis of 139,605 private sector firms in Belgium in 2012. The horizontal lines represent scale breaks on the vertical axis.

We summarize statistics on the distribution of the the direct and total export share by firm's sector in Table 11.

Table 11: Distribution of direct and total export share by firms' sector

Sector	Direct			Total		
	Mean	Weighted Mean	Median	Mean	Weighted Mean	Median
Agriculture	0.04	0.23	0	0.31	0.43	0.22
Construction	0.00	0.08	0	0.08	0.19	0.02
Manufacturing	0.10	0.60	0	0.25	0.68	0.10
Wholesale and Retail	0.04	0.22	0	0.10	0.30	0.01
Other Services	0.00	0.14	0	0.09	0.26	0.01
Total	0.03	0.36	0	0.12	0.45	0.02

Notes: The numbers for the weighted mean are calculated using total sales of firms as the weight.

D.3   Sectoral composition

Table 12 shows the sectoral composition of our selected sample. Values for value added and output are in billion Euro.

Table 12: Sectoral composition in 2012

Sector	Count	V.A.	Output	Imports	Exports
Agriculture	3,704	1.49	9.97	1.71	2.26
Construction	26,364	18.3	46.5	5.00	3.65
Manufacturing	20,385	55.5	322	147	194
Wholesale and Retail	42,999	31.8	245	85.3	54.5
Other Services	43,495	50.3	125	17.6	17.0
Other	2,658	12.7	80.5	39.8	24.3
Total	139,605	170	829	296	295

D.4   Link survival

E   Transmission of shocks along production chain

E.1   Constructing the exogenous trade shocks

Below, we explain the construction of the variables $\Delta \log M_{it}$, $\Delta \log M_{it}^S$, and $\Delta \log M_{it}^{PS}$.

Table 13: 2002 Link Survival

Occurred In...	Count	Col %	Cum %
2002	3,570,077	65.5	65.5
2002 & 2007	912,028	16.7	82.2
2002 & 2012	191,566	3.5	85.7
2002 & 2007 & 2012	778,734	14.3	100.0

$\Delta \log M_{it}$, an exogenous shock affecting the imports of firm i, is constructed as follows:

$$\Delta \log M_{it} = \log \underbrace{\sum_{k,c} s_{ic,t-1}^{k,M} \text{WES}_{k,c,t}}_{M_{it,t-1}} - \log \underbrace{\sum_{k,c} s_{ic,t-1}^{k,M} \text{WES}_{k,c,t-1}}_{M_{it-1,t-1}},$$

where $s_{ic,t-1}^{k,M}$ is the share of imports of firm i at the initial year $t-1$ that falls on product k from country c, and $\text{WES}_{k,c,t}$ is the world export supply (excluding sales to Belgium) of country c for product k.

The import supply shock that i's suppliers received, $\Delta \log M_{it}^S$, is constructed as:

$$\Delta \log M_{it}^S = \log \sum_k s_{ki,t-1} M_{kt,t-1} - \log \sum_k s_{ki,t-1} M_{kt-1,t-1}.$$

Finally, the import supply shock to potential customers of firm i is:

$$\Delta \log M_{it}^{PS} = \log \sum_u s_{ui,t} M_{ut,t-1}^{-i} - \log \sum_u s_{ui,t-1} M_{ut-1,t-1}^{-i}.$$

Where the variables on the RHS are constructed analogous to footnotes 10 and 11.[31] [32]

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[31]The input share from sector u for firm i, $s_{iu,t}$, is defined as the share of inputs of i that came from firms producing sector u goods:

$$s_{uit} = \sum_{j \in Z_{it}^u} \frac{\text{Sales}_{jit}}{\text{TotalInputs}_{it}},$$

where Z_{it}^u denotes the set of suppliers of i producing sector u goods at time t.

