

Technology, Competition, and Multi-Product Firms*

Akira Sasahara[†]
Hitotsubashi University

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Abstract

This paper examines the impacts of trade on firms' behavior and welfare by introducing multi-product firms into Krugman's (1979) homogeneous firm model. In a world consisting of two otherwise identical countries differing only in technology level of their firms, the impact of bilateral and unilateral trade liberalization is considered. It is shown that, in a high-tech country, bilateral trade liberalization induces firms to drop less efficient products from the domestic market and start exporting most efficient products. On the other hand, in a low-tech country, it is suggested that firms' within-firm-adjustment of product mix is less flexible. Moreover, it is shown that unilateral trade liberalization hurts the liberalizing country but benefits its trading partner. However, it is suggested that the result is not valid once heterogeneity across firms is introduced.

Key Words: Multi-product firms; Technology; Trade liberalization; New economic geography

JEL codes: F12, F13, F15, L11

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[†]Corresponding Address: Graduate School of Economics, Hitotsubashi University, 2-1 Naka, Kunitachi, Tokyo, Japan 186-8601.
e-mail: akirasasahara@gmail.com

1 Introduction

A considerable development in international theory during last twenty years has added microeconomic foundation on the theory of international trade. Krugman (1979, 1980, 1991) illustrates trade between nations that share same factor endowment and technology by incorporating increasing returns to scale. Melitz's (2003) seminal work make it possible to explain how aggregate trade flow constitutes of exports by individual firms. Besides, a recent growing body of literature in trade theory focuses on how aggregate export of each firm constitutes of exports of individual products within firms.¹ They incorporate multi-product firms and examine how tougher competition due to trade affects firms' optimal product-scope, the amount of production and exports of individual products within firms. However, due to the complexity of the model on multi-product firms, most of the papers fail to capture how a bilateral or unilateral reduction of trade costs between asymmetric countries affect individual firms' behavior and welfare.

This paper succeeds in introducing multi-product firms into a model of new economic geography and examines how progressive trade liberalization affect firms' product mix, location decision, price index, and welfare. Most of literature on multi-product firms introduces two-dimensional firm heterogeneity into their model. For example, Bernard et al. (2010) embed heterogeneous firm' ability and product attributes, and numerous models incorporate different marginal costs across products within firms and firm heterogeneity (e.g., Nocke and Yeaple, 2008; Arkolakis and Muendler, 2009; Mayer et al., 2010). However, they fail to capture the mechanism that the interaction of asymmetric countries induced by falling trade costs affects the behavior of multi-product firms. On the other hand, a simple homogeneous firm model developed in this paper makes it possible to analyze the impacts of the asymmetric trade liberalization between asymmetric countries.

Related Literature

This paper contributes to three strands of literature. First, it relates to existing theoretical research on multi-product firms in international trade literature. Bernard et al. (2010), Eckel and Neary (2010) show that trade liberalization induces firms to drop less productive products and makes them 'leaner and meager.' These work emphasis new gains from trade that is attained by reallocating resources from less productive products to productive products within firms. Regarding the within-firm product selection, Arkolakis and Muendler (2009) and Mayer et al. (2008) examines the impact of competition in the export markets on the product mix and show that only the most productive products are exported to competitive markets. Chatterjee et al. (2011) examines the effect of exchange rate shocks on the markups of products produced by multi-product firms. And Feenstra and Ma (2010), and Agur (2010) theoretically suggests variety proliferation due to trade by introducing multi-product firms even though the result is cannot obtained in a single-product firm models.²

¹Eckel and Neary (2010) call the within-firm adjustments in the range of goods produced by multi-product firms as "intra-firm extensive margin."

²Baldwin and Forslid (2010) and Arkolakis et al. (2009) show that the mass of available variety for a consumer decreases due to trade when fixed export cost is greater than the fixed cost for domestic operation.

Second, this work contributes to a literature on new economic geography. Ottaviano (2011) argues that future research on new economic geography should look more deeply into finer micro-heterogeneity across people and firms. Heterogeneous firms are introduced in new economic geography models by Baldwin and Okubo (2006, 2009), Okubo et al. (2008), and Okubo (2009). Even though this paper introduces homogeneous firms, they produce a continuum of vertically differentiated products. Therefore, in the sense that each firm produces heterogeneous products, this paper offers an alternative way to add heterogeneity across products to a new economic geography model.

Finally, this paper relates to a literature on the role of technology in international trade. The traditional Ricardian model explained how a technology difference causes trade between nations. Lately, Eaton and Kortum (2002) embed technology difference across countries and investigate the impact of trade on welfare and labor reallocation. Furusawa (2011) identifies a flying geese type technology development using a North-South trade model. The model developed in this paper makes it possible to introduce technology difference across countries, and examine how firms in high-tech and low-tech countries respond to progressive trade liberalization.

Model description and results

The model embodies a representative consumer that has a CES preference and two sectors: the monopolistically competitive differentiated goods sector and the perfectly competitive homogeneous goods sector. Contrary to the traditional models such as Krugman (1980), each firm produces a mass of vertically differentiated products. The mass of products produced by each firm is affected by a degree of competition that is represented in the price index. The tougher competition, a decrease in price index, reduces the mass of products produced by each firm.

In a world consisting two symmetric countries, it is shown that a bilateral reduction of trade costs decreases the CES price index in both countries since it bring a tougher competition. The fierce competition induces firms in each country continuously drop less efficient products from the domestic market and start exporting most efficient products to other country. Moreover, welfare that is defined by the inverse of CES price index increases due to the bilateral reduction of trade costs.

On the other hand, in a world consisting two countries differing only the technology level of their firms, it is shown that a bilateral reduction of trade costs continuously decreases the CES price index in the high-tech country, which induces firms in the high-tech country to continuously drop less efficient products from the domestic market and continuously add most efficient products as export products. However, interestingly, the behavior of low-tech country's variables is non-monotonic if the technology gap is sufficiently large. In the low-tech country, the CES price index (welfare) firstly decreases (increases) because of a reduction of trade cost, but after further liberalization, it increases (decreases). This non-monotonicity of the price index bring a non-monotonic behavior of firms in the low-tech country: firstly, they drop less efficient domestic products and start exporting most efficient products, but

after further liberalization, they start adding and dropping previously dropped domestic and previously added exporting products, respectively. These results can explain existing empirical evidences that firms in U.S. and Japan (that are high-tech countries) significantly reduces the number of domestic products after the tariff reduction (see Bernard et al., 2010; Kawakami and Miyagawa, 2010), and firms in India (that would be a low-tech country) do not reduce the number of domestic products after the reduction of tariff at statistically significant level (see Goldberg et al., 2010).³

In addition, surprisingly, it is shown that mass of operating firms in each country does not depend on trade cost. Therefore, in equilibrium, the mass of firms doesn't change due to trade liberalization. All the effects that affect the mass of firms are completely absorbed by the within-firm-adjustment of product mix.

The tractableness of the model makes it possible to examine the impact of a unilateral reduction of trade cost on firms' product mix and welfare. One might think that a unilateral reduction of trade costs by a country bring a tougher competition to the country that unilaterally reduce its trade cost. However, it is not the case in this general equilibrium model. Suppose that country 1 reduces its trade cost but country 2 does not. The unilateral reduction of trade cost by country 1 make country 2 better export base, which offer an incentive firms to agglomerate country 2. As a result of tougher competition induced by agglomeration, country 2 gains from trade. Firms in country 2 drop their domestic products and increase the mass of exported products. Firms in country 1 add domestic products and reduce mass of exported products. The unilateral reduction of trade cost reduces welfare of the liberalizing country but its trading partner experiences welfare gain. This result is similar to previous work such as Melitz and Ottaviano (2008).

These effects of a unilateral reduction of trade cost are crucially depends on the setting that firms can instantly enter to, and exit from the market. A simple way to introduce the friction that makes firms to be slow to move across countries is making firms heterogeneous in productivity. In heterogeneous firm model, firms do not have an incentive move across countries because they earn strictly positive profits contrary to the homogeneous firm model that each firm earn zero profit. This simple extension of Krugman-type multi-product firm model into Melitz-type multi-product firm model gives different results. In this heterogeneous firm model, it is shown that unilateral trade liberalization benefits both countries, and the increase in welfare in the liberalizing country is greater than that in its trading partner. Therefore, this paper may contributes to a literature in the line of Chaney (2008) that contrasts the difference between Krugman-model and Melitz-model with regard to the impact of elasticity of substitution on the trade flow.

The remainder of the paper is organized as follows. Next section describes the basic structure of the model. Section 3 derives equilibrium solutions of the closed-economy model. Section 4 extends the model into open-economy model and the comparative statics are exercised.

³Though Iacovone and Javorcik (2010) shows that firms in Mexico, possibly a low-tech country, significantly reduces their number of domestic products after trade liberalization, it is important that this model points out the possibility of product proliferation in a low-tech country if the technology gap between nations is sufficiently large.

Section 5 introduces heterogeneous firms and contrasts the results with homogeneous firm model. Section 6 concludes.

