Trade, Unemployment, and Reallocation with Search Frictions[*]

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Abstract

The impact of international trade on unemployment can be affected by inter-sectoral labor reallocation as well as intra-sectoral one. To see the effect of trade liberalization on unemployment with both intra-sectoral and inter-sectoral labor movements, we construct a two-country, two-sector model of international trade where one sector, called a differentiated-good sector, is characterized by heterogeneous firms and the search frictional labor market. The analysis shows that trade liberalization increases both wages and the labor market tightness in a differentiated-good sector through a decrease in hiring costs. These labor market reactions result in job creation within a differentiated-good sector which has a positive effect on economy-wide employment, and labor inflow from the outside sector which has a negative effect. These opposite effects can be totally cancelled out each other. The net impact of international trade on an economy-wide unemployment rate depends on the country's characteristics of the labor market such as matching quality between firms and workers and the skill distribution of workers.

1 Introduction

The impact of trade liberalization on unemployment is a controversial issue in academics as well as in public debates. Although this issue has attracted a great deal of attention, no concensus has yet emerged among academics, among public statements, and as a whole. Articles such as Krugman (1993) and Mussa (1993) claim that trade does not affect the rate of unemployment. When we look at empirics, while Attanasio et al. (2004) reveal no evidence of relationship between trade and the likelihood of unemployment, many studies show an increase in worker displacement after trade liberalization (Goldberg and Pavcnik (2003), Revenga (1997), Menezes-Filho and Muendler (2007)).[1] These mixed insights naturally give rise to the following questions. What is the impact of trade liberalization on unemployment? How and through which does trade liberalization affect unemployment? Providing a rational explanation for these questions is important in order for policy makers not to make a wrong decision based on false perceptions.

Toward these questions, many recent studies are based on the Melitz (2003) type heterogeneous firm trade model. By introducing firm heterogeneity, these can explain employment expansion or shrinkage through intra-sectoral labor reallocation without labor movement across sectors (Egger and Krieckmeier 2009, Felbermayr, Prat, and Schmerer 2008, 2011, Janiak 2006). However, trade liberalization affects inter-sectoral labor reallocation as well as intra-sectoral labor reallocation, which is supported by Kambourov (2009). In other words, one sector analyses of the impact of trade liberalization on unemployment seem to be inadequate.

Based on these background, we construct a two-country, two-sector model of international trade where one sector is characterized by heterogeneous firms and the search frictional labor market. Because of our two-sector setting where labor markets are also separated like the Harris-Todaro model, trade liberalization leads to both intra-sectoral

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[1]Public debate also has both pros and cons of trade liberalization. For instance, a report published by a NGO association *Policy Matters Ohio* shows the NAFTA resulted in 52,265 job losses in Ohio state (http://www.policymattersohio.org/trade.htm, accessed on 12 January 2011) and disagrees with trade liberalization. On the other hand, Egypt's Ambassador has an optimistic viewof the impact of trade on unemployment, saying: "The question is why all of a sudden, when third world labor has proved to be competitive, why do industrial countries start feeling concerned about our workers?"

and inter-sectoral labor reallocations. In other words, our model can analyze the effect of trade liberalization on unemployment by taking into account both worker compositions across sectors (how many workers are in the search frictional sector) and sectoral unemployment rates. Under these settings, we obtain the following results.

First, trade liberalization through a reduction of transport costs increases both wages and the labor market tightness in the differentiated-good sector. Although this result itself is the same as Felbermayr et al. (2008, 2011), the mechanism is different. In their papers, trade liberalization heightens the marginal revenue of hiring additional workers, encouraging remaining firms to employ more workers.[2] This causes an increase in wages and the labor market tightness. In our model, on the other hand, the effect on the marginal revenue is cancelled out by an opposite force generated by fiercer competition in the product market. Therefore the marginal revenue of hiring a worker does not change. Instead, trade liberalization reduces the marginal cost of hiring workers which leads to increases in wages and the labor market tightness.

Second, trade liberalization does not cause a clear reaction of unemployment, as the net effect on unemployment depends on the country's characteristics such as the matching quality between firms and workers and the skill distribution of workers. This ambiguity is derived from two competing forces. While trade liberalization creates job in the differentiated-good sector (job creation effect), which reflects an increase in the labor market tightness, this job expansion encourages some workers in outside sector to change their occupations (labor inflow effect). These two factors may cancell out. This result is generated because of the two-sector framework, and so is in contrast to Felbermayr et al. (2008, 2011) where higher productivity always creates jobs in one and only one sector.

