Domestic subsidies, cooperation and coalition formation

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Introduction

- With reduction in tariffs, domestic subsidies attracting a lot of attention (Bagwell and Staiger, AER, 2006)
- GATT was relatively relaxed about subsidies, WTO is not
- We examine the desirability of subsidies as well as possibility of cooperation in reducing those in an oligopoly setting.

Questions

1. Can unilaterally optimal subsidies be excessive from global welfare point of view? (Part I)
2. Is [1] just a possibility or fairly robust? (Part II)
3. To what extent countries can cooperate in reducing subsidies? (Part III)

Papers

- Papers
  1. Export/Domestic Subsidies: Brander and Spencer (1985, JIE)), Eaton-Grossman (1986, QJE), Dixit(1984, EJ), Horstmann and Markusen (1986, JIE)
  2. Subsidies/domestic policies in context of trade agreements: Bagwell and Staiger (2006 AER; 2001, QJE), Lee (2011 SMU working paper)
  3. Preferential Trade Agreements: Most speakers in the conference + many more!
  4. Coalition formation: To be mentioned later
- Step backward from [2], get rid of all instruments but domestic subsidies
- Build on [1], [3], and [4] and move towards [2]

Preliminaries

- $n(\geq 2)$-countries, each with unit mass of identical consumers
- Numeraire good y and n differentiated products
- Country $i \rightarrow$ firm $i \rightarrow$ variety $i \in \{1, 2, ..., n\} \equiv \mathcal{N}$
- Representative consumer in country i maximizes

$$U(q_{1i}, q_{2i}, ..., q_{ni}) + y_i$$

  subject to $\sum_{j \in \mathcal{N}} p_{ji} q_{ji} + y_i \leq I_i$
- Demand: $q_{ji} = q_{ji}(p_{1i}, p_{2i}, ..., p_{ni})$
- Inverse Demand: $p_{ji} = p_{ji}(q_{1i}, q_{2i}, ..., q_{ni})$

An example: Linear, Differentiated Duopoly

$$U(.) = a\sum_{j\in\mathcal{N}} q_{ji} - \frac{1}{2}\sum_{j\in\mathcal{N}} q_{ji}^2 - b\sum_{j\in\mathcal{N}}\sum_{k>j} q_{ji}q_{ki}$$

- Inverse demand: $p_{ji} = a - q_{ji} - b\sum_{k\neq j} q_{ki} = p_{ji}(\mathbf{q_i})$
- Products are
  - homogenous if $b = 1$,
  - independent if $b = 0$,
  - imperfect substitutes if $b \in (0, 1)$

Production

- Markets are segmented
- m_{ji}: constant marginal cost of firm j to sell in market i
- $m_{ji} = m$ for all $i, j \in \mathcal{N}$ if there are no subsidies
- Government i gives s^i per unit of q_{ii}
- Effectively, $m_{ji} = m - s^i$ if $j = i$ and $m_{ji} = m$ if $j \neq i$

Profit maximization

- Segmented markets + constant marginal cost $\Rightarrow$ n separate profit maximization problems for each firm.
- In market i, firm j chooses q_{ji} to maximize

$$\pi_{ji} = [p_{ji}(\mathbf{q_i}) - m_{ji}]q_{ji}$$

- Equilibrium outcomes for $i, j \in \mathcal{N}$:
  - Quantities: $q_{ji}(s^i)$
  - Prices: $p_{ji}(s^i)$

Comparative statics

- **RESULT 1:** Let $q_{ji}(s^i)$ and $p_{ji}(s^i)$ respectively denote equilibrium output and price of variety j in market i. An increase in s^i leads to higher q_{ii}. All other prices and quantities decline.

$$\frac{dq_{ii}(s^i)}{ds^i} > 0, \frac{dq_{ji}(s^i))}{ds^i} < 0$$
$$\frac{dp_{ii}(s^i)}{ds^i} < 0, \frac{dp_{ij}(s^i)}{ds^i} < 0$$