2 The Model

Consider an economy that produces homogeneous goods and differentiated goods with only labor. The market of the homogeneous goods is perfectly competitive and that of differentiated goods is monopolistically competitive. One unit of a homogeneous good is produced by one unit of labor, which pin down the wage rate. The wage rate is chosen as numéraire: $w = 1$. The model does not need homogeneous goods sector in a closed and an open-economy consists of symmetric countries. However, the model of asymmetric countries discussed later needs homogeneous goods sector in order to satisfy the balance of trade condition by assuming that homogeneous goods are freely traded.

Preference

The representative consumer has a two tier utility function:

$$U = u^\mu x_0^{1-\mu},$$

where x_0 is the consumption of homogeneous goods and

$$u = \left(\int_{\omega \in \Omega} x(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}, \quad \sigma > 1$$

is the utility from the consumption of differentiated products where $x(\omega)$ is the consumption of a variety ω of a differentiated good; Ω is the mass of available varieties; and σ is the elasticity of substitution. The utility maximization problem subject to budget constraint gives us the demand for a variety ω as:

$$x(\omega) = \frac{p(\omega)^{-\sigma} \mu L}{P^{1-\sigma}},$$

where P is the CES price index dual to the utility function defined as follows:

$$P = \left(\int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}}.$$

Production

Each firm produces a mass of products, h . The marginal cost of the production of a product ω is: $1/\lambda(\omega)$ where $\lambda(\omega)$ is the efficiency of the product ω . Therefore, the profit maximizing firms set their price of product $\lambda(\omega)$ as:

$$p(\omega) = \frac{1}{\rho \lambda(\omega)},$$

where ρ relates to elasticity of substitution σ as following: $\sigma = 1/(1 - \rho)$. The larger $\lambda(\omega)$ enables firms to produce the product with lower cost, which enables them to set lower price.

As products with the same efficiency behave symmetrically, we index products from now on by λ alone. The efficiency of a product, λ is distributed within firms according to Pareto distribution:

$$G(\lambda) = 1 - \left(\frac{T}{\lambda}\right)^\theta,$$

where T and θ are the technology parameters that are to be asymmetric across countries in the later section. T captures the minimum efficiency of products within firms. Therefore, if a firm have higher T , the firm has superior technology.⁴ The introduction of the vertical product differentiation within firms is motivated by existing empirical studies. For example, Iacovone and Javorcik (2010) find an intense product churning within firms due to trade, suggesting the existence of within-firm product heterogeneity.⁵ For simplicity, it is assumed that all firms in a country share same technology.

Given the marginal cost and the optimal price, the revenue of producing product ω is written as:

$$r(\lambda) = \frac{\mu L}{(\rho \lambda P)^{1-\sigma}}.$$

Now the profit function of each firm can be written as:

$$\pi = \int_{\lambda^*}^{\infty} \left[\frac{\mu L}{\sigma(\rho \lambda P)^{1-\sigma}} - f \right] dG(\lambda) - F,$$

where f is fixed cost for the production of each product and F is the fixed cost for operation. Each firm produces a product if its profitability is greater than the fixed cost, f . The profit is increasing in the price index P , which corresponds to an inverse measure of the degree of competition in a market. Product-cutoff of the product λ is given by setting the inside of the bracket zero:

$$\lambda^* = \frac{1}{\rho P} \left(\frac{\mu L}{\sigma f} \right)^{\frac{1}{1-\sigma}}. \quad (1)$$

The product-cutoffs are decreasing in P , which means that if the market is competitive (P is lower), each firms produces fewer products. This mechanism is crucial in deriving main results in this paper. Once λ^* is determined, the mass of products produced by each firm is determined by:

$$h \equiv 1 - G(\lambda^*).$$

The production technology is very similar to that of Bernard et al. (2010). However, they interpret λ as the preference of consumers which is exposed to idiosyncratic shock from the viewpoint of firms. Contrary to their model, λ is interpreted as product efficiency which is perfectly known by firms. Therefore, profit maximizing firms choose to produce products with higher λ that give them non-negative profit. In the sense, this model embed ‘core efficiency’ that implies the existence of product ladders: firms are more efficient in the production of products near to the core efficiency and less efficient in the production of products far from

⁴Arkolakis (2010) introduces technology difference across countries using the exactly same way. Eaton and Kortum (2002) embed technology of a country by a similar way. The latter assumes that each country’s efficiency is distributed following Fréchet distribution function.

⁵See also, Baldwin and Gu (2009) and Bernard et al. (2010) for similar empirical results.

their core efficiency as in Mayer et al. (2009), Arkolakis and Muendler (2009), and Eckel and Neary (2010).⁶

3 Closed economy

This section characterizes the equilibrium of a closed-economy model. The model has three unknown variables, the mass of firms, M , the CES price index, P , and the product-cutoff of products, λ^* . Therefore, we need three equilibrium conditions. Equation (1) is the one of three equilibrium conditions. The second condition is free entry condition that sets the profit of each firm to zero. Applying Pareto distribution to the above profit function and letting it zero, free entry condition is written as:

$$\pi = \frac{T^\theta \gamma L}{\sigma (\rho P)^{1-\sigma}} \lambda^{*-\theta+\sigma-1} - f T^\theta \lambda^{*-\theta} - F = 0, \quad (2)$$

where $\gamma \equiv \frac{\theta}{\theta-\sigma+1}$. The last equilibrium condition is labor market clearing condition that equates labor demand with labor supply L :

$$M \left[\underbrace{\frac{\sigma-1}{\sigma} \frac{T^\theta \gamma L}{(\rho P)^{1-\sigma}} \lambda^{*-\theta+\sigma-1}}_{\text{labor demand for production}} + \underbrace{f T^\theta \lambda^{*-\theta}}_{\text{fixed costs of products}} + \underbrace{F}_{\text{fixed cost}} \right] = \mu L, \quad (3)$$

where inside of the bracket represents labor demand from each firm; the first term in the bracket is the labor demand for production,⁷ the second term is the labor demand for the fixed cost for products, and last term is the fixed operation cost. The labor demand from each firm multiplied by the mass of operating firms must equal to total labor supply, L .

There are three unknowns, M , P , and λ^* and three equilibrium conditions. Therefore, the model is solvable. A unique set of equilibrium values is summarized as follows:

$$M = \frac{\mu L}{\frac{F \sigma \gamma}{\gamma-1}}, \quad (4)$$

$$P = \frac{1}{\rho T} \left(\frac{\mu L}{\sigma f} \right)^{\frac{1}{1-\sigma}} \left(\frac{F}{f} \frac{1}{\gamma-1} \right)^{\frac{1}{\theta}}, \quad (5)$$

$$\lambda^* = T \left[\frac{f}{F} (\gamma-1) \right]^{\frac{1}{\theta}}. \quad (6)$$

The mass of operating firms, M does not depend on the technology parameter T . The intuition behind the result is following. The higher T increases the profit of each firm, which decreases the available labor force and decreases the mass of firms. On the other hand, the higher T

⁶Mayer et al. (2010) pin down the optimal range of production by assuming each firm faces additional production cost for a new variety. In the paper by Arkolakis and Muendler (2009), incremental local entry cost and costs from declining efficiency determine the optimal product scope. Eckel and Neary (2010) assume that marginal cost varies across products and the combination of the product ladder and the cannibalization effect determines the optimal range of product scope.

⁷Let the revenue and the cost for production of each firm as r and l , respectively. From the basic property of monopolistic competition models, it follows that:

$$r - l = \frac{r}{\sigma}.$$

Therefore, the labor demand for production can be written as $l = \frac{\sigma-1}{\sigma} r$ because wage rate is one.

decrease the profit of each firms because the higher T make the market more competitive (see the equilibrium price index). This increases the available resource and increases mass of firms. These oppositely working forces are in balance in this model.

Technology parameter T is appeared in P and λ^* . Because the higher T make each firms more efficient in production, it make the market more competitive. Therefore, the equilibrium price index has negative correlation with T . This effect of T on P is also appeared in the equilibrium product-cutoff, λ^* . The competitiveness of the market because of the higher T , make the product-cutoff higher, then reducing the mass of products produced by each firms.

The results establish following proposition on the relationship between market sides, L_i and $(M_i, P_i, \lambda_{ii}^*$ and $\lambda_{ij}^*)$.

Proposition 1: *The larger country has the larger mass of firms and the lower price index, but the market size does not affect mass of products produced by each firm.*

Proof. It immediately follows, from (4), (5) and (6) that $\partial M/\partial L > 0$, $\partial P/\partial L < 0$, and $\partial \lambda^*/\partial L = 0$, respectively. \square

In the next section, the model is extended to the two country-open-economy model. The equilibrium values obtained here are compared with those in the open-economy model.