[32]

$$M_{ut,t-1}^{-i} = \sum_{j \in U_{t-1}, j \neq i} \frac{V_{jHt-1}}{\sum_{k \in U_{t-1}, k \neq i} V_{kHt-1}} M_{jt,t-1}$$

$$M_{ut-1,t-1}^{-i} = \sum_{j \in U_{t-1}, j \neq i} \frac{V_{jHt-1}}{\sum_{k \in U_{t-1}, k \neq i} V_{kHt-1}} M_{jt-1,t-1},$$

where U_t is the set of firms producing sector u good at t, and V_{iHt} is firm i's sales to domestic final demand

E.2 Other reduced form results

Here we report the reduced form results where firms' changes in domestic sales and domestic inputs are on the LHS variable. The first column shows the results for firms' changes in totals are on the LHS, and is identical to the third column in Table 2.

Controlling for own shocks and shocks that potential customers and suppliers have received, both positive demand shock on a firm's actual customers and actual suppliers lead to an increase of the firm's domestic sales and domestic inputs.

Table 14: Additional reduced form results

	$\Delta \ln$ Total Sales	$\Delta \ln$ Domestic Sales	$\Delta \ln$ Domestic Inputs
$\Delta \ln X_{it}$	0.089***	0.021	0.073***
	(0.009)	(0.013)	(0.010)
$\Delta \ln M_{it}$	0.156***	0.105***	0.093***
	(0.012)	(0.018)	(0.014)
$\Delta \ln X_{it}^{PC}$	0.025***	0.037***	0.024***
	(0.003)	(0.005)	(0.004)
$\Delta \ln M_{it}^{PS}$	0.039***	0.015**	0.085***
	(0.005)	(0.007)	(0.006)
$\Delta \ln X_{it}^{C}$	0.122***	0.127***	0.086***
	(0.013)	(0.022)	(0.016)
$\Delta \ln M_{it}^{S}$	0.041***	0.106***	0.078***
	(0.018)	(0.027)	(0.022)
N	87100	85795	87363

Notes: Standard errors are clustered at the firm level. All variables are in terms of yearly log differences from 2002 to 2012. All specifications include year fixed effects. We truncate outliers of each variables at the top and bottom 1% level. Firms' domestic sales are the sum of their sales to other domestic firms, and sales to domestic final demand. Firms' domestic inputs are the sum of their labor costs and input purchases from other domestic firms.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

---

at t.

F   Ordering algorithm

In this section we describe the implementation of the ordering algorithm to solve the feedback arc set problem. We begin by defining some terms and notation.

F.1   Terms and notation

- *graph / network*, $G = (V, E)$ - A collection of a set of edges E and set of vertices V. Edges describe the relationship between vertices. Two basic classifications of graphs are based on whether the edges are *directed* or *undirected* and whether they are *weighted* or *unweighted*

- $n = |V|$, $m = |E|$

- *cycle* - A path within a graph where a vertex is reachable from itself

- $d^+(u)$ - For a vertex $u \in V$ in a directed graph, number of outgoing edges

- $d^-(u)$ - For a vertex $u \in V$ in a directed graph, number of incoming edges

- $w^+(u)$ - For a vertex $u \in V$ in a directed graph, cumulative sum of weights of outgoing edges

- $w^-(u)$ - For a vertex $u \in V$ in a directed graph, cumulative sum of weights of incoming edges

- *sink* - A vertex $u \in V$ in a directed graph with $d^+(u) = 0$

- *source* - A vertex $u \in V$ in a directed graph with $d^-(u) = 0$

- *feedback arc set* - A set of edges from a directed cyclic graph that when removed make the graph acyclic

- $s = s_{left}s_{right}$ - Given 2 finite sequences s_{left} and s_{right} with the indicated notation we symbolize the *concatenation* operation. For example, if $s_{left} = (A, B, C)$ and $s_{right} = (X, Y, Z)$, then $s = s_{left}s_{right} = (A, B, C, X, Y, Z)$

