# "Super-Sizing" International Trade 

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#### Abstract

This paper develops a monopolistically competitive model of international trade where firms give consumers the option to "super-size" their consumption. Such a strategy is a response to consumer heterogeneity (e.g. income differences) and is implemented via second degree price discrimination (i.e. offering a menu of products to choose from). Allowing firms this option not only has implications for outcomes in an open economy, but also more broadly for models of competition and price discrimination. Compared to a monopoly model of second degree price discrimination it is shown that a firm always offers a complete product-line (i.e. all types are served in equilibrium). Hence, competition is associated with greater firm scope. However, the principle of "no distortion at the top" no longer applies, with high types offered an excessively lavish product at the same time low types receive an inferior product relative to the first best. Thus distortions are pervasive in this setting and occur in both directions. Allowing firms to offer "super-sized" products also has a number of important implications for how international integration is perceived. Relative to the standard model of single product firms it is shown that the volume of trade, number of varieties and welfare are all higher when firms use a product-line to indirectly discriminate. This suggests that the standard model substantially under-predicts the potential benefits of international integration. In addition, it is shown that a country's distribution of income is an important determinant of the gains from trade. In particular, countries with a "good" distribution of income receive magnified gains while those with a "bad" distribution have their gains diminished.


Key Words: Intra-industry trade, monopolistic competition
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## 1 Introduction

Recent studies have documented the important role played by multi-product firms in national economies and in international trade in particular. In the U.S. context, Bernard et al. (2009) find that 10 percent of exporters and 20 percent of importers trade 10 or more HS10 products and that these firms account for around 90 percent of exports and imports by value. These compelling figures are at odds with the standard models of international trade which assume that firms trade a single product. Given such a divergence it is natural to wonder whether the standard model is missing an important dimension by neglecting factors that influence the scope of a firm. To assess this issue a number of models have been constructed that provide a technological explanation for firm scope. ${ }^{1}$ The typical finding is that international trade can cause a reallocation of resources (potentially both within and across firms) but the welfare implications are similar to the standard model. ${ }^{2}$

In contrast, this paper approaches firm scope from a different perspective. Rather than appealing to purely technological factors, I couch the analysis in terms of a firm that uses multiple products to indirectly discriminate between consumers with different valuations. That is, firms are allowed to pursue second degree price discrimination. ${ }^{3}$ This makes consumer heterogeneity the principal determinant of firm scope rather than relying on purely technological factors. Consequently, second degree price discrimination and its associated menu of choices provides a natural motivation for multi-product firms where the length of a product-line is the dimension of interest. In this setting a product-line is associated with either products of different quality or firms offering quantity discounts.

This contrasts with the technology based models of firm scope which aim to motivate conglomerate type firms that produce across a number of different industries (Bernard et al. (2011)) or a range of horizontally differentiated goods (Eckel and Neary (2010), Feenstra and Ma (2007)). Instead our focus is on understanding the implications of trade in goods such as the best selling perfume in the world, Chanel $N^{o} 5$. This perfume is offered in not only a number of different concentrations (e.g. Eau de Parfum, Eau de Toilette and Eau de Cologne) but also in range of different sizes. In the case of the Eau de Parfum it is offered in .25 ounce, .50 ounce and 1 ounces bottles. Critically, the price per ounce of the largest size is discounted by over $30 \%$ compared to the price per ounce of the smallest size. Given the marginal cost of the largest and smallest bottles is essentially the same, this

[^0]is a clear example of quantity discounting - a key feature of a "supersizing" menu. These quantity discounts are generally interpreted as the implicit payment of information rents to the higher valuation customers. Hence, the implied distribution of prices can generate profoundly different welfare outcomes across consumers.

While Chanel $N^{o} 5$ offers a particularly clear example of "supersizing", these strategies are not limited to the selling of perfume. For example, the 2010 Corolla is offered in a base model, LE and XLE, with each successive upgrade associated with a higher price. Nevertheless, both the LE and XLE are associated with "extra value packages" not available on the base model. In this sense, these packages are consistent with an incentive to "super-size" your Corolla. ${ }^{4}$ Since this behavior is pervasive in the majority of globally integrated industries, yet it has not been studied in an international setting, it has the potential to alter our understanding of the implications of international trade. Indeed, as argued below, the gains from trade, their distribution and the volume of trade are modified by this behavior.

While this is not the first paper to develop a model of price discrimination and competition, the framework developed does address two shortcomings of the previous literature. ${ }^{5}$ First, by adopting a spatial model of competition, the previous literature restricts competition to occur over relatively low valuation consumers. This arises since consumers located close to one firm are simultaneously high value for that firm and low value for other firms. Such a negative correlation in valuations means that competition is focused on the consumer that is indifferent between their two closest firms. While this structure may be appropriate for some settings, it is questionable in others. For example, a consumer considering the purchase of a TV is likely to have a stronger preference over size ( 40 " versus 55 ") than over brand (Samsung, LG, Sony, Panasonic, etc). This suggests that all firms are likely to have a common ranking of a given consumers valuation. This implies that competition is likely to occur all along the product-line rather than just at the low end. The second shortcoming is that spatial models are not straightforward to integrate into an international setting.

To overcome these issues this paper introduces second degree price discrimination into a "love of variety" model of monopolistic competition. This is achieved by adopting the Spence-Dixit-Stiglitz preference structure that underlies the standard model of intra-industry trade. ${ }^{6}$ Under a "love of

[^1]variety" set-up, all firms agree on the rank of the high and low types, and consequently competition is more generalized across types. This places the focus on the residual demand curve and its location. As usual in the CES framework, an increase in the number of firms implies that residual demand is reduced for the typical firm, and along with it the potential rents extracted from all types served. This change in the structure of preferences produces a number of results that differ from both the standard monopoly model of indirect discrimination and the closely related spatial framework.

The first main result concerns the nature of consumption distortions. While low valuation types have their consumption options distorted below the first best (for the usual reasons), the high valuation types are now offered a consumption bundle in excess of the first best. That is, the option designed for the high types is excessively lavish. This result is in line with the perception of generalized excess for high end products (think of first class travel, high end cars, the starbucks venti size or super-sized meals), while low end products appear excessively mean or flimsy. Indeed the analysis of indirect price discrimination is often motivated by a quote from Dupuit (1849) who describes indirect discrimination as being characterized by a situation where firms: "having refused the poor what is necessary, they give the rich what is superfluous".