4 Open economy

Consider the world consisting two countries indexed by $i = 1, 2$ and populated by L_i identical households, each of which has a unit of labor supplied inelastically and immobile across countries. Suppose that the homogeneous goods are freely traded, which equates wage rates in two countries. Firms in each country are allowed to export their products to the other country. Firms from i have to pay additional iceberg trade cost τ_{ij} to sell one unit of a product in market j . The iceberg trade costs are $\tau_{ij} \geq 1$ for $i \neq j$ and assume that $\tau_{ii} = 1$ for $i = 1, 2$. Similarly, as the fixed cost for exporting, each products require firms in i to pay the fixed cost, $f_{ij} \geq f_{ii}$ to sell the products to market j .

Firms in each country has asymmetric technology represented by exogenous distribution function of product efficiency, $G_i(\lambda) = 1 - (T_i/\lambda)^\theta$. Here we assume that the countries have asymmetric technology parameters, T_i for $i = 1, 2$.

In the open-economy, the price of a product λ supplied by a firm in country i to country i , p_{ii} and the price of a product λ supplied by a firm in country i to country j is written as:

$$p_{ii} = \frac{1}{\rho\lambda}, \quad p_{ij} = \frac{\tau_{ij}}{\rho\lambda}, \quad i = 1, 2$$

respectively. The profit function of each firm in the open economy is given as:

$$\pi_i = \sum_{k=1}^2 \int_{\lambda_{ik}^*}^{\infty} \left[\frac{\tau_{ik}^{1-\sigma} \mu L_k}{\sigma(\rho\lambda P_k)^{1-\sigma}} - f_{ik} \right] dG_i(\lambda) - F, \quad i = 1, 2$$

where

$$\lambda_{ii}^* = \frac{1}{\rho P_i} \left(\frac{\mu L_i}{\sigma f_{ii}} \right)^{\frac{1}{1-\sigma}}, \quad i = 1, 2 \quad (7)$$

is the product-cutoff for the domestic market below which products would make negative profits if they are produced, and hence they are not produced.

$$\lambda_{ij}^* = \frac{\tau_{ij}}{\rho P_j} \left(\frac{\mu L_j}{\sigma f_{ij}} \right)^{\frac{1}{1-\sigma}}, \quad i = 1, 2 \quad (8)$$

is the product-cutoff for the domestic market below which products would make negative profits if they are exported, and hence they are not exported. Therefore, for sufficiently high values of fixed and variable trade costs, the model features selection into export markets. Only the most efficient products are exported, while intermediate efficiency products are supplied to only the domestic market, and least efficient products are not produced. However, if the foreign market is sufficiently large and less competitive, it might be the case that some products are exported by not supplied to the domestic market.⁸ These equations consists a subset of equilibrium conditions.

The remaining equilibrium conditions are, as in closed economy, free entry conditions and labor market clearing conditions. The former conditions are written as:

$$\pi_i = \sum_{k=1}^2 \left[\frac{\tau_{ik}^{1-\sigma} T_i^\theta \gamma \mu L_k}{\sigma (\rho P_k)^{1-\sigma}} \lambda_{ik}^{*-\theta+\sigma-1} - f_{ik} T_i^\theta \lambda_{ik}^{*-\theta} \right] - F = 0. \quad i = 1, 2 \quad (9)$$

Lastly, the labor market clearing requires:

$$M_i \left\{ \sum_{k=1}^2 \left[\frac{\sigma - 1}{\sigma} \frac{\tau_{ik}^{1-\sigma} T_i^\theta \gamma \mu L_k}{(\rho P_k)^{1-\sigma}} \lambda_{ik}^{*-\theta+\sigma-1} + f_{ik} T_i^\theta \lambda_{ik}^{*-\theta} \right] + F \right\} = \mu L_i. \quad i = 1, 2 \quad (10)$$

To summarize, there are eight unknown equilibrium variables: M_i , P_i , λ_{ii}^* , λ_{ij}^* for $i = 1, 2$. And we have eight equilibrium conditions: four equations that determine product-cutoff, (7) and (8), two free entry conditions, (9), and two labor market conditions, (10). Therefore, as in closed-economy model, the model is solvable. However, in order to simplify the solutions it is assumed that $\frac{f_{11}}{f_{12}} = \frac{f_{22}}{f_{21}}$.

An unique set of equilibrium values are summarized as follows:

$$M_i = \frac{\mu L}{\frac{F \sigma \gamma}{\gamma - 1}}, \quad i = 1, 2 \quad (11)$$

$$P_i = \frac{1}{\rho} \left(\frac{\mu L_i}{\sigma f_{ii}} \right)^{\frac{1}{1-\sigma}} \left[\frac{F}{f_{ii}} \frac{1}{1 - \tau_{ij}^{-\theta} \tau_{ji}^{-\theta} \Phi^2} \frac{1}{\gamma - 1} \left(\frac{1}{T_i^\theta} - \frac{\tau_{ij}^{-\theta} \Phi}{T_j^\theta} \right) \right]^{\frac{1}{\theta}}, \quad i = 1, 2 \quad (12)$$

$$\lambda_{ii}^* = \left[\frac{F}{f_{ii}} \frac{1}{1 - \tau_{ij}^{-\theta} \tau_{ji}^{-\theta} \Phi^2} \frac{1}{\gamma - 1} \left(\frac{1}{T_i^\theta} - \frac{\tau_{ij}^{-\theta} \Phi}{T_j^\theta} \right) \right]^{-\frac{1}{\theta}}. \quad i = 1, 2 \quad (13)$$

$$\lambda_{ij}^* = \tau_{ij} \left(\frac{f_{ii}}{f_{ij}} \right)^{\frac{1}{1-\sigma}} \left[\frac{F}{f_{jj}} \frac{1}{1 - \tau_{ji}^{-\theta} \tau_{ij}^{-\theta} \Phi^2} \frac{1}{\gamma - 1} \left(\frac{1}{T_j^\theta} - \frac{\tau_{ji}^{-\theta} \Phi}{T_i^\theta} \right) \right]^{-\frac{1}{\theta}}. \quad i = 1, 2 \quad (14)$$

⁸This is consistent with empirical evidence confirmed by Eckel et al. (2009). They show that multi-product firms sell fewer products in their export than their home markets, though they earn higher profits abroad when the foreign market is larger.

where $\Phi \equiv \left(\frac{f_{ij}}{f_{ii}}\right)^{\frac{\theta-\sigma+1}{1-\sigma}} \leq 1$ if $f_{ij} \geq f_{ii}$. Using λ_{ii}^* and λ_{ij}^* , the mass of products supplied by each country i 's firm to market i is expressed as:

$$h_{ii} = \frac{F}{f_{ii}} \frac{1}{1 - \tau_{ij}^{-\theta} \tau_{ji}^{-\theta} \Phi^2} \frac{1}{\gamma - 1} \left(\frac{1}{T_i^\theta} - \frac{\tau_{ij}^{-\theta} \Phi}{T_j^\theta} \right), \quad i = 1, 2$$

and the mass of products supplied by each country i 's firm to market j is expressed as:

$$h_{ij} = \tau_{ij}^{-\theta} \left(\frac{f_{ii}}{f_{ij}} \right)^{-\frac{\theta}{1-\sigma}} \frac{F}{f_{jj}} \frac{1}{1 - \tau_{ji}^{-\theta} \tau_{ij}^{-\theta} \Phi^2} \frac{1}{\gamma - 1} \left(\frac{1}{T_j^\theta} - \frac{\tau_{ji}^{-\theta} \Phi}{T_i^\theta} \right). \quad i = 1, 2$$

In order to make the mass of products non-negative, $h_{ii} \geq 0$ and $h_{ij} \geq 0$, we need the following assumption:

Assumption 1

$$\tau_{ji} \geq \frac{T_j}{T_i} \Phi^{\frac{1}{\theta}}. \quad i = 1, 2$$

In order to understand the meaning of this assumption, as later in this paper, suppose that country 1 has a superior technology than country 2: $T_1 > T_2$. To assure non-negative mass of products exported by country 2 to country 1, h_{21} , the model needs the assumption: $\tau_{12} \geq \frac{T_1}{T_2} \Phi^{\frac{1}{\theta}}$. However, if the technology gap between countries is sufficiently large and τ_{12} is low, it might be the case that $\tau_{12} < \frac{T_1}{T_2} \Phi^{\frac{1}{\theta}}$. In this case h_{21} is negative. The reason is following. Because country 1 has a very superior technology than country 2, firms in country 1 are more profitable than those in country 2. Moreover, the small τ_{12} make the country 1 a better export base, which make the country 1 more attractive than country 2 from the viewpoint of firms. These two offer a strong incentive to firms to agglomerate country 1. This competitive environment sharply decreases P_1 , which sharply decreases the exports by country 2 into country 1, h_{21} . Therefore, at an extreme case, h_{21} can be negative. In order to exclude such a strange situation, we need Assumption 1. The meaning of the assumption: $\tau_{21} > \frac{T_2}{T_1} \Phi^{\frac{1}{\theta}}$ can be explained by same logic.⁹

4.1 Bilateral reduction of the trade cost

Because our model allows asymmetry between countries, the impact of an asymmetric reduction of trade costs can be examined. However, before illustrating the consequences of asymmetric liberalization, we first quickly describe the case of symmetric liberalization among symmetric countries. Therefore, it is assumed that $\tau_{12} = \tau_{21} = \tau$, $L_1 = L_2 = L$, and $T_1 = T_2 = T$. The comparative statics with regard to a reduction of τ give following proposition:

Proposition 2: *In a world consisting two identical countries, bilateral trade liberalization in the form of reduction of τ :*

⁹In order to assure non-negative mass of products exported by country 1 to country 2, h_{12} , we need the assumption: $\tau_{21} \geq T_2/T_1 \Phi^{1/\theta}$. This condition always hold if the technology gap between countries is sufficiently large such that $\tau_{21} \geq 1 > T_2/T_1 \Phi^{1/\theta}$. In this case, the mass of products exported by country 1 to country 2, h_{12} cannot be negative for all $\tau \geq 1$. Therefore, the restriction on τ_{21} is non-binding. The inferior technology of country 2 cannot offer an incentive to firms to agglomerate in country 2, then, P_2 is relatively higher. This less competitive environment in country 2 assures positive h_{12} .