- $\lfloor x \rfloor$ is the greatest integer less than or equal to x

F.2　Overview

The Belgian B2B data describes a weighted directed graph $G = (V, E)$. Vertices are firms and edges are sales between firms. The goal of the ordering algorithm is to order firms in a way such that a given firm only sells to firms further along in the ordering and only buys from firms that precede it. The condition desired by this ordering is known in graph theory as a *topological ordering* (Black, 1999). A topological ordering exists if and only if a graph is directed and acyclic. The B2B data is cyclic. For the *unweighted* case our motivation is to find a feedback arc set of minimal cardinality, that is, what is the minimum number of transactions that we need to drop (i.e., the "violators") from our network to satisfy our ordering condition? For the *weighted case*, we seek to find a feedback arc set such that the cumulative weight of the violating transactions is minimized. Finding a minimum feedback arc set is computationally difficult but approximation algorithms exist.

F.3　Unweighted case

The algorithm we use for the paper was first presented by Eades et al. (1993). This algorithm was chosen because it has a linear run time complexity, $O(m + n)$, and because of its relative implementation simplicity. The algorithm uses a greedy heuristic through which it builds the proposed ordering $s = s_{left}s_{right}$.[33] Vertices are initialized into several buckets: sinks, sources, and δ buckets, where for a vertex $u \in V$, $\delta(u) = d^-(u) - d^+(u)$.[34] At each iteration, the algorithm removes all sinks from the network and prepends them to a sequence s_{right}, removes all sources and appends them to a sequence s_{left}, and then removes the vertex with the lowest δ score (the most "source"-like vertex) and appends it to s_{left}.[35] Each removal requires updating the buckets to reflect the modified graph. The algorithm stops when the graph is empty. There will be $2n - 1$ buckets, which can be formalized as follows:[36]

---

[33]According to Black (2005), a greedy algorithm is "an algorithm that always takes the best immediate, or local, solution while finding an answer. Greedy algorithms find the overall, or globally, optimal solution for some optimization problems, but may find less-than-optimal solutions for some instances of other problems."

[34]We have flipped the sign here compared to Eades et al. to be consistent with the diagrams elsewhere in our paper.

[35]Eades et al. (1993) take the vertex with the maximum δ score.

[36]Eades et al. (1993) assume that the graph G is simple (no bidirectional edges), and hence their original algorithm only requires $2n - 3$ buckets.

$$V_{-n+1} = V_{sources} = \{u \in V \mid d^-(u) = 0; \ d^+(u) > 0\}$$
$$V_{n-1} = V_{sinks} = \{u \in V \mid d^-(u) > 0; \ d^+(u) = 0\}$$
$$V_d = \{u \in V \mid d = \delta(u); \ d^-(u) > 0; \ d^+(u) > 0\}$$

The bucket V_{-n+1} contains all the vertices that are only the sources of edges. The bucket V_{n-1} contains all the vertices that are only the sinks of edges (in other words, vertices that are only receiving edges). Each V_d bucket contains vertices with d net incoming edges (conditional on these vertices having both outgoing and incoming edges).

F.4 Example execution on unweighted network

Consider the following network:
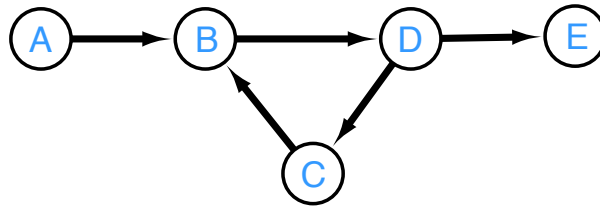
Let's trace the execution of the algorithm described by Eades et al.

F.4.1 Initialization

Buckets:

A			D	C	B			E
sources	-3	-2	-1	0	1	2	3	*sinks*

Ordering : $s = s_{left} = s_{right} = ()$

F.4.2 First iteration:

Remove sinks
Updated buckets:

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sources	−3	−2	−1	0	1	2	3	sinks
A				C, D	B			

Updated ordering : $s_{left} = ()$, $s_{right} = (E)$, $s = s_{left}s_{right} = (E)$