Profits, Consumer surplus and welfare

- Profit in own market i : $\pi_{ii}(s^i) = (p_{ii}(s_i) - (m - s^i))q_{ji}(s^i)$
- Profit in foreign market k: $\pi_{ik}(s^k) = (p_{ik}(s_k) - m)q_{ik}(s^k)$
- Consumer surplus:

$$cs^i(s^i) = U(q_{1i}(s^i), q_{2i}(s^i), ..., q_{ni}(s^i)) - \sum_{j \in \mathcal{N}} p_{ji}(s^i)q_{ji}(s^i)$$

- Corresponding to $\mathbf{s} \equiv (s^1, s^2, ..., s^n)$, welfare of country i is:

$$\begin{aligned} W^i(\mathbf{s}) &\equiv cs_i(s^i) + \pi_{ii}(s^i) + \sum_{k \neq i} \pi_{ik}(s^k) - s^i q_{ii}(s^i) \\ &= [U(\mathbf{q_i}(s^i)) - mq_{ii}(s^i) - \sum_{j \neq i} p_{ji}(s^i)q_{ji}(s^i)] + \sum_{k \neq i} \pi_{ik}(s^k) \end{aligned}$$

Nash subsidies

- Each country i chooses s^i to maximize $W^i(\mathbf{s})$
- First-order condition, $\frac{dW^i(\mathbf{s})}{ds^i} = 0$, does not involve s^k $(k \neq i)$
- $\Rightarrow$ for all s^k, the same $s^i = s^N$ (say) is the best response
- (Segmented markets strikes again!)

Nash subsidies (contd.)

- *Nash subsidy* s^N is implicitly given by

$$(p_{ii}(s^N) - m)\frac{dq_{ii}(s^N)}{ds^i} - \sum_{j \neq i} \frac{dp_{ji}(s^N)}{ds^i} q_{ji}(s^N) = 0. \qquad (1)$$

- Starting from $p_{ii}(s^i) - m = 0$, consider a small $\uparrow$ in s^i
  - second-order loss: $(p_{ii} - m)\frac{dq_{ii}}{ds^i} \approx 0$
  - first-order gain: $-\sum_{j \neq i} \frac{dp_{ji}}{ds^i} q_{ji} > 0$
- $\Rightarrow p_{ii}(s^N) - m < 0$

Efficient subsidies

- Efficient subsidy vector $\mathbf{s} \equiv \{s^1, s^2, ..., s^n\}$ maximizes $\sum_{i=1}^{n} W^i(\mathbf{s})$.
- Effectively *n* separate welfare maximization problems
- Choose s^i to maximize welfare *in* each market *i*:

$$cs^i(s^i) + \pi_{ii}(s^i) + \sum_{j \neq i} \pi_{ji}(s^i)$$

- Symmetry $\Rightarrow$ efficient s^i should be the same for all $i \in \mathcal{N}$.

Efficient subsidies (contd.)

- Efficient subsidy s^E is given by

$$(p_{ii}(s^E) - m)\frac{dq_{ii}(s^E)}{ds^i} + \sum_{j \neq i}(p_{ji}(s^E) - m)\frac{dq_{ji}(s^E)}{ds^i} = 0. \quad (2)$$

  - Starting from $p_{ii}(s^i) - m = 0$, consider a small $\uparrow$ in s^i:
  - second-order loss: $(p_{ii} - m)\frac{dq_{ii}}{ds^i} \approx 0$
  - first-order gain: $\sum_{j \neq i}(p_{ji} - m)\frac{dq_{ji}}{ds^i} > 0$
- $\Rightarrow p_{ii}(s^E) - m > 0$

Nash versus efficient outcomes

- **Result 2:** Nash subsidies are excessive, i.e. $s^N > s^E$.