- (i) monotonically decreases price indices, P_i and P_j , which improves welfare in both countries;
- (ii) monotonically reduces (does not affect) mass of available variety for a consumer if $f_{ij} > f_{ii}$ ($f_{ij} = f_{ii}$);
- (iii) induces firms to consecutively drop less efficient products from domestic market;
- (iv) induces firms to start exporting most efficient products.

Proof. See the Appendix. \square

Result (i) is driven by a similar mechanism with Melitz (2003). An increase in the domestic product-cutoff and the import of foreign inexpensive products lower the price index, which improves welfare in both countries. Result (ii) is very similar to the result obtained by Arkolakis et al. (2009) and Baldwin and Forslid (2010). In their single-product Melitz-type models, it is shown that trade liberalization decreases the mass of available variety if the fixed cost for exporting is greater than the fixed cost of domestic operation.¹⁰

Results (iii), (iv) are supported by several existing empirical studies of variety diversification during trade liberalization. For example, Baldwin and Gu (2009) and Barnard et al. (2010) find sharp decline in the number of products among Canadian and U.S. firms, respectively.

Next, the effects of bilateral liberalization in the asymmetric countries are examined. The asymmetric market size, $L_1 \neq L_2$, does not make a new result, therefore, it is assumed that $L_1 = L_2 = L$. However, we assume that the two countries have asymmetric technology parameters. More specifically, let's assume that country 1 has a superior technology than country 2: $T_1 > T_2$. In this world, a bilateral reduction of trade costs gives us following results:

Proposition 3: *If firms in country 1 has a superior technology than those in country 2, i.e., $T_1 > T_2$, and the technology gap is sufficiently large, bilateral trade liberalization in the form of reduction of $\tau_{12} = \tau_{21} = \tau$:*

- (i) monotonically decreases P_1 , which monotonically improves welfare of country 1;
- (ii) firstly decreases P_2 then increases it, which firstly improves then reduces welfare of country 2;
- (iii) induces firms in country 1 consecutively drop less efficient products from domestic market;
- (iv) induce firms in country 1 consecutively add most efficient products as exporting products;
- (v) induces firms in country 2 firstly drop domestic less efficient products, but after further deeper liberalization, start producing previously dropped products in the domestic market;
- (vi) induces firms in country 2 firstly add most efficient products as exporting products, but after further liberalization, drop previously added products from the export market.

Proof. See the Appendix. \square

These results are essentially driven by free entry conditions of two countries. Because firms in country 1 have a superior technology, their profitability is higher than firms in country 2.

¹⁰Baldwin and Foslid (2010) call this effect as “anti-variety effect” of trade liberalization. However, it is shown that the positive effect owing the productivity improvement of the industry dominates the negative “anti-variety” effect.

Moreover, bilateral trade liberalization makes firms in country 1 more profitable than firms in country 2 (even though 2's firms are also benefited by trade liberalization). Therefore, in order to restore equilibrium, that is, in order to equalize firms' expected profits in country 1 and 2, P_1 must sharply decrease after the reduction of τ . The sharp reduction of P_1 also has negative effect on the profitability of firms in country 2. Therefore, again, in order to equalize expected profits of firms in country 1 and 2, P_2 must slightly increase. This effect is strong when τ is low because the effect of τ on firms' profit is magnified as τ decreases.¹¹

The non-monotonic behavior of firms in country 2 is explained by same logic. In the country 2's market, when τ is sufficiently low, because P_2 is increased by the reduction of τ , now that 2's firms can produce larger mass of products due to the relaxed competition. Therefore, they can add previously dropped domestic products. On the other hand, the progressive decrease in τ continuously has induced 2's firms to export, however, due to the sharp decrease in P_1 , firms in country 2 begin to stop exporting previously exported products.

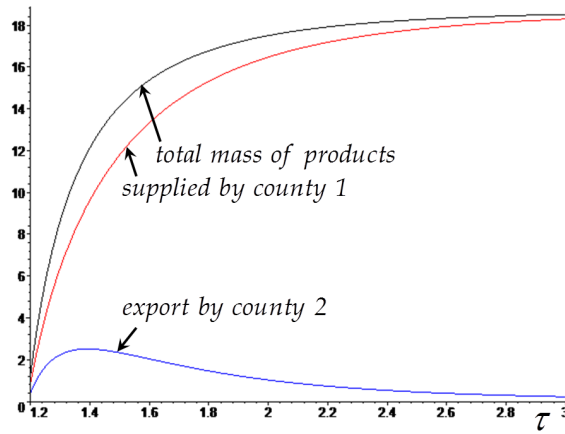


Figure 1: Mass of products in country 1

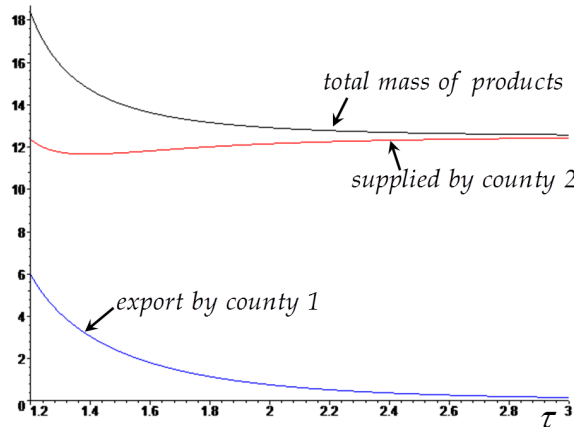


Figure 2: Mass of products in country 2

¹¹Recall that $\theta > 4$. This means that the value of $\tau^{-\theta}$ is not linear with respect to τ .

4.2 Unilateral reduction of the trade cost

We now describe the effects of a unilateral reduction of trade cost by country 1 (a decrease in τ_{21} , holding τ_{12} constant). Since asymmetric market sizes and technologies do not make the results insightful, we assume $L_1 = L_2 = L$ and $T_1 = T_2 = T$. A unilateral reduction of τ_{21} gives following results:

Proposition 4: *At any values of T_1, T_2, L_1 and L_2 , the progressive unilateral trade liberalization by country 1 in the form of reduction of τ_{21} :*

- (i) *monotonically increases P_1 , which monotonically reduces welfare of country 1;*
- (ii) *monotonically decreases P_2 , which monotonically raises welfare of country 2;*
- (iii) *induces firms in country 1 consecutively add products for the domestic market;*
- (iv) *induce firms in country 1 consecutively drop products from the export market;*
- (v) *induces firms in country 2 consecutively drop products from the domestic market;*
- (vi) *induces firms in country 2 consecutively add products for the export market.*

Proof. See the Appendix. \square

Again, the essence of the results is in the free entry conditions. A reduction of τ_{21} makes country 2 better export base, which increases the expected profit from being country 2. Therefore, the reduction of τ_{21} offers an incentive firms to agglomerate in country 2. Even though mass of firms in country 2 does not change in equilibrium, this threat in country 2 decreases P_2 . The decrease P_2 make the market in country 2 more competitive, inducing 2's firms to drop domestic products and 1's firms to quite exporting previously exported products. This competitive environment inhibits the formation of agglomeration that should occur in a new economic geography model with single-product firms.

On the other hand, a decrease in τ_{21} increases P_1 in order to restore equilibrium: country 1 must be more profitable by relaxing competition (with an increase in P_1). Because firms in country 1 have an incentive to move to country 2, this raises P_1 , which increases mass of products produced by firms in country 1. Now since firms in country 1 can produce larger mass of products, it prevents 1's firms from leaving country 1. Therefore, in this multi-product firm model, the de-industrialization that occurs in single-product firm model does not happens. All of the effects on mass of firms are absorbed by within-firm adjustment.

In addition, it is clear that the liberalizing country experiences a welfare loss while its trading partner experiences a welfare gain.¹² These results are driven by the setting that firms can freely move across countries. The lowering trade costs in a country make the other country better export base, which generates an incentive to agglomerate in the country. Even though they do not move in the equilibrium, the possibility of moving across countries affects price indices, and then affecting welfare.