Remove sources
Updated buckets:

sources	−3	−2	−1	0	1	2	3	sinks
				C, D, B				

Updated ordering : $s_{left} = (A)$, $s_{right} = (E)$, $s = s_{left}s_{right} = (A, E)$
Remove vertex with lowest delta score
Updated buckets:

sources	−3	−2	−1	0	1	2	3	sinks
B								D

Updated ordering : $s_{left} = (A, C)$, $s_{right} = (E)$, $s = s_{left}s_{right} = (A, C, E)$
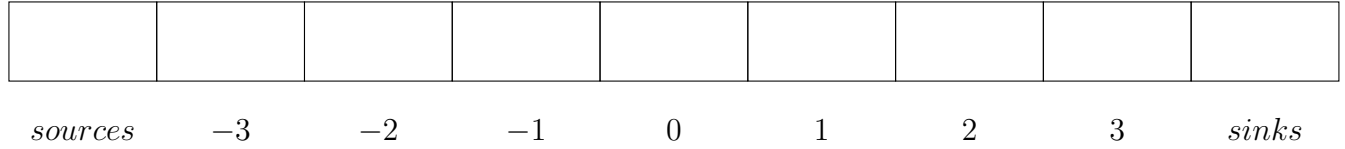
F.4.3 Second iteration

Remove sinks
Updated buckets:

sources	−3	−2	−1	0	1	2	3	sinks
B								

Updated ordering : $s_{left} = (A, C)$, $s_{right} = (D, E)$, $s = s_{left}s_{right} = (A, C, D, E)$

Remove sources
Updated buckets:

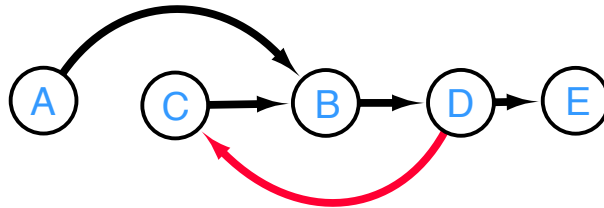*sources* -3 -2 -1 0 1 2 3 *sinks*

Updated ordering : $s_{left} = (A, C, B)$, $s_{right} = (D, E)$, $s = s_{left}s_{right} = (A, C, B, D, E)$

F.4.4   Final output

Ordering : $s = s_{left}s_{right} = (A, C, B, D, E)$, *Violator edge set*: $\{(D, C)\}$

F.5   Weighted case

Simpson et al. (2016) have proposed a modification to adapt the Eades et al. (1993) algorithm to solve the weighted problem. The required changes are:

1. In the initialization step, all edge weights need to be normalized to be between 0 and 1.

2. $\delta(u)$ is redefined as $\delta(u) = \lfloor w^-(u) - w^+(u) \rfloor$.

The key motivation behind these steps is to reformat the network so that the unweighted version of the algorithm could be used in an identical fashion as before, specifically without increasing the number of buckets. Without the floor in step 2, for any given network the number of buckets could become large.

G Algorithm to solve for the firm's sourcing strategy and export participation

A firm is solving the problem described in (25), where profits are defined in equation (24) and variable profits are defined in equation (13). For convenience, we re-state the problem of firm j here:

$$\max_{Z_j, I_{jF}} \pi_j(Z_j, I_{jF}) \quad \text{s.t.} \quad Z_j \subseteq \mathbf{Z}_j , \ I_{jF} \in \{0, 1\},$$

where

$$
\begin{aligned}
\pi_j(Z_j, I_{jF}) = & \frac{1}{\sigma} \beta_{jH}^{\sigma-1} \mu^{1-\sigma} \phi_j^{\sigma-1} \Theta_j(Z_j)^{(\sigma-1)/(\rho-1)} \frac{E}{P^{1-\sigma}} \\
& + I_{jF} \frac{1}{\sigma} \beta_{jF}^{\sigma-1} \mu^{1-\sigma} \phi_j^{\sigma-1} \Theta_j(Z_j)^{(\sigma-1)/(\rho-1)} \tau^{1-\sigma} \frac{E_F}{P_F^{1-\sigma}} \\
& - \sum_{k \in Z_j} f_{kj} w_\ell - I_{jF} f_{jF} w_\ell. \\
= & \pi_j^{var}(Z_j, I_{jF}) - \sum_{k \in Z_j} f_{kj} w_\ell - I_{jF} f_{jF} w_\ell
\end{aligned}
$$