  Proof:

  - (i) $p_{ii}(s^E) > m > p_{ii}(s^N)$
  - (ii) p_{ii} is $\downarrow$ in s^i
  - $(i) + (ii) \Leftrightarrow s^E < s^N$

Nash versus efficient outcomes

- Compared to efficient outcomes, under Nash we have
  - Excessive subsidies: $s^N > s^E$
  - Lower trade volume: $q_{ji}^N < q_{ji}^E$;

Part II: Robustness, the usual suspects

- Our results are robust with respect to
  1. functional forms
  2. mode of competition
  3. number of firms
- Just as domestic subsidy is optimal under (1)-(3) in a closed economy setting, our results concerning excessive domestic subsidies are also robust under (1)-(3)

Cost heterogeneity: the new suspect in town

- Uniform subsidy no longer optimal
  1. $s \uparrow \Rightarrow$ allocative efficiency improves
  2. $\Rightarrow$ production efficiency worsens
- For each national variety i, suppose there are two firms: L(ow-cost) and H(igh-cost).

$$\frac{q_{Li}}{q_{Hi}} = \frac{p_i - c_{Li} + s}{p_i - c_{Hi} + s} = 1 + \frac{c_{Hi} - c_L}{p_i - c_{Hi} + s}$$

- $s \uparrow \Rightarrow p_i \downarrow \Rightarrow p_i + s \uparrow \Rightarrow \frac{q_L}{q_H} \downarrow$ market share of low-cost type decreases.
- Nash subsidies are more likely to be excessive in a Melitz-type world when only L type serves the foreign market
- Country i takes the productive efficiency margin into account to the extent that its H_i steals business from L_i, but it ignores business stealing from foreign L. .

From s^N to s^E?

- Can countries agree on s^E?
- To answer that we need to let the countries do something.
- In trade agreements countries have bilateral instruments t_{ij}
- Here each country i has only one instrument s^i
  - Coalition formation
  - Each coalition chooses the subsidy level of its members to maximize the sum of welfare of coalition members
  - Product market competition

Broad brush

- Interested in
  - environments with externalities
  - possible group formation to tackle the externalities
  - role of rules/institution
- Examples
  1. Public goods/bads and voluntary contribution (Public Economics),
  2. Mergers, Research joint ventures (IO)
  3. Preferential/regional trade agreements (primarily on tariff reduction)
- Domestic subsidies, works differently from [3]. Similarity with [1] and [2] will be clear in that later part of the talk.

Definitions

- Partition n countries into m sets of countries $C_1, C_2, ..., C_m$
  - Non-empty: $C_k \neq \phi$,
  - Non-overlapping: $\bigcap_{k=1}^{m} C_k = \phi$
  - Exhaustive: $\bigcup_{k=1}^{M} C_k = \mathcal{N}$.
- Each nonempty $C \subseteq \mathcal{N}$ is a *coalition* and each partition $\sigma = \{C_1, C_2, ..., C_m\}$ represents a particular *coalition structure*.
- $\sigma(i)$: coalition that country i belongs to
- $|\sigma(i)|$: size of $\sigma(i)$.
- Example: $\mathcal{N} \equiv \{1, 2, 3, 4, 5, 6\}$
  - $(C_1, C_2, C_3) \equiv \{\{1\}, \{2, 3\}, \{4, 5, 6\}\}$
  - $\sigma(3) = C_2; |\sigma(3)| = |C_2| = 2$

- Let Σ denote the set of all possible coalition structures. Consider a $\sigma \in \Sigma$ and pick a $C \in \sigma$. Let $c = |C|$ denote the number of countries in C. Each coalition C chooses $(s^1, s^2, ..., s^c)$ to maximize

$$W^C(\mathbf{s}, \sigma) \equiv \sum_{i \in C} W^i(\mathbf{s}, \sigma)$$

Symmetry + segmented markets imply

1. s^i is the same for all $i \in C$
2. s^i is independent of coalition structure though W^i depends on subsidies offered by non-members of C.