Even though the direct effect of a decrease in τ_{21} lowers P_1 , the indirect effect through within-

¹²The similar results are derived in Melitz and Ottaviano (2008) with a setting of linear demand and Demidova (2008) with a setting of CES preference. The mechanism of the welfare loss of liberalizing country in their model is essentially same with this model.

firm-adjustment increases P_1 . The source of the within-firm-adjustment is the footlooseness of firms. Therefore, if the model introduces a friction that sticks firms in the locating countries, the effect of unilateral trade cost could be changed: the direct effect would dominate the indirect effect. With this motivation, I introduce heterogeneity in productivity across firms because it is a way of introducing the friction that weakens the footlooseness of firms.

5 Heterogeneous Firms

This section shows that, once we introduce heterogeneity across firms into the model, unilateral reduction of trade cost benefits a liberalizing country as in Demidova and Rodriguez-Clare (2011) and also benefits its trading partner.

Suppose that, by paying fixed entry cost, f_e , ex ante identical firms draw their productivity, φ , from the Pareto distribution function:

$$H(\varphi) = 1 - \varphi^{-k},$$

where k is the shape parameter that is assumed to be greater than θ to make the firms' profit finite. Assume that φ is the inverse of firm's marginal cost that affects all of the products within a firm in a same way. Therefore, firm's pricing rules are:

$$p_{ii} = \frac{1}{\rho\varphi\lambda}, \quad p_{ij} = \frac{\tau_{ij}}{\rho\varphi\lambda}, \quad i = 1, 2$$

which implies that the efficient firm can set the lower price. Now the profit of a firm whose productivity is φ is written as:

$$\begin{aligned} \pi_i(\varphi) &= \sum_{k=1}^2 \int_{\lambda_{ik}^*(\varphi)}^{\infty} \left[\frac{\tau_{ik}^{1-\sigma} \mu L_k}{\sigma(\rho\varphi\lambda P_k)^{1-\sigma}} - f_{ik} \right] dG_i(\lambda) - F \\ &= (\gamma - 1) \sum_{k=1}^2 f_{ik} \left(\frac{T_i}{\lambda_{ik}^*(\varphi)} \right)^\theta - F, \quad i = 1, 2 \end{aligned}$$

where

$$\lambda_{ii}^*(\varphi) = \frac{1}{\rho\varphi P_i} \left(\frac{L}{\sigma f_{ii}} \right)^{\frac{1}{1-\sigma}}, \quad i = 1, 2 \quad (15)$$

is the product-cutoff in domestic market below which products would make negative profits if they are produced, and hence they are not produced. Similarly,

$$\lambda_{ij}^*(\varphi) = \frac{\tau_{ij}}{\rho\varphi P_j} \left(\frac{L}{\sigma f_{ij}} \right)^{\frac{1}{1-\sigma}}, \quad i = 1, 2 \quad (16)$$

is the product-cutoff for foreign markets below which products would make negative profits if they are exported hence they are not exported. These imply that the more productive firms can produce and export larger mass of products.

It is clear that all firms participate in exporting because any firms have at least some very efficient products that can make the positive profit even in the foreign market. ¹³

¹³Contrary to the model developed here, in the model by Bernard et al. (2010), a subset of firms participate in exporting as in Melitz (2003) because each firm have to pay fixed exporting cost, F_x . The introduction of such cost is straightforward. However, in order to compare the heterogeneous model with the homogeneous firm model developed in previous sections, we assume that the fixed exporting cost is only appeared in f_{ij} and we do not introduce F_x .

Firms' profit function is re-written as:

$$\pi_i(\varphi) = F \left[\left(\frac{\varphi}{\varphi_i^*} \right)^\theta - 1 \right], \quad i = 1, 2$$

where

$$\varphi_i^{*\theta} = \frac{\frac{F}{\gamma-1}}{\sum_{k=1}^2 \left[\frac{T_i \rho P_k}{\tau_{ik}} \left(\frac{\sigma f_{ik}}{L_k} \right)^{\frac{1}{1-\sigma}} \right]^\theta}, \quad i = 1, 2$$

is the cutoff productivity above which firms can make non-negative profits. Now we can obtain expected profit of firms as:

$$\begin{aligned} \bar{\pi}_i &= \int_{\varphi_i^*}^{\infty} F \left[\left(\frac{\varphi}{\varphi_i^*} \right)^\theta - 1 \right] \frac{dH(\varphi)}{1 - H(\varphi_i^*)} \\ &= F \frac{\theta}{k - \theta}. \quad i = 1, 2 \end{aligned}$$

It is clear that firms' expected profit is same in country 1 and country 2 even though they are asymmetric. As pointed out in Helpman et al. (2003) and Baldwin and Forslid (2010), free entry of firms ensure that the expected profit is same in both markets.

Now I have done all preparations to derive equilibrium. The heterogeneous multi-product firm model has 10 unknowns: P_i , M_i , φ_i^* , λ_{ii}^* , and λ_{ij}^* for $i = 1, 2$. There are 10 equilibrium conditions: four product-cutoff conditions ((15) and (16)), two free entry conditions, two labor market conditions, plus two CES price index equations. The free entry conditions are written as:

$$[1 - H(\varphi_i^*)] \bar{\pi}_i / \delta = f_e, \quad i = 1, 2 \quad (17)$$

where LHS is expected profit from entry and RHS is the cost of entry. Labor market clearing requires:

$$M_i \left\{ \underbrace{\sum_{k=1}^2 \left[\frac{(\sigma - 1) \tau_{ik}^{1-\sigma} T_i^\theta \gamma \mu L_k}{\sigma (\rho \tilde{\varphi} P_k)^{1-\sigma} [\lambda_{ik}^*(\tilde{\varphi})]^{\theta - \sigma + 1}} + f_{ik} \left(\frac{T_i}{\lambda_{ik}^*(\tilde{\varphi})} \right)^\theta \right]}_{\text{labor demand for production plus fixed costs of products}} + \underbrace{\frac{F}{\text{fixed cost}}}_{\text{fixed cost}} + \underbrace{\frac{F\theta}{k - \theta}}_{\text{fixed entry fee}} \right\} = \mu L_i, \quad (18)$$

for $i = 1, 2$.¹⁴ LHS is labor demand and RHS is labor supply. $\tilde{\varphi}$ is the average productivity of heterogeneous firms written as: $\tilde{\varphi} = \int_{\varphi^*}^{\infty} \varphi \frac{dH(\varphi)}{1 - H(\varphi^*)}$. Lastly, the CES price indices are:

$$P_i^{1-\sigma} = \sum_{k=1}^2 M_i \int_{\varphi_k^*}^{\infty} \int_{\lambda_{ki}^*(\varphi)}^{\infty} \left(\frac{\tau_{ki}}{\rho \varphi \lambda} \right)^{1-\sigma} dG_k(\lambda) \frac{dH(\varphi)}{1 - H(\varphi_i^*)}, \quad i = 1, 2 \quad (19)$$

Therefore, the model is solvable. However, it is too complicated to obtain the analytical result. Then, a sketch of a way of solving the model is described and a numerical example is shown.

¹⁴The labor demand for fixed entry fee, L_e , is written as: $L_e = f_e M_e$ where M_e is mass of entrants. Using the free entry condition, it can be written that:

$$L_e = M_e f_e = \frac{\delta f_e M}{1 - H(\varphi^*)} = M \bar{\pi} = M F \frac{\theta}{k - \theta}.$$

The calculation process would be following:

Step 1: Using (17), derive equilibrium cutoff productivities, φ_i^* for $i = 1, 2$.

Step 2: Using equilibrium cutoff productivities, (15), (16), (18) and (19), equilibrium mass of firms, M_i for $i = 1, 2$, are obtained.

Step 3: Using equilibrium cutoff productivities, equilibrium mass of firms, (15), (16) and (19), obtain equilibrium price indices, P_i for $i = 1, 2$.

Step 4: Using equilibrium price indices, (15) and (16), obtain product-cutoffs, λ_{ii}^* , and λ_{ij}^* for $i = 1, 2$.

Following these steps, equilibrium values are characterized. But only equilibrium cutoff productivities are analytically solvable. Regarding other endogenous variables, the simultaneous equations that give equilibrium values are presented in the following.

Step 1: Equilibrium cutoff productivities are given as:

$$\varphi_i^* = \left(\frac{F}{\delta f_e} \frac{\theta}{k - \theta} \right)^{\frac{1}{k}}, \quad i = 1, 2$$

which implies that the two countries have same cutoff productivity, then the subscript i is dropped from the cutoff productivity: $\varphi_1^* = \varphi_2^* = \varphi^*$. Equilibrium price indices are determined as solutions of following simultaneous equations:¹⁵

Step 2: Equilibrium mass of firms in each country is obtained by solving following simultaneous equations:

$$M_i \left[\frac{T_i^\theta [(\sigma - 1)\gamma + 1]}{\sigma \gamma \varphi^{*k}} \left(\frac{L_i}{T_i^\theta M_i + \tau_{ji}^{1-\sigma} T_j^\theta \Phi M_j} + \frac{\tau_{ij}^{-\theta} L_j \Phi}{T_j^\theta M_j + \tau_{ij}^{1-\sigma} T_i^\theta \Phi M_i} \right) + F + \frac{F\theta}{k - \theta} \right] = \mu L, \quad (20)$$

for $i = 1, 2$. It is difficult to obtain analytical solutions of M_i for $i = 1, 2$. Therefore, a numerical example is shown below.