In words, the firm is choosing its sourcing strategy, Z_j, and export participation, I_{jF}. We solve the firm's problem of choosing its sourcing strategy separately for $I_{jF} = 0$ and $I_{jF} = 1$. We then calculate the profits for these two cases and determine the firm is an exporter if and only if the profits under exporting are higher than under non-exporting.

Below we describe how we solve for the firm's optimal sourcing strategy for a given export participation choice.

G.1 Lower and upper bounds for the optimal sourcing strategy

We determine the lower and upper bound for the firm's sourcing strategy following the procedure in Jia (2008) and Antras et al. (2017).

G.1.1 Lower bound

We start from a guess of no sourcing from any other domestic supplier and no importing, $S_l^{(0)}$. We then check supplier by supplier whether the benefit of adding a supplier (given the current guess of not purchasing from any supplier) is positive. At iteration t, starting from $S_l^{(t)}$, we calculate the marginal benefit of adding supplier $k \notin S_l^{(t)}$, $k \in \mathbf{Z}_j$:

$$\pi_j^{var}(S_l^{(t)} \cup k, I_{jF}) - \pi_j^{var}(S_l^{(t)}, I_{jF}) - f_{kj} w_\ell.$$

If the marginal benefit to include supplier k is positive, in the next iteration we include supplier k in the guess for the sourcing strategy of firm j. Note that given $\sigma > \rho$, one is the least likely to determine the benefit of a supplier is positive when the current guess is no supplier. Hence if it is possible to include a supplier in this iteration, in all the next iterations the marginal benefit from this supplier will be positive as well.

Starting from $S_l^{(t)}$, we consider firm-by-firm if trading with a firm not contained in $S_l^{(t)}$ brings positive marginal benefit (i.e., the additional variable profits under this sourcing strategy exceed the additional fixed cost) or not. Then, we add all those firms which bring positive benefit to form $S_l^{(t+1)}$.

The process ends when $S_l^{(t)} = S_l^{(t+1)}$ or all eligible suppliers are in $S_l^{(t)}$ already. When the process ends (i.e., $S_l^{(t)} = S_l^{(t+1)}$), we denote the lower bound of the sourcing strategy for firm j as $S_l^* = S_l^{(t)} = S_l^{(t+1)}$.

G.1.2 Upper bound

To determine the upper bound we start from a guess of purchasing from every supplier (incl. foreign), $S_u^{(0)}$. We then check supplier-by-supplier whether the marginal benefit from dropping the supplier is positive. At iteration t, starting from $S_u^{(t)}$, we calculate the marginal benefit of dropping supplier $k \in S_u^{(t)}$ as:

$$\pi_j^{var}(S_l^{(t)} \setminus k, I_{jF}) - \pi_j^{var}(S_l^{(t)}, I_{jF}) + f_{kj} w_\ell.$$

The remainder of the procedure is very similar to the iteration for the lower bound but starting from the opposite direction (i.e., we drop from the next iteration $S_u^{(t+1)}$ all those suppliers for which the marginal benefit of dropping is positive). The ending criteria is the same. We denote the upper bound for the sourcing strategy as S_u^*.

G.2 From lower and upper bounds to optimal sourcing strategy

Once we obtain S_u^* and S_l^*, we consider 3 alternative cases. Let $D = \{x \in S_u^* \mid x \notin S_l^*\}$ denote the set with the elements that are in the upper bound but not in the lower bound for the sourcing strategy.

G.2.1 $S_u^* = S_l^*$

If the upper and lower bounds for the sourcing strategy are the same, then we have obtained the optimal sourcing strategy for the firm (for a given exporting choice).

G.2.2 S_u^* is close to S_l^*

When the cardinality of set D is less than or equal to 15, we consider S_u^* to be close to S_l^*.