Although this motivation is pervasive in the literature, an excessively extravagant bundle designed for the high type is not an outcome that arises in the standard monopoly model of second degree price discrimination or the spatial model of monopolistic competition. ${ }^{7}$ To gain intuition for this new result note that the rents extracted under second degree price discrimination are smaller than under perfect price discrimination (which is associated with an efficient outcome - see Spence (1976)). This has two immediate implications when there is free entry. First, since rent extraction is lower than under the first best, fewer firms have the capacity to cover the entry cost. Therefore, second degree price discrimination is associated with a sub-optimally low number of firms. Second, fewer firms implies less intense competition, and accordingly greater residual demand for the typical firm. Combining these two insights with a final characteristic of second degree price discrimination generates excessiveness at the top: the features of the high end bundle are chosen such that marginal cost equals perceived marginal benefit. However, since there are too few firms, this perceived marginal benefit is greater than the first best and as a consequence an excessively endowed option is offered to the high type. Therefore, it is the interaction between the generalized nature of

[^2]competition and market structure that plays a central role in this new result.
The second main result is that all types are served in equilibrium. Therefore the most egregious distortions associated with exclusion from the market are avoided in the "love of variety" framework. Once again this result differs from what happens in both the spatial and monopoly models. In the case of monopoly, supplying the low type is necessarily associated with the conceding of information rents to the high types. Thus, the trade-off faced by a monopolist revolves around the relative frequency of the various types in the population, the more low types, the more likely they are to be served. ${ }^{8}$ In the spatial model the mechanism is similar although it is augmented by entry costs. If entry costs are sufficiently high, then the market need not be covered with the implication that certain market segments are not served. Yet in a "love of variety" model the low types are always served in equilibrium, i.e. each firm offers a full product line. ${ }^{9}$ Once again the residual demand curve plays a central role. If all other firms choose not to supply the low types, then this presents a firm with the opportunity to be the sole supplier of the low types and consequently the firm faces an enormous residual demand. Such a profitable deviation undermines any proposed equilibrium involving lack of service to the low types. Note that once again the generalized nature of demand is critical for this result. If customers are not served by one firm in the spatial model, then only one other firm has a viable opportunity to serve them, as a result residual demand is far more restricted. Under the "love of variety" set-up the greater substitution possibilities imply that the residual demand is a more potent force for determining outcomes.

The extension of this model to an international environment also has important implications relative to the standard single product model. A key result relates to the equilibrium number of varieties. Since the ability to extract rents is greater than in the standard model, the number of firms that can cover the cost of entry is now greater. Consequently, indirect discrimination is associated with greater entry. This naturally translates into two other important results: both welfare and trade are higher when firms discriminate. Or stated another way, multi-product firms enhance the gains from trade. This suggests that our standard model of intra-industry trade not only under predicts the diversity in an economy but it also understates the volume and gains from trade.

An additional result emerges when countries differ in terms of their distribution of income. Since

[^3]the size of the consumption distortion is determined by the distribution of income, free trade alters the size of this distortion relative to autarky in both countries. In particular, it is shown that if a country has a better distribution of income (more high income types) then this country receives enhanced gains from trade since the consumption distortion is reduced when markets are integrated. However, the converse also holds - countries with a "bad" distribution of income receive diminished gains due to a larger distortion. As a result the distribution of income can inform a country's perspective on international integration.

To establish these results this paper has the following organization. Section 2 sets out two important benchmarks in the standard representative agent "love of variety" model: the equilibrium for the standard linear price model and the equilibrium for perfect price discrimination. While the pricing structure associated with each of these scenarios is very different, it is shown that the equilibrium outcomes have a number of common features that help establish the intuition for the later analysis. Section 3 adds a second consumer type with proportionately higher marginal utility. This heterogeneity makes little difference to the analysis if it is observable and firms can directly discriminate. However, if these differences are not observable then a firm has an incentive to take advantage of this heterogeneity by implementing an indirect discrimination mechanism where consumers self select from a menu of available options (second degree price discrimination). The monopolistically competitive equilibrium of this model is characterized and the main results derived for the closed economy. Section 4 considers the possibility of international trade and discusses the implication of the model for the number of varieties, the volume of trade and welfare. Section 5 explores the implications of asymmetric distributions of high and low types across countries (generated by differences in income distribution). In this case, the gains from free trade are shown to be greater for the country that has income concentrated in the upper end of the distribution. Section 6 concludes with a number of suggestions for possible extensions.

## 2 Preferences

At the heart of the standard model of intra-industry trade is a representative consumer with "love of variety" preferences. A particularly convenient representation of these preferences is given by: ${ }^{10}$

$$
\begin{equation*}
U=\frac{A}{\alpha} Q^{\alpha}+y, \quad A>0, \quad 0<\alpha<1 \tag{1}
\end{equation*}
$$

where $Q$ is an index of consumption in the differentiated goods sector and $y$ is consumption of a homogeneous good which is taken to be the numeraire (i.e. $p_{y}=1$ ). Note that $A$ can be interpreted as the inverse of the marginal utility of income. The implications of this interpretation are taken up in section 5. As is traditional in this literature $Q$ is assumed to have a CES form:

$$
\begin{equation*}
Q=\left[\int_{0}^{n} q(v)^{\rho} d v\right]^{\frac{1}{\rho}}, \quad 0<\rho<1 \tag{2}
\end{equation*}
$$

It follows that the marginal utility of differentiated variety $v$ is:

$$
\frac{\partial U}{\partial q(v)}=A Q^{\alpha-\rho} q(v)^{\rho-1}=A Q^{\frac{\epsilon-\sigma}{\epsilon \sigma}} q(v)^{-\frac{1}{\sigma}}
$$

where $\sigma=\frac{1}{1-\rho}$ and $\epsilon=\frac{1}{1-\alpha}$. Furthermore, assume that $\sigma>\epsilon \Leftrightarrow \rho>\alpha$. That is, varieties within a sector are more substitutable for each other than they are for $y$.

### 2.1 Benchmark 1: The Standard Linear Price Model

Under the usual linear price assumption a firm is constrained to set a fixed per unit price, with a consumer then choosing a quantity to maximize utility by equating their marginal utility to this price. A firm maximizes profits by setting a price that is a mark-up over marginal cost. Given the assumption on preferences, the elasticity is constant and consequently so is the mark-up. Furthermore we follow the literature and assume that marginal cost is also constant. Profit maximizing prices are then given by the following simple formula:

$$
p(v)=\frac{c}{\rho}=\left(\frac{\sigma}{\sigma-1}\right) c
$$

[^4]Based on this optimal behavior, the value function for a firm has the following form:

$$
\begin{equation*}
\pi(v)=\frac{c q(v)}{\sigma-1}-F \tag{3}
\end{equation*}
$$

where $F$ is a fixed cost of production. In line with the previous literature we assume, at this stage, that a firm only produces a single good.

In equilibrium, free entry requires that no firm makes positive profits. From (3) this implies that firm scale is

$$
\begin{equation*}
q=\frac{(\sigma-1) F}{c} \tag{4}
\end{equation*}
$$

While market clearing implies that the equilibrium number of firms under a linear price assumption is:

$$
\begin{equation*}
n^{\text {Linear }}=\left[\frac{\rho A}{c q^{\frac{1}{\epsilon}}}\right]^{\frac{\epsilon(\sigma-1)}{\sigma-\epsilon}}=\left[\frac{\rho A}{c q^{1-\alpha}}\right]^{\frac{\rho}{\rho-\alpha}} \tag{5}
\end{equation*}
$$

This equilibrium can be depicted from the perspective of a typical firm with the aid of a familiar diagram (see Figure 1). The firm optimum is defined by $M R=M C$ while the industry equilibrium is associated with $A R=A C$. While these are general conditions, the advantage of the functional form assumptions are apparent from the relationship between $A R$ and $M R$, in particular $M R=\rho A R$.