Within the literature, our model is closely related to the following three papers. Larch and Lechthaler (2009) extend Felbermayr et al. (2008) by incorporating the second sector which also has frictional labor market and monopolistically competitive product market. It has two factors (skilled and unskilled workers) according to Bernard, Redding and Schott (2007) in order to examine the effect of trade liberalization on distribution of gains

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[2]Technically speaking, this is due to their assumption that the price index $\mathcal{P}$ is set to one.

from trade as well as skill specific-unemployment rate. While these ample ingredients may enable them to derive many results, these complications seem to prevent us from intuitive understanding. Moreover, their calibration methodology also makes it difficult for us to trace the influence path of trade liberalization on unemployment.

Coşar, Guner and Tybout (2010) also construct a two-sector heterogeneous firm model, where one sector has a frictional labor market and the other has a friction-less labor market. Although their framework looks like ours, their main focus is on a firm's response to productivity shocks as well as the impact of trade liberalization on the labor market. In addition, their results are obtained by a calibration method, which makes it difficult for us to follow the logical relation. On the other hand, our model employs a simpler two-sector model with analytical solutions, which enables us to keep up with the influence path, therefore, contributing to better understanding on the labor market outcomes.

Helpman and Itskhoki (2007, 2009, 2010) use a similar framework to study the effect of trade liberalization on unemployment. Among these, Helpman and Itskhoki (2007) has the closest setting: the differentiated-good sector with a frictional labor market and the homogeneous product sector with a frictionless labor market.[3] An economy-wide unemployment rate is determined by the product of (1) a sectoral unemployment rate in the differentiated sector, and (2) the fraction of job seekers in that sector.

Even though this component of unemployment is the same as ours, the obtained results of trade liberalization are different due to the following points. On the one hand in Helpman and Itskhoki (2007), the sectoral labor market tightness is pinned down only by domestic labor market parameters such as matching quality, costs of vacancy, and market frictions, because hiring costs are paid in the unit of homogeneous good. With this structure, trade liberalization changes only composition of workers across two sectors, but does not change a sectoral unemployment rate. Since trade liberalization in a more frictional sector encourages workers to shift into that sector, it expands an economy-wide

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[3]Helpman and Itskihoki (2010) give a similar analysis to their paper in 2007 except for a homogeneous product sector having frictional labor market. Their paper in 2009 extends the model in 2010 into a dynamic analysis. Despite of these differences, all of these obtain the same implications.

unemployment rate.

On the contrary in our model, hiring costs are paid in the unit of final good in the differentiated-good sector. Hence, trade liberalization has an impact on a sectoral unemplorment rate through a change of the labor market tightness, as well as the proportion of workers in the differentiated-good sector. Since these two effects change an economy-wide unemployment rate to the opposite directions, the net result is ambiguous depending on the country's characteristics. In that sense, it can be said that our paper is testing the robustness of Helpman and Itskhoki (2010).[4]

The remainder of this paper is as follows: Section 2 gives the setup of the model. Section 3 also provides the basic structure of sector Y and occupational choice and labor market equilibrium. After product market equilibrium is exhibited in Section 4, the effect of trade liberalization is supplied in Section 5. The last section concludes.

2   The setup

The present model is based on Felbermayr et al. (2008, 2010) framework, where a sector has monopolistically competitive product market with firms differentiated with respect to productivity and labor market with search friction (hereafter, sector X). The only thing that is notably different from their work is to include a perfectly competitive homogeneous good sector (hereafter, sector Y), and therefore, workers choose their occupations in consultation with their skills. In addition, we suppose the world is composed of two symmetric countries, home and foreign, across which intermediate inputs in sector X and final good Y are traded. Symmetry enables us to focus on home country with omitting the subscript of the country.

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[4]Concerning the relationship between international trade and unemployment in the dual sector setting, our model is also related to extensions of Harris and Todaro (1970).

2.1 Production in the sector X

The preference of a representative consumer is given by the following quasi-linear form:

$$\mathcal{U} = \mu \ln C_X + C_Y, \quad \mu \in (0,1), \tag{1}$$

where C_X is the total amount of consumption of good X, C_Y is that of good Y. We assume that the representative consumer has enough income to consume both goods.