Stage 2: Size of coalition, subsidy and welfare

- **Result 3:** Optimal subsidy, welfare per member lower in larger coalition.
  Let $\mathbf{s}(\sigma)$ denote the subsidy vector generated from such coalitional choices, $s^i(\sigma)$ denote the subsidy chosen by country i and let $v^i(\sigma) \equiv W^i(s(\sigma), \sigma)$ denote the welfare of country i. Then

$$
\begin{aligned}
|\sigma(i)| < |\sigma(j)| \quad &\Leftrightarrow \quad s^i(\sigma) > s^j(\sigma) \\
&\Rightarrow \quad v^i(\sigma) > v^j(\sigma)
\end{aligned}
$$

  - More members in i's coalition $\Rightarrow$ more concern for π_{ji} $\Rightarrow$ lower s^i
  - Note, members outside of the coalition benefit as well

Asymmetric coalitions

- **Claim 1:** The equilibrium coalition structure cannot have two coalitions of same size.
- **Proof:**
  - Consider i, j such that $j \notin \sigma(i)$ but

    $$|\sigma(i)| = |\sigma(j)|.$$

    Then

    $$s^i(\sigma) = s^j(\sigma)$$

  - Apply revealed preference argument
  - Note subsidy offered by the other coalitions do not change $\square$

To merge or not to merge?

- **Claim 2:** Consider $S(mall)$, $B(ig) \subset \mathcal{N}$ where $|S| < |B|$.
  - If $|B| < 2|S|$, members in both coalitions are better off under a new coalition $S \cup B$ irrespective of the differentiation parameter $b \in (0, 1)$.
  - If $|B| > 2|S|$, members in smaller coalition S are better off by staying separate for $b < \frac{3}{4}$
- Thus the result is "if and only if" for $b \in (0, \frac{3}{4})$
- For the remainder of talk we focus on $b \in (0, \frac{3}{4})$

Ideal world

- Ideally, an equilibrium coalition structure should be
  - **Merger proof:** no two coalitions have incentive to merge together
  - **Split proof:** no individual country has an incentive to split from a coalition
- By definition
  - Grand coalition is merger proof since no further merger is possible
  - Coalition structure of all singleton is Split proof since no further split is possible

Non-existence

- $\mathcal{N} \equiv \{1, 2, 3, 4, 5\}$
- **Merger proof:** $\mathcal{N}; (\{1\}, \{2345\})$
  - But neither of the above is split-proof
  - Consider unilateral splitting by country 5
  - The group it splits from remains at least 3 times large (after splitting)than $\{5\}$
  - So, splitting gives higher payoff to country 5
- Split proof rules out grand coalition almost by definition (except for small n), while merger proof always involve grand coalition.
- We look at the set of merger proof coalition structures; so grand coalition remains a possibility
- Select one that survives an extensive-form based charaterization.

Towards selection

- $n = 1 \Rightarrow (i)\mathbf{N}$
- $n = 2 \Rightarrow (i)\mathbf{N}$
- $n = 3 \Rightarrow (i)\mathbf{N}$
- $n = 4 \Rightarrow (i)\mathcal{N}; (ii)\mathbf{(\{1\}, \{234\})}$
- $n = 5 \Rightarrow (i)\mathbf{N}; (ii)(\{1\}, \{2345\})$
- $n = 6 \Rightarrow (i)\mathcal{N}; (ii)\mathbf{(\{1\}, \{23456\})}$
- $n = 7 \Rightarrow (i)\mathcal{N}; (ii)(\{1\}, \{234567\}, (iii)\mathbf{(\{12\}, \{34567\})}$
- $n = 8 \Rightarrow (i)\mathbf{N;}(ii)(\{1\}, \{2345678\}), (iii)(\{12\}, \{345678\})$

Observe: Grand coalition forms for $n = 1, 2, 3, 5, 8, \ldots$.