In a free entry equilibrium profits are driven to zero and consumer surplus is the only source of welfare. The equilibrium level of consumer surplus is given by:

$$
\begin{equation*}
C S^{\text {Linear }}=\frac{(1-\alpha)}{\alpha} \frac{c}{\rho} n^{\text {Linear }} q \tag{6}
\end{equation*}
$$

### 2.2 Benchmark 2: First Degree Price Discrimination Model

From a firm's perspective setting a linear price forgoes surplus from two sources: consumer surplus and deadweight loss. However, if the firm can implement first degree price discrimination then it can capture both these sources of rent. With such an ability a firm is able to extract the entire surplus under the residual demand curve associated with its product. The marginal utility function


Figure 1: Equilibrium with linear pricing
derived above can be used to calculate the associated surplus functions:

$$
\begin{equation*}
S(q)=\theta \int_{0}^{q} z^{\rho-1} d z=\frac{\theta q^{\rho}}{\rho} \tag{7}
\end{equation*}
$$

where $\theta=\left(\frac{A}{Q^{\rho-\alpha}}\right)$. To implement first degree price discrimination the firm chooses both the transfer, $T(v)$, and the quantity, $q(v)$, subject to this surplus function (i.e. $T(v)=S(q(v))$ ).

$$
\pi(v)=\frac{\theta q(v)^{\rho}}{\rho}-c q(v)-F
$$

The first order condition yields:

$$
\frac{\partial \pi(v)}{\partial q(v)}=\theta q(v)^{\rho-1}-c=0
$$

This behavior implies the value function of the firm is:

$$
\pi(v)=\frac{c q(v)}{(\sigma-1)}-F
$$

Note that the reason that the value function has the same form as the linear price regime is that the implied per unit price is the same (i.e. $\frac{T(v)}{q(v)}=\frac{c}{\rho}$ ). ${ }^{11}$

Once again equilibrium implies that all firms behave symmetrically, setting the same transfer and quantity menu, and earning zero profits. This implies that the equilibrium scale is given by:

$$
\begin{equation*}
q=\frac{(\sigma-1) F}{c} \tag{8}
\end{equation*}
$$

Market clearing implies that the equilibrium number of firms under a first degree price discrimination scenario is:

$$
\begin{equation*}
n^{1 s t}=\left[\frac{A}{c q^{\frac{1}{\epsilon}}}\right]^{\frac{\epsilon(\sigma-1)}{\sigma-\epsilon}}=\left[\frac{A}{c q^{1-\alpha}}\right]^{\frac{\rho}{\rho-\alpha}} \tag{9}
\end{equation*}
$$

Therefore, the relative number of varieties under the two pricing regimes is given by:
LEMMA 1. $\frac{n^{\text {Linear }}}{n^{1 s t}}=\rho^{\frac{\rho}{\rho-\alpha}}<1$
This equilibrium is represented in Figure 2 (an analogue to Figure 1). Once again profits are maximized by $M R=M C$ and industry equilibrium requires $A R=A C$. The key difference between linear pricing and first degree price discrimination is that the $M R$ function is the same as the residual demand function under first degree price discrimination since adding an additional unit for sale has no impact on the revenue of the infra-marginal units. This also implies that the $A R$ function differs across the two scenarios. In the current setting $A R$ is simply derived by calculating $T R(q)=T(q)=\frac{\theta q^{\rho}}{\rho}$ and dividing by $q, A R=\frac{\theta q^{\rho-1}}{\rho}$. While the average revenue functions appear to differ across the two pricing regimes, they in fact coincide in each equilibrium. The element that coordinates this outcome is that the equilibrium number of firms differ across the two scenarios. In particular, since $n^{1 s t}>n^{\text {Linear }}$, the residual demand curve under first degree price discrimination is closer to the origin. Moreover, it is still the case that $M R=\rho A R$. Using lemma 1 it follows that $M R^{\text {Linear }}=\rho \theta^{\text {Linear }} q^{\rho-1}=M R^{1 s t}=\theta^{1 s t} q^{\rho-1}$ and $A R^{\text {Linear }}=\theta^{\text {Linear }} q^{\rho-1}=A R^{1 s t}=\theta^{1 s t} \frac{q^{\rho-1}}{\rho} . \mathrm{A}$ final implication is that the per unit price must be the same in both situations, $A R^{1 s t}=A R^{\text {Linear }}=$ $\frac{c}{\rho}$, since $q$ is the same under both regimes.

As Spence (1976) emphasizes, unlike the monopoly case, first degree price discrimination under monopolistic competition does not imply that consumers receive zero net benefits. In fact, if products

[^5]

Figure 2: Equilibrium with $1^{\text {st }}$ degree price discrimination
are close substitutes, then the contribution of any product to total surplus is likely to be small. This implies that profits will be small and consumer surplus will be large. In other words, while a firm can extract the marginal surplus associated with their product, they cannot extract the average surplus associated with a typical good (i.e. consumers are indifferent about consuming any given product, but they are not indifferent about the entire basket of goods). ${ }^{12}$ With this in mind consumer surplus can be calculated as:

$$
\begin{equation*}
C S^{1 s t}=\left(\frac{\rho-\alpha}{\alpha}\right) \frac{c}{\rho} n^{1 s t} q \tag{10}
\end{equation*}
$$

Moreover as noted by Spence (1976) this is also the first best outcome as both product scale and variety are efficient.

[^6]
## 3 Two types of Consumer

A simple extension of the previous framework is to move from a representative consumer setting to one where there is some generalized heterogeneity in the population. To achieve this assume that instead of a single representative consumer with a taste for differentiated goods given by $A$, that there are now two consumer types, a high taste type $\left(A_{H}\right)$ and a low taste type $\left(A_{L}\right)$, where $A_{H}>A_{L} \cdot{ }^{13}$ Assume that the fraction of high taste types is $\beta$ and low taste types is therefore $1-\beta$.

### 3.1 Complete Information

If a consumer's type is costlessly observable then the previous analysis is generalized in a straightforward manner for both pricing regimes. For a linear price firm there is no incentive to set different prices for the different types (third degree price discrimination) since both types possess the same perceived elasticity of demand, $\sigma$. A similar result also holds for first degree price discrimination.

Since it serves as a natural link between the previous analysis and the framework to follow the details of the first degree price discrimination equilibrium are represented in Figure 3. Total production, $q$, is the weighted average of the outputs designed for the two types, and in equilibrium this output is associated with an average cost that also equals average revenue. Average revenue is the weighted average of $A R_{L}$ and $A R_{H}$. Note that at $q_{L}, A R_{L}=c / \rho$ and similarly at $q_{H}$, $A R_{H}=c / \rho$. Therefore both types pay the same average price despite having differing preference intensities. This per unit price neutrality follows directly from the functional form assumptions that imply $A R=M R / \rho$. Therefore, given that $M R=c$ for profit maximization for both types, it follows that $A R=c / \rho$. Note that this is also the same result that emerges under third degree price discrimination. This represents a sufficiently useful benchmark that we record it in the following proposition.

PROPOSITION 1. If preferences take the form of (1) and (2), and consumer heterogeneity is represented by differences in the shift parameter $A_{j}$, then an equilibrium where firms can implement a direct price discrimination mechanism involves all consumers being charged a per unit price of $c / \rho$. That is, there is no discrimination in terms of the average price across customers.

[^7]

Figure 3: Equilibrium with two types of consumers

So far we have reviewed the standard monopolistic competition model under both linear prices and first degree price discrimination and shown that adding consumer heterogeneity does not alter the results in any fundamental way. However, a critical assumption is that consumer type is observable. Once we relax this assumption the analysis is fundamentally altered. In particular, the ex post terms of supply and the quality of supply vary quite dramatically across types if the firm does not have access to complete information.