While good Y is produced only by labor, good X is produced from continuously differentiated intermediate goods, according to the following Blanchard and Giavazzi (2003) type production function:

$$Q_X = \left[(M)^{-\frac{1}{\sigma}} \int_{j \in J} [x(j)]^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{\sigma}{\sigma-1}}, \quad \sigma > 1 \tag{2}$$

where $x(j)$ is the quantity of intermediate variety j, the measure of the set J is the mass M of available intermediate inputs, and σ denotes the elasticity of substitution between each variety. This Blanchard and Giavazzi type specification enables us to eliminate scale expansion effect (love of variety effect) due to trade liberalization on production, and hence to focus our attention only on trade-induced-firm-reallocation-effect developed by Melitz (2003). This can be seen in a symmetric demand case: $x(j) = Q/M$ where Q denotes an aggregate input demand. By substituting it, we obtain $Q_X = Q$, which means that the total output is independent of the number of firms M.[5]

With the price index of good X defined $\mathcal{P} \equiv \left[(M)^{-1} \int_{j \in J} [p(j)]^{1-\sigma} dj \right]^{\frac{1}{1-\sigma}}$, where $p(j)$ is the price of a variety j, we obtain the demand for a variety j:

$$x_H(j) = \left(\frac{\mathcal{P}}{p_H(j)} \right)^{\sigma} \left(\frac{\mu}{\mathcal{P}M} \right), \tag{3}$$

where $x_H(j)$ indicates the demand for an intermediate variety j by a domestic final good producer. Similarly, if an intermediate firm can engage in exporting, it encounters the

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[5]This specification is also used in Egger and Kreickemeier (2009), Ebell and Haefke (2009), and Felbermayr et al.(2010). The detailed chacracteristics of this fucntion is dealt in these papers.

following demand by a foreign final good producer:

$$x_F(j) = \left(\frac{\mathcal{P}}{p_F(j)}\right)^{\sigma} \left(\frac{\tau\mu}{\mathcal{P}M}\right),$$ (4)

where $\tau \geq 1$ is the iceberg transport cost to ship one unit of good internationally. In addition to this variable costs, a firm serving domestic market has to incur fixed costs f_H per period in the unit of homogeneous good, and a firm engaging in export activity incurs additionally f_F per preriod in the same way. These fixed costs are supposed to $f_F \geq f_H$ so as to focus on the case where within domestically active firms only a fraction of firms engage in exporting like Melitz (2003).[6]

The production function of an intermediate good j simply depends on the amount of labor inputs $h(j)$ and firm's productivity $\varphi(j)$, $x(j) = h(j)\varphi(j)$. As each firm produces a different variety from the others and firms with the same productivity behave similarly, we, hereafter, denote each firm as its productivity φ. If a firm can afford to export its good to the foreign market, the total output of that firm has to be optimally allocated so as to equalize the marginal revenue from home and foreign, which requires $p_H(\varphi)\tau = p_F(\varphi)$ clearly shown later (in equation (26)).

Given these demand functions, now we derive the revenue function of a firm:

$$R(h;\varphi) = \mathcal{P}^{\frac{\sigma-1}{\sigma}} \left(\frac{\mu}{M}\right)^{\frac{1}{\sigma}} \left[1 + I(\varphi)\tau^{1-\sigma}\right]^{\frac{1}{\sigma}} (\varphi h(\varphi))^{\frac{\sigma-1}{\sigma}} \equiv D(\varphi)(\varphi h(\varphi))^{\frac{\sigma-1}{\sigma}},$$ (5)

where $h(\varphi) = h_H(\varphi) + I(\varphi)h_F(\varphi)$ is the total amount of employment for a firm with productivity φ, $I(\varphi)$ is the indicator function which takes one if a firm engages in export activity or otherwise zero, and $D(\varphi)$ controls for aggregate variables.

According to Melitz (2003), average productivity $\tilde{\varphi}$ is such that $x_H(\tilde{\varphi}) = \frac{Q_X}{M}$: output of a firm with average productivity is equal to average output in this economy.

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[6]More exactly, to focus on the separation case, we have to impose $\tau^{\sigma-1}f_F \geq f_H$.

2.2 Labor search in the sector X

The labor market in sector X is modeled by following Pissarides (2000), being imperfectly competitive because of search and matching friction. This frictional matching between à labor and a firm is summarized by the constant returns to scale and concave matching function. By using these properties, the rate of filling a vacancy for a firm is denoted by $q(\theta)$ where θ denotes the labor market tightness. It is worth noting that the rate is decreasing in θ, meaning that if the number of vacancies relative to that of unemployed workers is larger, it is more difficult for a firm to find a worker. On the contrary, the rate of finding a job for a worker is denoted $\theta q(\theta)$, which is increasing in θ.