Step 3: Once equilibrium mass of firms, M_i for $i = 1, 2$ are determined, equilibrium price indices are obtained as functions of M_i for $i = 1, 2$:

$$P_i = \left[\frac{1}{\gamma(\rho\varphi^*)^\theta} \frac{k - \theta}{k} \left(\frac{L_i}{\sigma f_{ii}} \right)^{-\frac{\sigma - \theta - 1}{1 - \sigma}} \frac{1}{T_i^\theta M_i + \tau_{ji}^{1-\sigma} T_j^\theta \Phi M_j} \right]^{1/\theta} \cdot \quad i = 1, 2 \quad (21)$$

Step 4: Once equilibrium price indices, P_i for $i = 1, 2$ are determined, equilibrium product-cutoffs are determined by (15) and (16).

Because our primary interest is the effects of unilateral trade liberalization, the effect of unilateral reduction of trade costs by country 1 (a decrease in τ_{21} , holding τ_{12}) is examined. Using

¹⁵These equations are obtained by inserting (15) and (16) into (19).

(21), it is numerically shown that, when the technology levels are similar between countries, unilateral trade liberalization by country 1 increases M_1 and M_2 as opposed to standard Melitz (2003)-type models.¹⁶ In addition, the increase in M_1 is greater than the increase in M_2 .

Figure 3 shows the result of the numerical exercise.¹⁷ The dashed lines are the equations (21) when $\tau_{12} = \tau_{21} = 5$ and the solid lines are those when $\tau_{12} = 5$ and $\tau_{21} = 2$. It is shown that a decrease in τ_{21} increases M_1 and M_2 and the increase in M_1 is greater than the increase in M_2 .

The intuition behind the result is following. A decrease in τ_{21} make the market in country 1 more competitive, which reduces the sales of each products and the mass of products produced by each firm in country 1. Therefore, labor required for production decreases. This decrease in labor demand allows additional entry. Then, a decrease in τ_{21} increases mass of firms in country 1.

On the other hand, in country 2, a decrease in τ_{21} reduces sales in country 1 and mass of exported products because the decreases in τ_{21} makes country 1 more competitive. This effect reduces labor demand as in firms in country 1. However, a decrease in τ_{21} gives firms in country 2 export opportunity. That is, as a result of a decrease in τ_{21} , more products are exported to country 1, which increases labor demand from each firm in country. This effect reduces mass of firms in country 2. Therefore, the increase in M_2 is smaller than the increase in M_1 .¹⁸

One might think that decreases in revenue of each firm decreases profit of each firm, which is inconsistent with the result that expected profit of firms is constant: $\pi = \frac{F\theta}{k-\theta}$. However, a decrease in revenue does not mean decrease in expected profit. Because firms drop products, now firms pay fewer fixed costs for production, which has positive impacts on firms' profit. The positive impact and the negative impact from tougher competition is in balance. Therefore, a decrease in revenue is not against the fact that firms' expected profit is constant.

Now the effect of a decrease in τ_{21} on equilibrium price indices can be identified. From equation (21), it follows that $\frac{\partial P_1}{\partial M_1} < 1$, $\frac{\partial P_1}{\partial M_2} < 1$, and $\frac{\partial P_2}{\partial \tau_{21}} > 1$. In addition, it is suggested by the numerical exercise that $\frac{\partial M_1}{\partial \tau_{21}} < 1$ and $\frac{\partial M_2}{\partial \tau_{21}} < 1$. Therefore, a decrease in τ_{21} reduces P_1 . This implies that liberalizing country benefits from the unilateral trade liberalization. Moreover, since a decrease in τ_{21} increases M_1 and M_2 , the unilateral reduction of trade costs by country 1 improves welfare in country 2.

Result 1: *In the heterogeneous multi-product firm model, if the technology level is similar between countries, unilateral trade liberalization by country 1 in the form of reduction of τ_{21} :*

- (i) *monotonically decreases P_1 and P_2 , which monotonically raises welfare of country 1 and 2;*
- (ii) *country 1 benefits more than country 2;*

¹⁶In standard Melitz (2003)-type models, a decrease in trade costs induces firms to begin exporting, which requires additional resources for paying higher fixed cost for exporting. However, in this model, such effect is excluded because all firms are exporting some most efficient products even if trade costs are very high.

¹⁷It is assumed that $L_1 = L_2 = 400$, $F = 1$, $\mu = 0.5$, $[(\sigma - 1)\gamma + 1]/\sigma\gamma\varphi^{*k} = 0.5$, $T_1 = T_2 = 1$, $k = 5$, and $\theta = 4$.

¹⁸If the liberalizing country has a inferior technology than its trading partner and the technology gap is sufficiently larger, it could be the case that unilateral trade liberalization increases mass of firms in the country that is not liberalizing. This is because the increase in labor demand due to additional export opportunity dominates the decrease in labor demand due to intensified competition in the liberalizing country since country 1 is less competitive due to their inferior technology. Conversely, the liberalizing country has a superior technology than country 1, it could be the case that mass of firms in the country that is not liberalizing sharply increases.

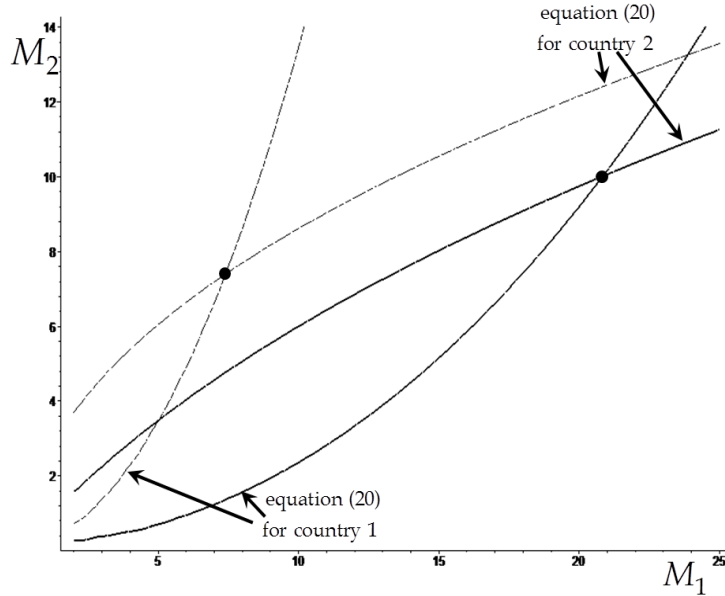


Figure 3: The effect of unilateral reduction of trade costs

- (iii) induces firms in country 1 and 2 consecutively drop products for the domestic market;
- (iv) induce firms in country 1 and 2 consecutively drop products from the export market.

From this numerical exercise, it is suggested that the introduction of heterogeneity across firms alters the results obtained in homogeneous firm model developed in previous section. That is, it is suggested that unilateral reduction of trade costs benefits liberalizing country. In addition, not only the liberalizing country but also its trading partner could benefits from a unilateral reduction of trade costs.

One might be interested in the impacts of bilateral trade liberalization among asymmetric countries. However, its impact on welfare is ambiguous. For the discussion on the effects of bilateral trade liberalization, see the Appendix.

6 Conclusion

This paper has examined the impact of trade on the market and firms' behavior by introducing multi-product firms into a model of new economic geography. The simple model developed here enables us to investigate how the interaction of asymmetric countries induced by trade liberalization influences the firms' behavior, price index and welfare.

It is shown that symmetric bilateral trade liberalization affects differently firms in high-tech and low-tech countries. In a high-tech country, it is suggested that firms could significantly reduces their domestic products and start exporting the most efficient products. On the other hand, in the low-tech country, it is suggested that firms do not reduce domestic products so much. In addition, in a setting of new economic geography with homogeneous firms, firms in a low-tech country take a non-monotonic behavior: they first drop and add their domestic

and exporting products, respectively, but after further liberalization, they add and drop previously dropped domestic and added exporting products, respectively. These difference in firms' behavior in high-tech and low-tech countries can explain different existing empirical results in developed and developing countries.

Moreover, regarding the effects of unilateral trade liberalization, results differ in Krugman-type homogeneous multi-product firm model and Melitz-type heterogeneous firm model. Contrary to the homogeneous firm model that unilateral reduction of trade costs hurts the liberalizing country, in the heterogeneous firm model, it is shown that not only the liberalizing country but also its trading partner gains from trade and the gains in the liberalizing country is greater.

This framework develops a new and very tractable way of describing how difference in technology and trade costs across countries affects firms' behavior and welfare differently. I hope that this provides a useful foundation for future empirical investigations on firms' behavior and its international comparison.