### 3.2 Incomplete Information

To design an indirect discrimination mechanism a firm can utilize the information on the demand and surplus functions associated with each consumer type. With two types these surplus functions follow directly from (7) and are labeled as follows:

$$
\begin{equation*}
S_{j}(q)=\theta_{j} \frac{q^{\rho}}{\rho}=\theta_{j} u(q) \quad j=L, H \tag{11}
\end{equation*}
$$

where $\theta_{j}=\left(\frac{A_{j}}{Q_{j}^{\rho-\alpha}}\right)$ which the firm is assumed to take as given. ${ }^{14}$

[^8]
### 3.3 Firm Behavior

Since monopolistic competition assumes an absence of strategic interactions, the profit maximizing program resembles that of an indirectly discriminating monopolist. Therefore, using the surplus functions from above and the information on the distribution of types in the population a firm chooses a menu of $\left\{\left(q_{L}, T_{L}\right),\left(q_{H}, T_{H}\right)\right\}$ to maximize

$$
\pi=\beta\left(T_{H}-c q_{H}\right)+(1-\beta)\left(T_{L}-c q_{L}\right)-F
$$

subject to

$$
\begin{array}{r}
\theta_{L} u\left(q_{L}\right)-T_{L} \geq \theta_{L} u\left(q_{H}\right)-T_{H} \\
\theta_{H} u\left(q_{H}\right)-T_{H} \geq \theta_{H} u\left(q_{L}\right)-T_{L} \\
\theta_{L} u\left(q_{L}\right)-T_{L} \geq 0 \\
\theta_{H} u\left(q_{H}\right)-T_{H} \geq 0 \tag{15}
\end{array}
$$

where (12) and (13) are the incentive compatibility constraints for the low and high types, respectively, while (14) and (15) are the corresponding participation constraints. In a standard monopoly problem, the ordering of the taste shift parameters would be enough to ensure that the single crossing property holds; implying that only two of these constraints bind, the incentive constraint for the high type, (13), and the participation constraint for the low type, (14). However, since the $\theta_{j}^{\prime} s$ are part of an equilibrium outcome we cannot simply impose that $\theta_{H}>\theta_{L}$. Nevertheless we conjecture that this ordering holds and will check that it is in fact satisfied in equilibrium. Under this conjecture the relevant constraints can be rewritten as:

$$
\begin{align*}
T_{H} & =\theta_{H} u\left(q_{H}\right)-\theta_{H} u\left(q_{L}\right)+\theta_{L} u\left(q_{L}\right)  \tag{16}\\
T_{L} & =\theta_{L} u\left(q_{L}\right) \tag{17}
\end{align*}
$$

This allows the profit maximization problem to be simplified to become:

$$
\begin{equation*}
\max _{q_{L}, q_{H}} \pi=\beta\left(\theta_{H} u\left(q_{H}\right)-\theta_{H} u\left(q_{L}\right)+\theta_{L} u\left(q_{L}\right)-c q_{H}\right)+(1-\beta)\left(\theta_{L} u\left(q_{L}\right)-c q_{L}\right)-F \tag{18}
\end{equation*}
$$

Taking first order conditions gives:

$$
\begin{align*}
\frac{\partial \pi}{\partial q_{L}} & =(1-\beta)\left(\theta_{L} u^{\prime}\left(q_{L}\right)-c\right)-\beta\left(\theta_{H}-\theta_{L}\right) u^{\prime}\left(q_{L}\right)=0  \tag{19}\\
\frac{\partial \pi}{\partial q_{H}} & =\theta_{H} u^{\prime}\left(q_{H}\right)-c=0 \tag{20}
\end{align*}
$$

Therefore as in the standard monopolistically competitive model an individual firm acts just like a monopolist. If the firm were a monopolist, then the $\theta_{j}^{\prime} s$ are preference parameters and the high type is offered an efficient quantity $\left(\theta_{H} u^{\prime}(q)=c\right)$. This is the well known "no distortion at the top" result. A monopolist would also distort the low types quantity away from the efficient level (since $\theta_{L} u^{\prime}(q)>c$ ) by selecting a product quantity that is too small. While the essence of these results carry over to the monopolistic competition setting it is important to keep in mind that the $\theta_{j}^{\prime} s$ are not preference parameters but are in fact equilibrium outcomes. As will be seem below, this distinction has important implications for the characterization of the outcomes.

Using the constant elasticity functional forms the profit maximizing quantities are:

$$
\begin{align*}
q_{H} & =\left(\frac{\theta_{H}}{c}\right)^{\sigma}  \tag{21}\\
q_{L} & =\left(\frac{\theta_{L}-\beta \theta_{H}}{(1-\beta) c}\right)^{\sigma} \tag{22}
\end{align*}
$$

Inspection of (22) delivers the following result:
LEMMA 2. If $\frac{\theta_{L}}{\theta_{H}} \leq \beta$, then $q_{L}=0$ and only the high types are served in equilibrium.
Note that if the firm were a monopolist then this outcome could arise in equilibrium if $\beta$ is sufficiently large since $\theta_{L}$ and $\theta_{H}$ would be parameters from some underlying utility function. However, under monopolistic competition no definitive conclusion can be drawn at this point. Nevertheless, if both types of goods are produced in equilibrium, then a relationship between the two goods can be summarized by the following lemma:

LEMMA 3. If $q_{L}>0$, then $q_{L}$ is proportional to $q_{H}$. Hence, the ratio $\frac{q_{L}}{q_{H}}$ is a constant, $\phi_{2}$, where

$$
\begin{equation*}
\phi_{2}=\frac{q_{L}}{q_{H}}=\left(\frac{\frac{\theta_{L}}{\theta_{H}}-\beta}{(1-\beta)}\right)^{\sigma} \tag{23}
\end{equation*}
$$

Regardless of whether or not the firm offers a full product-line, the value function can be written
as:

$$
\begin{aligned}
\pi^{*} & =\beta\left(\theta_{H} \frac{q_{H}^{\rho}}{\rho}-c q_{H}\right)+\left(\theta_{L}-\beta \theta_{H}\right) \frac{q_{L}^{\rho}}{\rho}-(1-\beta) c q_{L}-F \\
& =\beta\left(\frac{c q_{H}}{\rho}-c q_{H}\right)+(1-\beta)\left(\frac{c q_{L}}{\rho}-c q_{L}\right)-F \quad \text { using }(19) \text { and }(20) \\
& =\frac{\left(\beta q_{H}+(1-\beta) q_{L}\right) c}{(\sigma-1)}-F \\
& =\frac{c q}{\sigma-1}-F
\end{aligned}
$$

Hence the value function for a typical firm takes exactly the same form as in the other pricing regimes.