In that case we evaluate the profits at all possible combinations of sourcing strategies in between S_u^* and S_l^*, including S_u^* and S_l^* themselves. We choose the combination that yields the highest total profit as the optimal sourcing strategy for the firm.

G.2.3 S_u^* is far from S_l^*

When the cardinality of set D is larger than 15, then evaluating the profits at all combinations of feasible sourcing strategies in between the two bounds would be too computationally intensive. For that case, we have developed the following greedy algorithm to determine the firm's sourcing strategy:

Starting from S_l^*, we calculate the marginal benefit from adding separately each supplier in D to the sourcing strategy S_l^*. Note that by construction the marginal benefit from adding each of these suppliers individually to S_l^* is negative (otherwise the algorithm in Section G.1 would have already added those suppliers to the lower bound). We order the suppliers in D by their marginal benefit of being added to S_l^*. If the cardinality of D is K, we consider $K - 1$ alternative sourcing strategies. We first add the top 2 suppliers in D (those with the highest marginal benefit of being added evaluated at S_l^*) to S_l^*, then add the top3 suppliers to S_l^*, and so forth. Hence, we calculate the profits for $K - 1$ alternative sourcing strategies.

In addition, we also follow a similar process starting from S_u^* and calculate the marginal benefit from dropping separately each supplier in D from the sourcing strategy S_u^*. Again, by construction, the benefit from dropping each of the suppliers individually is negative. We order the suppliers in D by their marginal benefit of being dropped from S_u^*. We then consider $K - 1$ alternative sourcing strategies, in which $2, 3, ..., K$ suppliers are removed from S_u^* (following the ranking of their marginal benefit of dropping individually at S_u^*).

Then, out of these $2K - 2$ sourcing strategies we choose the one with the highest total profit for the firm.

Note that, using the approach in Section G.2.2, the number of sourcing strategies we would need to calculate profits for would be 2^K (growing exponentially in K). The greedy algorithm developed here, requires evaluations of alternative sourcing strategies that grow

linearly in K, making it feasible even in the rare case that the difference between S_u^* and S_l^* is large.

We present statistics on the cardinality of the differences in the bounds in Table 15.

G.3   Statistics on the algorithm

Table 15: Cardinality of differences in the upper and lower bounds

Number of firm draws		Percent of cases in which	
× parameter iterations	Bounds are	Differences in bounds	Differences in bounds
	perfectly overlapping	≤ 15	> 15
51,029,600,000	98.95	0.91	0.14

Notes: During the estimation we have to solve for each firm and parameter guess the firm's optimal sourcing strategy and exporting choice. This table presents aggregate statistics on the cardinality of the differences in the upper and lower bounds for the sourcing strategy summing over the outcomes for each firm, parameter guess, and exporting choice.

H Algorithm for network formation

Below we describe the algorithm to solve for the network formation in three contexts: In Section H.1, we describe the algorithm to solve for the network formation and equilibrium for a given set of parameters. In Section H.2, we describe the algorithm to estimate the parameters of the model. In Section H.3, we describe the algorithm to solve for network formation and equilibrium in a closed economy.

H.1 Network formation given parameters

Given a set of parameters, size of the labor force, price of foreign goods, and foreign demand, we follow the steps below to simulate the network formation.

1. Firms with productivities ϕ_i are randomly sorted, and indexed with $i = 1, 2, 3, \cdots$. A firm's index determines the firm's set of eligible suppliers, $\mathbf{Z}_i$. The set of eligible suppliers is such that all feasible networks will be acyclic. Firms' draws of firm-pair-specific fixed cost of sourcing, fixed cost of importing and exporting, export demand, and benefits of importing, and firm-pair-specific cost shifters are also known at this point.

2a. All firms make a common guess of the wage level w_ℓ.

2b. All firms make a common guess of aggregate demand term: $A = EP^{\sigma-1}$.