A Appendix

A.1 Proof of Proposition 2

Since two countries share same technology parameter, let $T_1 = T_2 = T$.

Proof of (i) and (iii)

The partial differentiation of h_{ii} with respect to $\tau_{12} = \tau_{21} = \tau$ is:

$$\frac{\partial h_{ii}}{\partial \tau} = \frac{\tau^{-\theta-1}\theta\Phi}{T^\theta(\gamma-1)} \frac{F}{f_{ii}} \left(\frac{1-\tau^{-\theta}\Phi}{1-\tau^{-2\theta}\Phi^2} \right)^2 > 0. \quad \text{for all } \tau \geq 1$$

Therefore, the mass of products supplied by firms in country i to county i monotonically decreases as τ decreases. This proves part (iii) of Proposition 2. Then part (i) follows because h_{ii} has a perfect negative relationship with λ_{ii}^* that has a perfect negative relationship with P_i .

Proof of (iv)

The partial differentiation of h_{ij} with respect to $\tau_{12} = \tau_{21} = \tau$ is:

$$\frac{\partial h_{ij}}{\partial \tau} = - \left(\frac{f_{ii}}{f_{ij}} \right)^{-\frac{\theta}{1-\sigma}} \frac{\tau^{-\theta-1}\theta}{T^\theta(\gamma-1)} \frac{F}{f_{ii}} \left(\frac{1-\tau^{-\theta}\Phi}{1-\tau^{-2\theta}\Phi^2} \right)^2 < 0. \quad \text{for all } \tau \geq 1$$

Therefore, the mass of products exported by firms in country i to country j increases as τ decreases.

Proof of (ii)

In the equilibrium, the mass of operating firms in each country does not change due to a

reduction of τ . Moreover, all firms are engaging in exporting in an open-economy. Therefore, the change in the mass of variety for a consumer is fully captured by the change in the mass of products supplied by domestic firms and foreign firms:

$$h_{ii} + h_{ji} = \left[1 + \tau^{-\theta} \left(\frac{f_{ji}}{f_{ii}} \right)^{-\frac{\theta}{1-\sigma}} \right] \frac{1}{T^\theta(\gamma-1)} \frac{F}{f_{ii}} \frac{1 - \tau^{-\theta}\Phi}{1 - \tau^{-2\theta}\Phi^2}.$$

The partial differentiation of $h_{ii} + h_{ji}$ with respect to $\tau_{12} = \tau_{21} = \tau$ is:

$$\frac{\partial}{\partial \tau}(h_{ii} + h_{ji}) = \left[\Phi - \left(\frac{f_{ji}}{f_{ii}} \right)^{-\frac{\theta}{1-\sigma}} \right] \frac{\tau^{-\theta-1}\theta}{T^\theta(\gamma-1)} \frac{F}{f_{ii}} \left(\frac{1 - \tau^{-\theta}\Phi}{1 - \tau^{-2\theta}\Phi^2} \right)^2.$$

Therefore, $h_{ii} + h_{ji}$ decreases as τ decreases if $\Phi > \left(\frac{f_{ji}}{f_{ii}} \right)^{-\frac{\theta}{1-\sigma}}$; and τ does not affect $h_{ii} + h_{ji}$ if $\Phi = \left(\frac{f_{ji}}{f_{ii}} \right)^{-\frac{\theta}{1-\sigma}}$. Recall that $\Phi \equiv \left(\frac{f_{ij}}{f_{ii}} \right)^{\frac{\theta-\sigma+1}{1-\sigma}}$. Hence it follows that $\frac{\partial}{\partial \tau}(h_{ii} + h_{ji}) > 0$ if $\frac{f_{ji}}{f_{ii}} > 1$; and $\frac{\partial}{\partial \tau}(h_{ii} + h_{ji}) = 0$ if $\frac{f_{ji}}{f_{ii}} = 1$.

Now all statements in Proposition 2 are proved. \square

A.2 Proof of Proposition 3

Proof of (i), (ii), (iii) and (v)

Suppose that, as in the main text, country 1 has superior technology than country 2: $T_1 > T_2$. I show that the mass of domestic products produced by each firm in country 1, h_{11} , monotonically increases and that in country 2, h_{22} , has non-monotonic behavior under a moderate parameter restriction. By showing this, the results, (i), (ii), (iii) and (v) in Proposition 3 follows because h_{ii} and P_i have a perfect positive correlation, and h_{ii} and λ_{ii}^* have a perfect negative correlation for $i = 1, 2$.

The partial differentiation of h_{ii} with respect to $\tau_{ij} = \tau_{ji} = \tau$ is:

$$\frac{\partial h_{ii}}{\partial \tau} = \frac{1}{\gamma-1} \frac{F}{f_{ii}} \frac{\theta\Phi\tau^{-\theta-1}}{(1 - \tau^{-2\theta}\Phi^2)^2} \left(-\frac{2\tau^{-\theta}\Phi}{T_i^\theta} + \frac{1 + \tau^{-2\theta}\Phi^2}{T_j^\theta} \right).$$

Therefore, the sign of $\partial h_{ii}/\partial \tau$ is determined as:

$$\frac{\partial h_{ii}}{\partial \tau} \begin{cases} \geq 0 \\ \leq 0 \end{cases} \Leftrightarrow \frac{T_j^\theta}{T_i^\theta} \begin{cases} \leq \\ \geq \end{cases} \frac{\tau^\theta\Phi^{-1} + \tau^{-\theta}\Phi}{2}.$$

Therefore, in country i , the non-monotonic behavior of firms arises when:

$$\begin{aligned} \frac{T_j^\theta}{T_i^\theta} &< \frac{\tau^\theta\Phi^{-1} + \tau^{-\theta}\Phi}{2}, & \text{for high value of } \tau \\ \frac{T_j^\theta}{T_i^\theta} &> \frac{\tau^\theta\Phi^{-1} + \tau^{-\theta}\Phi}{2}. & \text{for low value of } \tau \end{aligned}$$

Recall that $\Phi \equiv \left(\frac{f_{ij}}{f_{ii}} \right)^{\frac{\theta-\sigma+1}{1-\sigma}}$. Therefore, $\Phi^{-1} \geq 1 \geq \Phi$ when $f_{ij} \geq f_{ii}$. Then $\frac{\tau^\theta\Phi^{-1} + \tau^{-\theta}\Phi}{2}$ is increasing in τ and f_{ij} . As $\tau \rightarrow 1$ and $f_{ij} \rightarrow f_{ii}$, $\frac{\tau^\theta\Phi^{-1} + \tau^{-\theta}\Phi}{2} \rightarrow 1$.

Now suppose that country 1 has superior technology than country 2. The results, (i), (ii), (iii) and (v) arise if the following condition is satisfied:

$$\frac{\tau^\theta \Phi^{-1} + \tau^{-\theta} \Phi}{2} > \frac{T_1^\theta}{T_2^\theta} > \frac{T_2^\theta}{T_1^\theta}, \quad \text{for high value of } \tau \quad (22)$$

$$\frac{T_1^\theta}{T_2^\theta} > \frac{\tau^\theta \Phi^{-1} + \tau^{-\theta} \Phi}{2} > \frac{T_2^\theta}{T_1^\theta}. \quad \text{for low value of } \tau \quad (23)$$

This condition implies that non-monotonic behavior is more likely to occur when technology gap is larger and the difference between f_{ij} and f_{ii} is smaller. On the contrary, the non-monotonicity in country 2 would not occur when f_{ij} is extremely high or the technology gap is sufficiently small. However, P_1 and h_{11} always monotonically decrease as τ decreases because $\frac{\tau^\theta \Phi^{-1} + \tau^{-\theta} \Phi}{2} \geq 1 > \frac{T_2^\theta}{T_1^\theta}$ for all $\tau \geq 1$.

Proof of (iv) and (vi)

I show that the mass of products exported by country 1, h_{12} , monotonically increases as τ decreases: $\frac{\partial h_{12}}{\partial \tau} < 0$ for all $\tau \leq 1$, and the mass of exported products by country 2, h_{21} , has the non-monotonic behavior: $\frac{\partial h_{21}}{\partial \tau} < 0$ for high τ and $\frac{\partial h_{21}}{\partial \tau} > 0$ for low τ .