Fibonacci sequence and Fibonacci decomposition

- *Fibonacci* numbers form a sequence defined by the following recursion relation: $F_0 = 1,\ F_1 = 2$ and

$$F_{k+2} = F_{k+1} + F_k$$

- Let $\mathcal{F} = \{F_1 = 1, F_2 = 2, F_3 = 3, F_4 = 5, F_5 = 8, F_6 = 13, F_7 = 21, F_8 = 34, \ldots\}$ denote the set of Fibonacci numbers.
- The *Fibonacci decomposition of any positive integer m*, is a unique (in some sense minimal) expression of the form

$$m = F_{p(1)} + \ldots + F_{p(L)}$$

where $F_{p(l)} \in \mathcal{F}$ for all $l \in \{1, \ldots, L\}$ and $p(l+1) \geq p(l) + 2$ for all $l \in \{1, \ldots, L-1\}$.
- To illustrate take the number of attendees and find the decomposition.

A characterization

- **Claim:** Fibonacci decomposition of n is merger proof for all n.
- **Proof:**
  - Fibonacci decomposition of n does not contain consequent Fibonacci numbers (as it would not have been minimal then)
  - e.g., $18 = 13 + 5$ (skips 8), $27 = 21 + 5 + 1$(skips 13,8,3,2)
  - Adjacent (non-adjacent) numbers in Fibonacci sequence differ by less (more) than a factor of two

$$
\begin{aligned}
F_{k+2} &= F_{k+1} + F_k < F_{k+1} + F_{k+1} < 2F_{k+1} \\
&= F_{k+1} + F_k > F_k + F_k > 2F_k
\end{aligned}
$$

□

- **Tying Back:** A coalition of size F_k always merges with a coalition of size F_{k+1} but not with the one with size F_{k+2} or greater.

Sequential formation of coalitions in presence of externalities

- Technique
  - Bloch (GEB, 1996)
  - Ray and Vohra (JET 1997, GEB 1999)
- Applications
  - Bloch (Rand,1995), Ray and Vohra (2001)
- Similar ideas in simultaneous form games (Yi, 1996 JIE; ? GEB)

Game 1: A bargaining game

- One country (say 1) first proposes a coalition C and each prospective member responds
- If all accept, the coalition is formed; and the procedure is repeated among the remaining firms and country with lowest i chosen as the new initiator
- If one of the prospective members of C rejects, then she initiates the next proposal and so on...
- Countries while accepting or rejecting makes a prediction regarding complementary coalitions(which in equilibrium must be correct)
- Coalition structure forms when all agreements conclude
- If bargaining continues forever, countries get zero

The bargaining game (contd.)

- Restrictions
  1. Unanimity
  2. Once a coalition is formed, it cannot change its composition(restrictive)
  3. Sequential formation
- Richness
  - Forward looking nature of the game
  - Payoffs in our game derived from a game with a optimal policy choice and market competition.
- Pay-off relevant state at any stage of the game: ongoing proposal, the structure already formed
- Consider Markov strategies and find Markov perfect equilibrium

Game 2: Choice-of-coalition size game

- Denote the game by $\Delta(n, v)$
  - n: number of countries;
  - v is the vector of welfare valuations $v_i's$ where $i \in \mathcal{N}$.
- Country i is ranked i-th in the order of moves.
- The game $\Delta(n, v)$ is as follows:
  - Country 1 starts the game and chooses an integer $c_1 \in \{1, \dots, n\}$.
  - Country $c_1 + 1$ then moves and chooses $c_2 \in \{1, \dots, n - c_1\}$.
  - Country $c_1 + c_2 + 1$ chooses c_3 from $\{1, \dots, n - c_1 - c_2\}$...
- The game continues until the sequence of integers $(c_1, \dots, c_M)$ satisfies $\sum_{m=1}^{M} c_m = n$.
- Find the subgame-perfect equilibrium of this game
- Equilibrium coalition structures are identical upto a permutation of countries in Game 1 and Game 2.
- Outcome of $\Delta(n, v)$ is a decomposition of the number n.