### 3.4 Industry Equilibrium

Imposing the zero profit condition associated with free entry implies that a familiar scale is achieved in equilibrium:

$$
\begin{equation*}
q=\frac{F(\sigma-1)}{c} \tag{24}
\end{equation*}
$$

The next question to address is how this output is distributed across types.
For each type the quantity index becomes, $Q_{j}^{\rho-\alpha}=n^{\frac{\rho-\alpha}{\rho}} q_{j}^{\rho-\alpha}=\tilde{n} q_{j}^{\rho-\alpha}$, which implies $\theta_{j}=$ $\left(\frac{A_{j}}{\tilde{n} q_{j}^{\rho-\alpha}}\right)$. Therefore the high type quantity can be written as:

$$
\begin{equation*}
q_{H}=\left(\frac{A_{H}}{\tilde{n} c}\right)^{\epsilon} \tag{25}
\end{equation*}
$$

While the solution to $q_{L}$ is not as neat it is still possible to characterize its implicit properties. In particular, $q_{L}$ solves:

$$
\begin{equation*}
(1-\beta) c q_{L}^{1-\alpha}+\beta \theta_{H} q_{L}^{\rho-\alpha}=\frac{A_{L}}{\tilde{n}} \tag{26}
\end{equation*}
$$

Note that since the left hand side is equal to zero when $q_{L}=0$ and is strictly increasing in $q_{L}$, while the right hand side is positive, there is a unique solution to this equation that involves $q_{L}>0$. Consequently, the low type is never excluded from the market and is always served in equilibrium.

This leads to the following proposition:

PROPOSITION 2. In a setting where firms engage in monopolistic competition and employ an indirect price discrimination mechanism, then in contrast to the monopoly outcome both types are always served in equilibrium.

This result provides a stark contrast to the monopoly model where the extent of the product-line varies depending on the parameters in the model. ${ }^{15}$ In particular the fraction of low valuation types could be sufficiently small that the monopolist would optimally truncate the product-line and just serve the high valuations types (extracting all their surplus in the process), a scenario captured by lemma 2 when the $\theta_{j}^{\prime} s$ are interpreted as preference parameters. However, this result does not carry over to monopolistic competition as all firms provide a full product-line in equilibrium. ${ }^{16}$ Furthermore, the proposition emphasizes the role that competition plays in extending the length of the product-line. To see that it is indeed competition and not the "love of variety" specification, consider the length of the product-line a monopolist would offer in the same situation (when faced with "love of variety" preferences and heterogeneous consumers). In this case the equilibrium bundle for the low type is given by $q_{L}=\left(\frac{\left(A_{L}-\beta A_{H}\right) n^{\alpha / \rho}}{(1-\beta) c}\right)^{\epsilon}$. As is evident, even a multiproduct monopolist has an incentive to truncate the product-line when the share of high types is sufficiently large.

The intuition for this result is straight-forward and follows directly from the interpretation of $\theta_{j}$ as a measure of residual demand. In a symmetric equilibrium $\theta_{L}=\frac{A_{L}}{\left(n^{\frac{1}{\rho}} q_{L}\right)^{\rho-\alpha}}$. Consequently, if no other firm is producing output for the low type, then the firm producing variety $v$ faces an extremely large residual demand curve (since the denominator of $\theta_{L}$ approaches 0 ). In this case the residual demand curve for the low type would be larger than the residual demand curve for the high type, which implies that not only would the firm producing $v$ have an incentive to produce for the low type but so would all other firms. Since $q_{L}=0$ can't be part of a symmetric equilibrium, it follows that $q_{L}>0$ and all firms offer a full product-line.

Since both types are always served in equilibrium and output is symmetric, (23) implies that $\phi_{2}$ is the solution to:

$$
\begin{equation*}
\beta \phi_{2}^{\frac{1}{\epsilon}-\frac{1}{\sigma}}+(1-\beta) \phi_{2}^{\frac{1}{\epsilon}}=\frac{A_{L}}{A_{H}} \tag{27}
\end{equation*}
$$

Note that the solution to this equation has the following properties: when $\beta=0$, then $\phi_{2}=\phi_{1}=$

[^9]$\left(\frac{A_{L}}{A_{H}}\right)^{\epsilon}$, and that $\frac{d \phi_{2}}{d \beta}<0$ for $\beta \in(0,1)$. In particular, $\phi_{1}$ is the first best distribution of bundle size. This leads to the following lemma

LEMMA 4. The ratio of high to low type consumption of any given variety under second degree price discrimination is greater than the corresponding ratio under first degree price discrimination for $\beta \in(0,1)$.

When combined with the observations that first degree price discrimination achieves the first best outcome and that $q$ is the same across both price regimes then the following proposition is apparent:

PROPOSITION 3. In a model of monopolistic competition and second degree price discrimination, not only is the good designed for the low types below the efficient level (degraded), but the high type is offered a product that exceeds the efficient level (lavish).

Thus while the overall level of production of a variety is efficient, its distribution across the various types is not. ${ }^{17}$ This is a striking result given the robustness of the "no distortion at the top" finding in the previous literature. ${ }^{18}$ Nevertheless the intuition is straight-forward. Since a firm cannot capture all of the perceived rents associated with the optimal output, $q$, fewer firms enter in equilibrium than would occur under the first best (this is confirmed below). Consequently relative to the first best, a typical firm faces greater residual demand. However, just as under the first best a firm chooses output for the high type that equates the perceived marginal surplus to marginal cost. Since residual demand is greater than socially optimal so will be the equilibrium output chosen by a typical firm.

Moreover this result captures the situation described in the following famous quote that is typically used to motivated the study of second degree price discrimination:

It is not because of the few thousand francs which would have to be spent to put a roof over the third-class carriage or to upholster the third-class seats that some company or other has open carriages with wooden benches What the company is trying to do is prevent the passengers who can pay the second-class fare from traveling third class; it hits the

[^10]poor, not because it wants to hurt them, but to frighten the rich And it is again for the same reason that the companies, having proved almost cruel to the third-class passengers and mean to the second-class ones, become lavish in dealing with first-class customers.

Having refused the poor what is necessary, they give the rich what is superfluous.
Jules Dupuit, 1849 (as quoted in Tirole (1988)).

Hence this model produces both the degraded product targeted at the low types and the excessively lavish option designed for the high types.

The equilibrium outcome once again can be viewed from the familiar perspective of a typical firm, as shown in Figure 4. This diagram is almost identical to Figure 3 with the exception that $A R_{H} \neq M U_{H} / \rho$. Since a firm must concede information rents to the high type, they cannot capture all the surplus under the perceived residual demand curve. Consequently, $A R_{H}<M U_{H} / \rho$. This figure also confirms a number of other results. It is evident that quantity discounting is a feature of the equilibrium in the sense that the per unit price paid by the high type is lower than the per unit price paid by the low type. However, despite this disparity the average per unit price set by a firm must equal $c / \rho$ in equilibrium. Note that this is a direct consequence of profit maximizing behavior (i.e. follows from (24)) that is then transposed onto the equilibrium outcome. In addition, setting a common linear price would satisfy the incentive constraint but this is not a feature of the profit maximizing behavior. As a result, indirect discrimination offers a greater ability to extract rents than a common linear price.

As suggested above the number of firms that enter is below the first best. This follows since $\phi_{1}>\phi_{2}$ for $\beta \in(0,1)$ :

$$
\begin{equation*}
n^{2 n d}=\left[\frac{c}{(\sigma-1) F}\left(\frac{A_{H}}{c}\right)^{\epsilon}\left(\beta+(1-\beta) \phi_{2}\right)\right]^{\frac{\sigma-1}{\sigma-\epsilon}} \tag{28}
\end{equation*}
$$

Nevertheless there is greater entry than under linear pricing due to the greater rent extracting ability under indirect discrimination. This leads to the following proposition.