3. We assume that firms decide on their sourcing strategies in sequence of i. Firm 1 decides its sourcing strategy and determines c_1, then firm 2 decides its sourcing strategy and determines c_2, and so on. When firms make their sourcing decisions, we assume that all firms are able to use labor and foreign inputs, but firm i is only able to choose its suppliers from its eligible supplier set $\mathbf{Z}_i$. We determine which suppliers among $\mathbf{Z}_i$ firm i sources from, using the algorithm described in Section G, and compute c_i. After the final firm $i = N$ decides its sourcing strategy, the whole vector $\mathbf{c}$ and the supplier sets of all firms are determined. At this point we have also solved for the firm's optimal export participation choice and export sales.

4. Given the network, the guesses for A and w, we are able to compute the equilibrium variables.

   (a) Sales to domestic final demand of firm i is computed as $X_{iH} = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} c_i^{1-\sigma} A$ and to foreign final demand is computed as $X_{iF} = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} c_i^{1-\sigma} \beta_{kF}^{\sigma-1} \frac{E_F}{P_F^{1-\sigma}}$.

   (b) The cost of inputs used for firm i's sales to domestic final demand is thus $C_{iH} = \frac{\sigma-1}{\sigma} X_{iH}$ and to foreign final demand is $C_{iF} = \frac{\sigma-1}{\sigma} X_{iF}$.

(c) The total input costs of firms, C_i are calculated by solving the system of linear equations below:

$$C_i = C_{iH} + C_{iF} + \sum_j s_{ij} C_j,$$

$$\rightarrow \mathbf{C} = (I - S)^{-1} (\mathbf{C}_H + \mathbf{C}_F)$$

where $\mathbf{C}$, $\mathbf{C}_H$, and $\mathbf{C}_F$ are vectors of C_i, C_{iH}, and C_{iF}, respectively, and the i, j element of matrix S is s_{ij}.

(d) The total sales of firm i is then $X_i = X_{iH} + X_{iF} + C_i - C_{iH}$.

(e) Firm profits and total expenditure on fixed costs.

5. We solve for equilibrium variables of A and w_ℓ in the following way: In the outer loop, we solve for wages such that the labor market clearing condition (26) is solved. In the inner loop, we iterate over steps 2b-4, such that a fixed point for the market demand level, A, is found.

H.2 Parameter estimation and network formation

One possible approach to estimating the parameters of the model is to simulate the model for each parameter guess according to the algorithm outlined in Section H.1, calculate the objective function in equation (29), and vary the parameter guesses to maximize the objective function. However, this requires for each parameter guess finding a fixed point in both the market demand, A, and a wage level, w_ℓ. Below, we describe a more computationally attractive algorithm to estimate the model.

Throughout the estimation, we set the domestic wage, $w_\ell = 1$, as well as the domestic market demand, $A = 1$. We ensure labor market clearing condition and the fixed point in market demand in the following way:

1. Of the 8 parameters to estimate, the mean foreign demand parameter is implicitly pinned down to take the value that satisfies the trade balance condition.

2. Instead of treating the size of the labor force as data (note that the units are arbitrary), we choose its level by setting: $L = \frac{AP^{1-\sigma} - \sum_i \pi_i}{w_\ell}$.

Note that $A = w_\ell = 1$. Under this level of the size of the labor force, $L = \frac{AP^{1-\sigma} - \sum_i \pi_i}{w_\ell}$, the fixed point for the market demand, A is automatically satisfied. Also, since the trade balance holds, the domestic labor market clears as well.

Given a parameter guess for $\hat{\Phi}^{\alpha_{\text{dom}}}_{scale}$, $\hat{\Phi}^{\alpha_F}_{scale}$, $\hat{\Phi}^{\alpha,\beta}_{disp}$, $\hat{\Phi}^{f_{\text{dom}}}_{scale}$, $\hat{\Phi}^{f_{\text{imp}}}_{scale}$, $\hat{\Phi}^{f_{\text{exp}}}_{scale}$, and $\hat{\Phi}^{f}_{disp}$, we vary $\hat{\Phi}^{\beta_F}_{scale}$ and go through steps 3 and 4 in Section H.1 to calculate the aggregate trade

balance. Given the other parameters, the level of $\hat{\Phi}_{scale}^{\beta_F}$ is pinned down implicitly such that aggregate trade balance is equal to zero. Hence, instead of search for a fixed point in both A and w_ℓ, we hold those fixed throughout the estimation and only need to find one fixed point in $\hat{\Phi}_{scale}^{\beta_F}$, that satisfies trade balance, for each guess of the other seven parameters.