Strategy in the game of choice of coalition size

- A strategy τ_i for player i in the game $\Delta(n, v)$ is a mapping from the set Σ_{i-1} to the set of integers $\{1, \ldots, n - i + 1\}$.
- For any coalition structure σ_{i-1} of the first $i - 1$ countries, country i chooses a coalition size $\tau_i(\sigma_{i-1})$.
- Note that all players need not be called to announce coalition sizes in the game
- For any strategy profile τ, a single coalition structure $\sigma(\tau)$ is formed and country i receives a payoff of $v_i(\sigma(\tau))$.

Subgame-perfect equilibrium and equilibrium Coalition Structure

- A strategy profile τ^* is a **subgame perfect equilibrium** if and only if $\forall\ i \in \mathcal{N}$, $\forall\ \sigma_{i-1} \in \Sigma_{i-1}$ and $\forall\ \tau_i \in \{1, \ldots, n-i+1\}$, $v_i(\sigma(\tau_i^*(\sigma_{i-1}), \tau_{-i}^*)) \geq v_i(\sigma(\tau_i(\sigma_{i-1}), \tau_{-i}^*))$.
- A coalition structure $\sigma(\tau^*)$ generated by a sub game perfect equilibrium τ^* is called an **equilibrium coalition structure**

Equilibrium Coalition Structure: partial characterization

- **Result 5:** Assume quadratic utility function and Cournot competition among firms. For $b \in \left(0, \frac{3}{4}\right]$,
  - the unique numerical equilibrium coalition structure for any $n (\geq 2)$ is the *Fibonacci decomposition* for n, and
  - grand coalition forms in equilibrium if and only if n is a *Fibonacci number*.
- What about $b \in (3/4, 1)$?
- Ongoing: Last attempt at recurrence relationships (in particular, Lucas sequence) before we turn to simulations.
- The fact that calculations are falling in line with recurrence relationships is not completely coincidental..
  - Welfare expressions are quadratic
  - Fibonacci and few other popular sequences in number theory are second-order difference equations

Lessons from coalition formation game and beyond Fibonacci

- **Endogenous Majority:** There always exists a coalition with majority of members
- **Free-riders:** Almost always, there will always be some
- **Grand Coalition:** Always a subsequence where grand coalition forms (things fall in place)
- Things not efficient, but may not be too far off
- Plurilateral arrangements

1. Why are we concerned about subsidies now?

- Requires multiple instruments or at least trade cost
- Does $t \downarrow \Rightarrow s^N(t) - s^E(t) \uparrow$?
- More importantly, does $t \downarrow \Rightarrow W(s^E(t)) - W(s^N(t)) \uparrow$?
- At some level, yes, if one compare the two extremes (prohibitive tariff and trade)
  1. Is it possible that Nash subsidies were insufficient before but excessive now?
  2. If there is a choice of choosing one instrument first, and one later, is this the order — tariff negotiations first, subsidies next?
- Models with trade cost + heterogeneity show some promise

2. Entry

- Tricky; in presence of entry, subsidy is not necessarily positive
- Three things are at work
  1. Allocative efficiency
  2. Production efficiency
  3. Variety
- Subsidy improves [1] and usually [3] as well, but worsens [2] in any environment where $N \uparrow$ leads to $q \downarrow$ [business-stealing effect]
- Homogenous products Cournot: Optimal subsidy is zero for linear demand
- Entry in oligopoly trade models rare, more so in context of trade agreements
  - Venables (1985, JIE), Horstman and Markusen (1986, JIE), Bagwell and Staiger (forthcoming 2010, JIE)
  - Our conjecture: results will depend on the particular way the entry is modelled — entry of new varieties or entry of new firms for a given variety