The partial differentiation of h_{ij} with respect to $\tau = \tau_{ij} = \tau_{ji}$ is:

$$\frac{\partial h_{ij}}{\partial \tau} = \frac{1}{\gamma - 1} \frac{F}{f_{ii}} \frac{\theta \tau^{-\theta-1}}{(1 - \tau^{-2\theta} \Phi^2)^2} \left(\frac{f_{ii}}{f_{ij}} \right)^{-\frac{\theta}{1-\sigma}} \left(\frac{2\tau^{-\theta} \Phi}{T_i^{-\theta}} - \frac{1 + \tau^{-2\theta} \Phi^2}{T_j^\theta} \right).$$

Therefore, the sign of $\partial h_{ij} / \partial \tau$ is determined as:

$$\frac{\partial h_{ij}}{\partial \tau} \begin{matrix} \geq \\ < \end{matrix} 0 \Leftrightarrow \frac{T_j^\theta}{T_i^\theta} \begin{matrix} \geq \\ < \end{matrix} \frac{\tau^\theta \Phi^{-1} + \tau^{-\theta} \Phi}{2}.$$

It follows that country 1 monotonically increases the mass of exporting products because $\frac{T_2^\theta}{T_1^\theta} < 1 \leq \frac{\tau^\theta \Phi^{-1} + \tau^{-\theta} \Phi}{2}$ for all $\tau \geq 1$. By contrary, firms in country 2 first increase their mass of exporting products, h_{21} , but reduce it after further reduction of τ if technology gap between countries is large enough and f_{ij} is low enough so that:

$$\frac{\tau^\theta \Phi^{-1} + \tau^{-\theta} \Phi}{2} > \frac{T_1^\theta}{T_2^\theta}, \quad \text{for high value of } \tau$$

$$\frac{T_1^\theta}{T_2^\theta} > \frac{\tau^\theta \Phi^{-1} + \tau^{-\theta} \Phi}{2}. \quad \text{for low value of } \tau$$

Therefore, the results (iv) and (vi) hold if the results (i), (ii), (iii), and (v) hold.

Now I can conclude this proof by maintaining that all results in Proposition 3 occur when technology gap between the countries are large enough and f_{ij} is not so high so as to satisfy (15)-(16). \square

A.3 Proof of Proposition 4

Proof of (i) and (iii)

The partial differentiation of h_{11} with respect to τ_{21} is:

$$\frac{\partial h_{11}}{\partial \tau_{21}} = -\frac{1}{\gamma - 1} \frac{F}{f_{11}} \frac{\theta \tau_{12}^{-\theta} \tau_{21}^{-\theta-1} \Phi^2}{(1 - \tau_{12}^{-\theta} \tau_{21}^{-\theta} \Phi^2)^2} \left(\frac{1}{T_1^\theta} - \frac{\tau_{12}^{-\theta} \Phi}{T_2^\theta} \right).$$

Under Assumption 1, it always follows that $\frac{\partial h_{11}}{\partial \tau_{21}} < 0$. Therefore, a reduction of τ_{21} always increases the mass of products supplied by firms in country 1 to country 1. Now part (iii) of Proposition 4 has been proved. This also proves part (i) because h_{11} has a perfect negative relationship with λ_{11}^* and λ_{11}^* has a perfect negative relationship with P_1 .

Proof of (ii) and (iv)

The partial differentiation of h_{22} with respect to τ_{21} is:

$$\frac{\partial h_{22}}{\partial \tau_{21}} = -\frac{1}{\gamma - 1} \frac{F}{f_{22}} \frac{\tau_{21}^{-\theta} \theta \Phi}{(1 - \tau_{12}^{-\theta} \tau_{21}^{-\theta} \Phi^2)^2} \left(\frac{1}{T_1^\theta} - \frac{\tau_{12}^{-\theta} \Phi}{T_2^\theta} \right).$$

Under Assumption 1, it always follows that $\frac{\partial h_{22}}{\partial \tau_{21}} > 0$. Therefore, a reduction of τ_{21} always decreases the mass of products supplied by firms in country 2 to country 2. Now part (iv) of Proposition 4 has been proved. This also proves part (ii) because h_{22} has a perfect negative relationship with λ_{22}^* that has a perfect negative relationship with P_2 .

Proof of (v)

The partial differentiation of h_{12} with respect to τ_{21} is:

$$\frac{\partial h_{12}}{\partial \tau_{21}} = \tau_{12}^{-\theta} \left(\frac{f_{22}}{f_{12}} \right)^{-\frac{\theta}{1-\sigma}} \frac{1}{\gamma - 1} \frac{F}{f_{22}} \frac{\theta \Phi \tau_{21}^{-\theta-1}}{(1 - \tau_{21}^{-\theta} \tau_{12}^{-\theta} \Phi^2)^2} \left(\frac{1}{T_1^\theta} - \frac{\tau_{12}^{-\theta} \Phi}{T_2^\theta} \right).$$

Under Assumption 1, it always follows that $\frac{\partial h_{12}}{\partial \tau_{21}} > 0$. Therefore, a reduction of τ_{21} always decreases the mass of products exported by firms in country 2 to country 1.

Proof of (vi)

The partial differentiation of h_{21} with respect to τ_{21} is:

$$\frac{\partial h_{21}}{\partial \tau_{21}} = -\frac{1}{\gamma - 1} \left(\frac{f_{11}}{f_{21}} \right)^{-\frac{\theta}{1-\sigma}} \frac{F}{f_{11}} \frac{\tau_{21}^{-\theta-1} \theta}{1 - \tau_{12}^{-\theta} \tau_{21}^{-\theta} \Phi^2} \left(1 + \frac{\tau_{21}^{-\theta} \Phi^2}{1 - \tau_{12}^{-\theta} \tau_{21}^{-\theta} \Phi^2} \right) \left(\frac{1}{T_1^\theta} - \frac{\tau_{12}^{-\theta} \Phi}{T_2^\theta} \right).$$

Under Assumption 1, it always follows that $\frac{\partial h_{21}}{\partial \tau_{21}} > 0$. Therefore, a reduction of τ_{21} always decreases the mass of products exported by firms in country 2 to country 1.

Now all of the statements in Proposition 4 are proved. \square

A.4 Bilateral trade liberalization among asymmetric countries in heterogeneous firm model

Suppose that country 1 has a superior technology than country 2. It can be numerically shown that bilateral reduction of trade cost (a decrease in $\tau_{21} = \tau_{12} = \tau$) increases M_2 more than M_1 as shown in Figure 4. Figure 4 represents equations (20) by assuming $L_1 = L_2 = 400$, $F = 1$, $\mu = 0.5$, $(\sigma\gamma + 1)/\sigma\gamma\varphi^{*k} = 0.5$, $T_1 = 3.5$, $T_2 = 1$. The dashed lines are equations (20) in case of $\tau_{21} = \tau_{12} = 5$ and the solid lines are equations (20) in case of $\tau_{21} = \tau_{12} = 2$. The intuition is following:

Why there are more firms in country 2 than country 1?

Because firms in country 2 cannot make larger profits which reduces labor demand in country 2, then allowing more entry in country 2.

Why bilateral reduction of trade costs increases mass of firms?

A decrease in τ intensifies the competition in both markets, which reduces sales and then reduces labor demand, which increases M_1 and M_2 .

Why bilateral reduction of trade costs increases M_2 more than M_1 ?

Additional export opportunity by decreases in τ increases sales of firms in country 1 more than those in country 2 because country 1 has a superior technology. On the other hand, firms in country 2 demand fewer labor because they cannot make larger profit due to an inferior technology. Then, more firms can enter country 2 than country 1 as τ decreases.

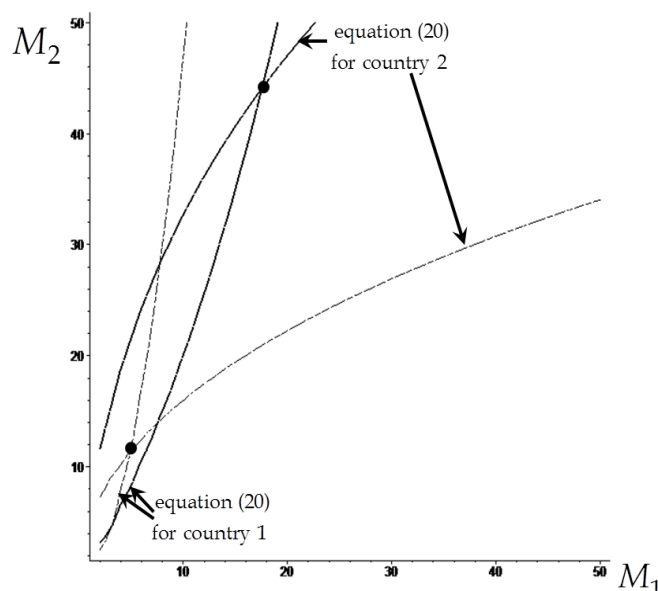


Figure 4: The effect of bilateral reduction of trade costs among asymmetric countries

P_1 and P_2 decrease due to bilateral trade liberalization, which improves welfare in both country 1 and 2. However, it is not clear whether country 1 benefits more or country 2 benefits more. Since M_2 increases more than M_1 , one might presume that bilateral trade liberalization benefits country 2 more than country 1. But it might not be the case. It is suggested from equation (21) that the effect of an increase in M_2 on P_2 is weak because T_2 is small and the effect of an increase in M_1 on P_1 is strong because T_1 is large. Intuitively, even if the mass of firms in country 2 increased, its effect on P_2 is weak because they produce expensive goods due to their inferior technology. Conversely, even if the increase in M_1 is smaller than the increase in M_2 , its impact on P_1 is strong because they produce low-priced goods. Therefore, the difference between the decrease in P_1 and P_2 is unclear. Then, the difference between the increase in P_1

and P_2 is also unclear.

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