PROPOSITION 4. For $\beta \in(0,1)$ the number of varieties across the three price regimes is as follows:

$$
n^{1 s t}>n^{2 n d}>n^{\text {linear }}
$$



Figure 4: Equilibrium under $2^{\text {nd }}$ degree price discrimination

Finally, all the previous results have been predicated on the assumption that the single crossing property holds, that is $\theta_{H}>\theta_{L}$ in equilibrium. To verify that this is in fact the case, note that $0<\phi_{2}<1$ which implies that $\phi_{2}^{\frac{1}{\epsilon}-\frac{1}{\sigma}}>\phi_{2}^{\frac{1}{\epsilon}}$. From (27) it follows that $\phi_{2}^{\frac{1}{\epsilon}-\frac{1}{\sigma}}>\frac{A_{L}}{A_{H}}$ which implies $\frac{A_{H}}{q_{H}^{\rho-\alpha}}>\frac{A_{L}}{q_{L}^{\rho-\alpha}}$. Since the definition of $\theta_{j}$ involves dividing both these previous quantities by $n^{(\rho-\alpha) / \rho}$ it follows directly that $\theta_{H}>\theta_{L}$ in equilibrium. Thus the single crossing property does in fact hold in equilibrium.

### 3.5 Consumer Surplus

Just as in the case of first degree price discrimination a firm can only extract the marginal surplus from the low type and not the average surplus. Consequently, and in contrast to the monopoly outcome under indirect discrimination, low types enjoy positive net surplus. This is given by:

$$
C S_{L}=\left(\frac{\rho-\alpha}{\alpha \rho}\right) A_{L} n^{\frac{\alpha}{\rho}} q_{L}^{\alpha}>0
$$

The calculation of net surplus for the high type also differs from the monopoly case. Under monopoly the surplus for the high type is given by $\left(\theta_{H}-\theta_{L}\right) u(q)$. Once again the competition between firms leaves surplus to the consumer over and above that associated with the individual
incentive compatibility constraints. The consumer surplus of the high type is:

$$
C S_{H}=\left(\frac{\rho-\alpha}{\alpha \rho}\right) A_{H} n^{\frac{\alpha}{\rho}} q_{H}^{\alpha}+\left(A_{H} \phi_{2}^{\rho-\alpha}-A_{L}\right) \frac{n^{\frac{\alpha}{\rho}}}{\rho} q_{L}^{\alpha}
$$

## 4 Free Trade with a Symmetric Country

To assess the implications for international trade consider the Krugman (1979) scenario of two symmetric countries. Since there are zero profits in equilibrium, any welfare implications are associated with consumption. Under free trade (24) continues to hold and the scale of a firm remains the same as in a closed economy, though each consumer halves their purchase of any given variety. However, while the firms total production is constant, the composition of this production may change in response to free trade. The following lemma shows that the composition of production is invariant to the number of firms.

LEMMA 5. A change in the number of firms, $n$, is associated with the same proportional response in $q_{L}$ and $q_{H}$. Consequently, the ratio $\frac{q_{L}}{q_{H}}=\phi_{2}$ is invariant to $n$.

Proof. Start by noting that $\frac{d\left(q_{L} / q_{H}\right)}{d n}=0$ if and only if $\frac{d \log \left(q_{L}\right)}{\operatorname{dlog}(n)}=\frac{\operatorname{dlog}\left(q_{H}\right)}{\operatorname{dlog}(n)}$. Note that from (25) $\frac{d \log \left(q_{H}\right)}{d l o g(n)}=-\left(\frac{\sigma-\epsilon}{\sigma-1}\right)$. To determine $\frac{\operatorname{dlog}\left(q_{L}\right)}{d \log (n)}$ note that (26) along with the assumption $\frac{d\left(q_{L} / q_{H}\right)}{d n}=0$ imply:

$$
\begin{equation*}
n^{\frac{\rho-\alpha}{\rho}} q_{L}^{1-\alpha}(1-\beta) c+K=0 \tag{29}
\end{equation*}
$$

Total differentiation of this equation gives:

$$
\frac{d q_{L}}{d n}=-\frac{(\rho-\alpha) q_{L}}{\rho(1-\alpha) n}=-\left(\frac{\sigma-\epsilon}{\sigma-1}\right) \frac{q_{L}}{n}
$$

Consequently, $\frac{\operatorname{dlog}\left(q_{L}\right)}{\operatorname{dlog}(n)}=\frac{\operatorname{dlog}\left(q_{H}\right)}{\operatorname{dlog}(n)}$.
Given that the composition of production is unchanged under free trade and that the overall level of a firm's production remains constant at $q$, the relative volume of trade follows directly from the ordering over the number of varieties.

PROPOSITION 5. For $\beta \in(0,1)$ the volume of trade varies across price regimes and has the
following ranking:

$$
V T^{1 s t}>V T^{2 n d}>V T^{\text {Linear }}
$$

The consumer surplus ranking across regimes also exhibits a similar pattern. Using lemma 5 allows the welfare function of the both types to be rewritten in the following manner:

$$
\begin{aligned}
C S_{L} & =\left(\frac{\rho-\alpha}{\alpha \rho}\right) A_{L} \phi_{2}^{\alpha} n^{\frac{\alpha}{\rho}} q_{H}^{\alpha} \\
C S_{H} & =\left[\left(\frac{\rho-\alpha}{\alpha \rho}\right) A_{H}+\frac{1}{\rho}\left(A_{H} \phi_{2}^{\rho-\alpha}-A_{L}\right) \phi_{2}^{\alpha}\right] n^{\frac{\alpha}{\rho}} q_{H}^{\alpha}
\end{aligned}
$$

Using (25) these surpluses can be expressed as:

$$
\begin{equation*}
C S_{j}=\psi_{j} n^{\frac{\epsilon-1}{\sigma-1}} \quad j \in L, H \tag{30}
\end{equation*}
$$

where

$$
\begin{aligned}
\psi_{L} & =\left(\frac{\rho-\alpha}{\alpha \rho}\right) A_{L} \phi_{2}^{\alpha}\left(\frac{A_{H}}{c}\right)^{\frac{\alpha}{1-\alpha}} \\
\psi_{H} & =\left[\left(\frac{\rho-\alpha}{\alpha \rho}\right) A_{H}+\frac{1}{\rho}\left(A_{H} \phi_{2}^{\rho-\alpha}-A_{L}\right) \phi_{2}^{\alpha}\right]\left(\frac{A_{H}}{c}\right)^{\frac{\alpha}{1-\alpha}}
\end{aligned}
$$

Therefore consumer surplus is proportional to the number of varieties. Consequently, since the number of varieties increases by $2^{\frac{\sigma-1}{\sigma-\epsilon}}$ consumer surplus increases by $2^{\frac{\epsilon-1}{\sigma-\epsilon}}$. Thus a move to free trade generates the same proportional change in consumer surplus across all three pricing regimes.