In addition, these rates also have the following feaures : $lim_{\theta \to 0} q(\theta) = lim_{\theta \to \infty} \theta q(\theta) = \infty$, and $lim_{\theta \to \infty} q(\theta) = lim_{\theta \to 0} \theta q(\theta) = 0$. These properties mean that as the labor market tightness is close to zero, matching is quite difficult for a worker, and easy for a firm. In the case that the labor market tightenss is close to infinity, the situation is the opposite.

Because of this frictional matching process, firms are forced to incur hiring costs per vacancy c_v in the unit of final good X.[7] Taking the matching probability into consideration, a firm has to pay total hiring costs $\frac{c_v \mathcal{P}}{q(\theta)}$ to fill a vacancy.

In each period, an employed worker encounters job destruction at the probability s, which consists of two elements. One is match-specific job destruction, denoted by χ, and the other is the job destruction as a result of a firm itself being disrupted, denoted by δ. The reason of existing δ is that without this kind of firm destruction, every firm could easily pay the entry fixed costs in installments over the infinite periods. Since these two probabilities are independent, the total job destruction rate for a worker in each period is defined by $s \equiv \delta + \chi - \delta\chi$.

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[7]Justification for this assumption is as follows. An intermediate-good firm faces resource allocation problem between production and recruiting activity, and if it uses additional resourse for recruiting activity, it has to give up one unit of production. In that sense, the cost of hiring additional workers is the price of intermediate-good. In our setting, this cost is paid by the weighted average of the prices of each intermediate-good, that is, the price index. By doing so, hiring cost per vacancy does not depend on productivity of a firm.

2.3 Maximization of an intermediate firm

Knowing or expecting the demand, costs and wages, a firm maximizes its expected discount value with respect to the number of vacancies posted.

$$
J(h; \varphi) = max_{v(\varphi)} \frac{1}{1+r} \left[R(h; \varphi) - w(h; \varphi)h(\varphi) - c_v \mathcal{P} v(\varphi) - f_H - I(\varphi)f_F + (1 - \delta)J'(h'; \varphi) \right],
$$
$$
s.t. \quad R(h; \varphi) = D(\varphi)(\varphi h(\varphi))^{\frac{\sigma-1}{\sigma}},
$$
$$
h'(\varphi) = (1 - \chi)h(\varphi) + q(\theta)v(\varphi),
$$
$$
\tag{6}
$$

where prime denotes the level of a variable in the next period, and w is the wage level determined, as we will see, by bargaining game.

The first order condition for the vacancy posting v is

$$
\frac{c_v \mathcal{P}}{q(\theta)} = (1 - \delta) \frac{\partial J'(h'; \varphi)}{\partial h'(\varphi)}, \tag{7}
$$

which indicates that a firm determines the amount of job vacancies so as to equalize the marginal cost of posting a vacancy in this period with the marginal benefit obtained in the next period. In other words, if the right hand side is greater than the left hand side, a firm would be better off by posting more vacancies. Also, by envelop theorem, differentiating with respect to $h(\varphi)$ yields

$$
\frac{\partial J(h; \varphi)}{\partial h(\varphi)} = \frac{1}{1+r} \left[\frac{\partial R(h; \varphi)}{\partial h(\varphi)} - w(h; \varphi) - \frac{\partial w(h; \varphi)}{\partial h(\varphi)}h(\varphi) + (1 - \delta)(1 - \chi)\frac{\partial J'(h'; \varphi)}{\partial h'(\varphi)} \right].
$$
$$
\tag{8}
$$

By combining these two equations, we obtain the optimal employment condition for the intermediate producer,

$$
\frac{\partial R(h; \varphi)}{\partial h(\varphi)} = w(h; \varphi) + \frac{\partial w(h; \varphi)}{\partial h(\varphi)}h(\varphi) + \Delta \frac{c_v \mathcal{P}}{q(\theta)}, \tag{9}
$$

where $\Delta \equiv \frac{r+s}{1-\delta}$ is a parameter that controls job destruction rates. Clearly, a firm opti-

mally equalizes the marginal revenue of hiring a worker with the marginal costs. Differences from the optimal condition in the usual frictionless labor market are the existence of second and third term in the right hand side. The former indicates the effect of marginal employment on the wage level of all employed workers and the latter reflects the hiring costs due to the search and matching friction.