H.3   Network formation in the closed economy

In the closed economy, we can normalize the domestic wage $w_\ell = 1$. We therefore only have to follow steps 2b-4 in Section H.1 to pin down the level of domestic market demand, A, in the closed economy.

I Sensitivity results under exogenous network

Figure 10: Distributions of $\log \hat{c}$ from banning imports, different parameter values
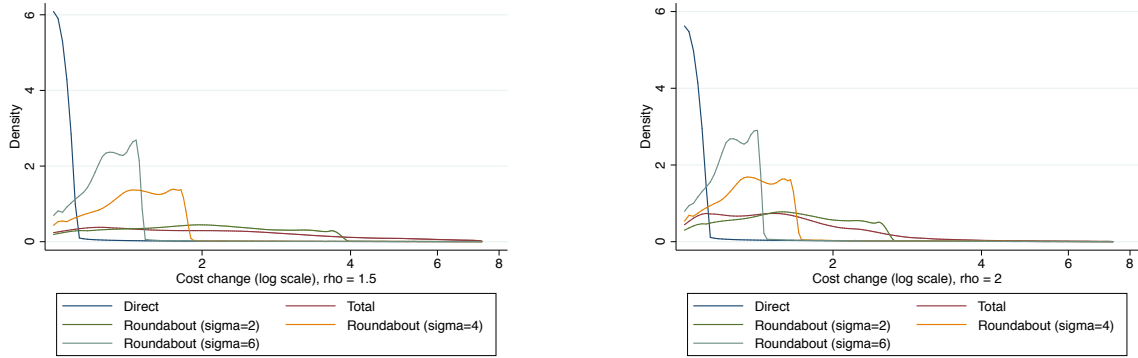

Table 16 shows the median value of cost change $\hat{c}_i$ under different parameter values.

Table 16: Median $\hat{c}_i$ under different values of σ and ρ

| | Direct | Total | Roundabout | | |
			$\sigma = 2$	$\sigma = 4$	$\sigma = 6$
$\rho = 1.5$	1	2.89	2.37	1.53	1.33
$\rho = 2$	1	1.70	1.78	1.41	1.28

Table 17: 90th percentile $\hat{c}_i$ under different values of σ and ρ

| | Direct | Total | Roundabout | | |
			$\sigma = 2$	$\sigma = 4$	$\sigma = 6$
$\rho = 1.5$	1.08	11.93	3.76	1.85	1.50
$\rho = 2$	1.04	3.45	2.52	1.67	1.42

Table 18: Change in price index $\hat{P}$ under different values of σ and ρ

ρ	σ	$\hat{P}\mid_{direct}$	$\hat{P}\mid_{total}$	$\hat{P}\mid_{roundabout}$
1.5	2	1.71	3.84	3.80
1.5	4	1.28	2.25	1.85
1.5	6	1.17	1.83	1.50
2	2	1.48	2.35	2.56
2	4	1.23	1.77	1.67
2	6	1.15	1.56	1.43

Table 19: Change in price index $\hat{P}$ under acyclic network

ρ	σ	$\hat{P}\mid_{total}$	$\hat{P}\mid_{total}$, acyclic
1.5	2	3.84	3.93
1.5	4	2.25	2.28
1.5	6	1.83	1.84
2	2	2.35	2.37
2	4	1.77	1.78
2	6	1.56	1.57

Notes: The fourth column shows the change in price index from banning imports when taking into account the acyclic network structure. We obtain the acyclic network from the algorithm explained in Appendix F, for the weighted case where we minimize the value of violating transactions.