This result parallels work by Arkolakis et al. (2010) who find that models with a CES demand structure produce the same percentage gains from trade. While their paper deals with firms that price with a simple mark-up over marginal cost and shows that the underlying details of the model (perfect competition, heterogeneous firms, multi-product firms etc) produce the same proportional gains from trade, the above shows the result also extends to a non-linear price setting as well. However it is important to note that while the proportional change in consumer surplus is the same under linear and non-linear pricing, the levels of consumer surplus can still be very different. In particular, the following welfare ranking emerges:

PROPOSITION 6. For $\beta \in(0,1)$ the following consumer surplus ranking holds across the three

$$
C S^{1 s t}>C S^{2 n d}>C S^{\text {Linear }}
$$

While the ranking of $C S^{1 s t}$ and $C S^{2 n d}$ needs no explanation, the ranking of $C S^{2 n d}$ and $C S^{\text {Linear }}$ requires some comment. Clearly as $\beta \rightarrow 0, C S^{2 n d} \rightarrow C S^{1 s t}>C S^{\text {Linear }}$. Therefore the greatest concern is when $\phi_{2}$ deviates the furtherest from $\phi_{1}$, i.e. when the distortion on the low types is the greatest. However, this occurs as $\beta \rightarrow 1$, right at the point when the low types are given the least weight in social welfare. This negative correlation between $\phi_{2}$ and $\beta$ ensures that the final inequality holds. Finally, it is worth noting that the focus on consumer surplus captures the within sector changes in welfare. A multi-sector index of welfare is constructed from the indirect utility function: $W^{i}=C S^{i}+y$ for $i \in\{1 s t, 2 n d$, Linear $\}$ and where $y$ is income. Since $y$ is common across all pricing regimes, then from proposition 6 it follows immediately that the proportional gains from trade follow the same ranking: $\% \Delta W^{1 s t}>\% \Delta W^{2 n d}>\% \Delta W^{\text {Linear }}$.

Consequently, the nature of the pricing regime can have important implications for outcomes. In particular, both the volume of trade and the gains from trade are greater under second degree price discrimination than under the standard linear price assumption. This implies that multi-product firms can have a profound effect on outcomes that is not captured by the standard international trade model. Given that second degree price discrimination is likely to be a pervasive practice (implemented through product-lines of varying length), this suggests that we have been understating the gains available from trade.

## 5 Asymmetric Countries: Different $\beta$ 's

While countries can differ in many ways, an interesting dimension to consider relates to variation in $\beta$. Broadly this can be seen as differences in the distribution of income, since $A_{i}$ can be interpreted as the inverse of the marginal utility of income. ${ }^{19}$ To analyze this situation we will make two additional assumptions. First, assume that $\beta_{1}$ (the proportion of high types in country 1) and $\beta_{2}$ (analogous proportion in country 2) have the following ranking:

[^11]\[

$$
\begin{equation*}
\beta_{1}>\beta^{w}=\frac{\beta_{1}+\beta_{2}}{2}>\beta_{2} \tag{31}
\end{equation*}
$$

\]

Second, to facilitate comparisons we will focus on the special case where $\alpha=0$ in the utility function. This implies that utility of type $j$ is given by: ${ }^{20}$

$$
\begin{equation*}
U_{j}=A_{j} \ln Q+y, \quad A_{j}>0 \tag{32}
\end{equation*}
$$

Consequently, even though there is variation in the relative frequency of types across the two countries, the underlying types (high and low) are the same.

Under symmetry the gains from trade are associated with the same proportional increase in consumer surplus both within and across countries. However, when $\beta$ varies across countries this conclusion no longer holds. To some extent this is due to the difference in $\beta$ 's generating a difference in the relative size of the differentiated sector across countries (i.e. the higher $\beta$ the greater the equilibrium number of varieties). As a result, the gains from trade will be related to the number of additional varieties a country has access to - in the case of free trade, the low $\beta$ country experiences the largest increase in varieties and therefore the largest proportional gain in consumer surplus. This type of mechanism operates regardless of the pricing regime implemented. However, if second degree price discrimination is implemented then an additional factor shapes the distribution of gains both within and across countries. In particular, the size of the low types consumption distortion changes when asymmetric countries are integrated through free trade. In particular, the country with the high $\beta$ will gain from a decrease in the low types consumption distortion, while the reverse will be true for the country with the relatively low $\beta$. That is, under integration the global $\beta$ differs from the autarky $\beta$ in both countries, with the change in the consumption distortion governed by $\frac{d \phi_{2}}{d \beta}<0$. Since $\phi_{1}$ is invariant to $\beta$, it follows that the consumption distortion is reduced in country 1 but increased in country 2.

The implications of the change in $\phi_{2}$ can be assessed via the change in consumer surplus associated

[^12]with a move from autarky to free trade. For a high type indirect utility is given by:
\[

$$
\begin{align*}
V_{H} & =A_{H} \ln \left(n^{\frac{1}{\rho}} q_{H}\right)-n T_{H}+y \\
& =A_{H} \ln \left(\frac{A_{H}}{c} n^{\frac{1-\rho}{\rho}}\right)-\frac{A_{H}}{\rho}\left(1-\phi_{2}^{\rho}-\phi_{1}\right)+y \tag{33}
\end{align*}
$$
\]

It is apparent from this representation that changes in the size of the low types consumption distortion, $\phi_{2}$, directly impacts the expenditure of the high types.

While indirect utility for the low types is given by:

$$
\begin{align*}
V_{L} & =A_{L} \ln \left(n^{\frac{1}{\rho}} q_{L}\right)-n T_{L}+y \\
& =A_{L} \ln \left(\frac{A_{H}}{c} \phi_{2} n^{\frac{1-\rho}{\rho}}\right)-\frac{A_{L}}{\rho}+y \tag{34}
\end{align*}
$$

In this case, the expenditure of the low types remains constant and welfare increases through the improved product design associated with the increase in $\phi_{2}$.

Using (33) and (34), and denoting free trade quantities by " $f$ " and autarky ones by " $a$ ", the change in consumer surplus is given by:

$$
\begin{align*}
\Delta C S_{H} & =\frac{A_{H}}{\rho}\left[(1-\rho) \ln \left(\frac{n_{f}}{n_{a}}\right)+\left(\phi_{f}^{\rho}-\phi_{a}^{\rho}\right)\right]  \tag{35}\\
\Delta C S_{L} & =\frac{A_{L}}{\rho}\left[(1-\rho) \ln \left(\frac{n_{f}}{n_{a}}\right)+\rho \ln \left(\frac{\phi_{f}}{\phi_{a}}\right)\right]  \tag{36}\\
& \approx \frac{A_{L}}{\rho}\left[(1-\rho) \ln \left(\frac{n_{f}}{n_{a}}\right)+\left(\phi_{f}^{\rho}-\phi_{a}^{\rho}\right)\right] \tag{37}
\end{align*}
$$

Clearly, gains are generally greater for the high types in both countries. However, the gains are also generally asymmetric across countries since the sign of $\left(\phi_{f}-\phi_{a}\right)$ is determined by $\beta_{i} \gtrless \beta^{w}$. Since $\beta_{1}>\beta^{w}$, the consumption distortion is reduced for the low types in country 1 , which naturally translates into a consumption gain for these consumers. This also confers a benefit for the high types in country 1 since the incentive constraint means that they now receive greater information rents. For country 2 the logic is reversed with a greater consumption distortion for the low types reducing the surplus for both types. This outcome is summarized in the following proposition:

PROPOSITION 7. If a country's share of high types is greater than the global average, then not only do consumers in this country gain from increased variety but all consumer types gain from a reduced consumption distortion for the low types. In contrast, if a country's share of high types is
below the global average, then all consumers gain from an increase in variety from free trade, but these gains are reduced for all types by a higher consumption distortion imposed on the low types relative to autarky.

This immediately leads to the following corollary:
COROLLARY 1. If trade is possible but markets are segmented internationally, then country 2 gains from segmentation while country 1 no longer gets the benefit of a reduced consumption distortion from trade. Moreover, welfare for both types is higher in country 2 than country 1.

The type of segmentation referred to is one that does not change the marginal cost but does restrict arbitrage opportunities. For example, variation in voltage or frequency segments markets internationally for electrical goods though the impact on marginal cost is relatively minimal. In such situations only the variety gains are available with the consumption distortions the same as in autarky. Since consumption distortions vary across countries, so will the level of welfare even though both countries have access to the same number of varieties. This suggests that the role of standards may be far more important then previously recognized.

## 6 Conclusion

The recent international trade literature has focused extensively on the role that firm heterogeneity plays in shaping trade flows and its subsequent implications. This paper introduces heterogeneity on the other side of the market and demonstrates that heterogeneity among consumers can have important implications for the operation of a firm and the volume of trade. In particular it is shown that firms have an incentive to utilize mechanisms that indirectly discriminate between consumers by designing different product bundles for different consumer types. Thus firms are multi-product in equilibrium since they operate product-lines. While the essential components of the model are separately familiar the implications of their interaction has not been previously explored, generating important new results. In relation to competition and price discrimination two results emerge. First, competition is associated with an expansion of product diversity within a firm. Second, while the degraded option offered to a low type is well documented in the literature, the "excess at the top" result has not been. This bi-directional distortion arises naturally in a competitive environment and squares with the folklore surrounding indirect discrimination. The implications for international trade are also intriguing and suggest that the standard model under-predicts the available varieties
and therefore the volume of trade and welfare. Moreover, the distribution of income now plays an important role in determining the gains from trade due to its impact on the size of the consumption distortion.

One feature of the model is its tractability which suggests that the basic framework can be extended in a number of ways. The motivation for setting trade policy is likely to be different in this setting, especially since there is a bi-directional distortion. In particular, McCalman (2010) finds that in a monopoly setting that trade policy is associated with international externalities that are an order of magnitude larger than the standard model. The current framework provides an opportunity to investigate these issues in more depth. Variation in the underlying details of the model would also be interesting to explore. In particular the formulation of the "love of variety" preferences followed the CES approach and it would be worthwhile to examine whether similar results emerge with quadratic preferences. Also consideration of how a consumer is served appears to be an important issue. In the current setting a firm deals directly with a consumer, rather than through an intermediary such as a retailer. Once a retailer is introduced, then issues related to product-line design are influenced by both consumer preferences and also the incentive faced by the retailer. This seems like a particularly interesting topic given technological changes that are occurring in the retail sector.

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[^0]:    ${ }^{1}$ See Arkolakis and Muendler (2009), Bernard et al. (2011), Eckel and Neary (2010), Feenstra and Ma (2007) and Nocke and Yeaple (2006).
    ${ }^{2}$ See Arkolakis et al. (2010).
    ${ }^{3}$ For an introduction to price discrimination see Tirole (1988).

[^1]:    ${ }^{4}$ Note this behavior has not always been present in the automobile industry. A standardized product is exemplified by the model-T and the claim "Any customer can have a car painted any color that he wants so long as it is black."
    ${ }^{5}$ See for example Spulber (1989) and Stole (1995). Recent surveys of price discrimination and competition include Stole (2007) and Armstrong (2006).
    ${ }^{6}$ See Spence (1976) and Dixit and Stiglitz (1977)

[^2]:    ${ }^{7}$ Models of countervailing incentives can also generate distortions at the top and the bottom (see Lewis and Sappington (1989) and Maggi and Rodriguez-Clare (1995)). Such a mechanism is explicitly ruled out as the model developed below is constructed around two types only.

[^3]:    ${ }^{8}$ This trade-off has an analogue in the multi-product international trade literature where the introduction of a another product cannibalizes revenue from other products offered by a firm (see Eckel and Neary (2010) and Feenstra and $\mathrm{Ma}(2007)$ ). This reflects the horizontal differentiation of products within a firm. However, second degree price discrimination involves vertical differentiation in terms of either quality or bundle size.
    ${ }^{9}$ For an early model of product line rivalry see Brander and Eaton (1984).

[^4]:    ${ }^{10}$ See Antràs and Helpman (2004) and Helpman and Krugman (1989) for other applications of this quasi-linear framework in an international setting.

[^5]:    ${ }^{11}$ Though the optimal $q(v)$ will differ across price regimes when confronted with the same market level characteristics.

[^6]:    ${ }^{12}$ To see this note that the total benefit that the consumer gets from the entire basket is $T B=\frac{A}{\alpha} n^{\frac{\alpha}{\rho}} q^{\alpha}$, which implies that the benefit from the average variety is $\frac{T B}{n}=\frac{A}{\alpha} n^{\frac{\alpha-\rho}{\rho}} q^{\alpha}$. While the payment that the consumer makes for the marginal variety is only $\frac{\theta q^{\rho}}{\rho}=\frac{A}{\rho} n^{\frac{\alpha-\rho}{\rho}} q^{\alpha}=\frac{c}{\rho} q$. Intuitively since the marginal utility of consuming from the differentiated sector is declining (i.e. $\frac{\partial^{2} U}{\partial Q^{2}}<0$ ) the average utility is also declining but is greater than the marginal utility (i.e. $\frac{U}{Q}>\frac{\partial U}{\partial Q}>0$ ).

[^7]:    ${ }^{13} \mathrm{~A}$ possible source of this heterogeneity is differences in income. Section 5 examines the implications of this interpretation in more detail. For an intriguing empirical analysis that relaxes the standard assumption of a representative agent see Broda and Romalis (2009).

[^8]:    ${ }^{14}$ Note that we are implicitly assuming that the firm will and can separate consumers by type. Of course this needs to confirmed as equilibrium behavior.

[^9]:    ${ }^{15}$ As discussed above this result also differs from the spatial model of monopolistic competition
    ${ }^{16}$ This strong result follows from the CES functional form. Nevertheless, a quadratic preference specification also is associated with a longer product-line under monopolistic competition than under monopoly (i.e. $q_{L}=0$ occurs at a higher $\beta$ under monopolistic competition).

[^10]:    ${ }^{17}$ If there are more than two types it is possible for at least one type to receive the efficient bundle.
    ${ }^{18}$ All the models of indirect discrimination reviewed in Stole (2007) feature the "no distorts at the top" outcome. While other models have generated an "enhancement at the top" result, this typically requires a departure from the standard setup where the single crossing property does not hold (see Srinagesh and Bradburd (1989)) and instead there is no distortion at the bottom.

[^11]:    ${ }^{19}$ See Tirole (1988) page 143.

[^12]:    ${ }^{20}$ Formally, (32) is derived from:

    $$
    \lim _{\alpha \rightarrow 0} \frac{A_{j}}{\alpha} Q^{\alpha}-\frac{A_{j}}{\alpha}+y
    $$