This equation also can be interpreted as the Euler equation of a firm, which is readily understood by rewriting as follows:

$$\frac{c_v \mathcal{P}}{q(\theta)} = \frac{1-\delta}{1+r} \left[\frac{\partial R(h;\varphi)}{\partial h(\varphi)} - w(h;\varphi) - \frac{\partial w(\varphi)}{\partial h(\varphi)} h(\varphi) + (1-\chi) \frac{c_v \mathcal{P}}{q(\theta)} \right].$$

The right hand side represents costs of hiring in this period. The left hand side represents discounted benefit of hiring in this period.

2.4 Wage bargaining

Before introducing wage bargaining structure, we define the value of a worker in each status. An employed worker has the following value function,

$$E(h;\varphi) = \frac{1}{1+r} \left[w(\varphi) + sU + (1-s)E(h;\varphi) \right], \tag{10}$$

where E is the expected discount value of an employed worker, and U is that of an unemployed worker. Since a wage is assumed to pay at the end of each period, the wage of the current period is also discounted. As we saw, a worker is faced with job destruction at the rate of s, resulting in being unemployed in the next period. There is no on-the-job search. The counterpart for an unemployed worker is,

$$U = \frac{1}{1+r} [\theta q(\theta)\tilde{E} + (1 - \theta q(\theta))U], \tag{11}$$

where an unemployed worker earns no income in the current period. In the next period, an unemployed worker obtaining a job at the rate of $\theta q(\theta)$, can earn the expected value

$\tilde{E}\left(=\int_0^\infty E(\varphi)dG(\varphi)\right)$, where $G(\varphi)$ is the productivity distribution defined later.

After matching a firm with a worker or at the begining of every period while the matching remains, they bargain only about wages according to the Stole and Zwiebel (1996) solution to the bargaining problem. In other words, they determine wage rates so as to maximize the following equation:

$$E(h;\varphi) - U = \frac{\partial J(h;\varphi)}{\partial h(\varphi)}, \tag{12}$$

where $E - U$ is the surplus of the matching for a worker, and $\frac{\partial J}{\partial h}$ is the surplus for a firm by firing the marginal worker. For simplicity, we assume that the bargaining power of a worker is 0.5.[8]

By solving differential equation, we can obtain the following wage equation and job creation equation respectively:[9]

$$
\begin{aligned}
\text{Wage curve} \quad & w = \Delta \frac{c_v \mathcal{P}}{q(\theta)} + \theta \frac{c_v \mathcal{P}}{1 - \delta}, \\
\text{Job creation curve} \quad & w = \frac{2(\sigma - 1)}{2\sigma - 1} \tilde{\varphi} \mathcal{P} - \Delta \frac{c_v \mathcal{P}}{q(\theta)}.
\end{aligned} \tag{13}
$$

The wage equation, increasing in $w - \theta$ plane, indicates worker's and firm's optimal wage setting condition: if the labor market tightness is large, the firm is willing to pay high wages above the outside option in order to save high hiring costs $\frac{c_v \mathcal{P}}{q(\theta)}$ which has to be paid if the firm could not have been succeed in the bargaining. Without any friction in the labor market ($c_v = 0$), the wage curve is equalized to zero which is the outside option of an unemployed worker who commits to this sector. Clearly, the intercept of wage curve does not change in any case.

On the other hand, the job creation curve, decreasing in $w - \theta$ plane, shows a firm's optimal hiring behavior in the frictional labor market, which is a counterpart of the labor demand curve in the frictionless case. In other words, an increase of the labor

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[8]Abowd and Allain (1996) examine that worker's bargaining power in an individual bargaining case is close to 0.5.

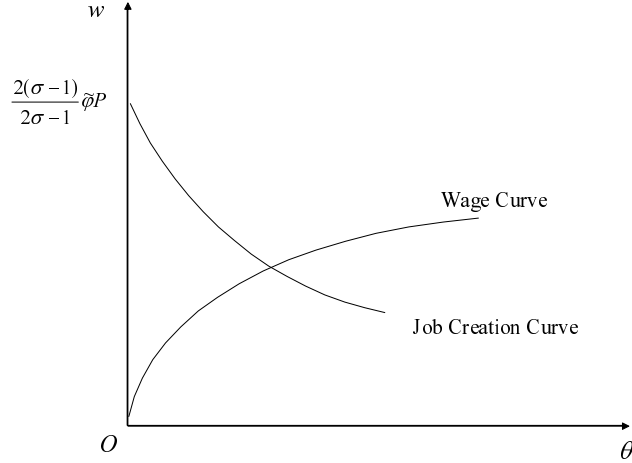[9]Derivation of these equations is given in appendix.

Figure 2.1: Labor market equilibrium

market tightness leads to a dominance of the marginal costs of hiring over the marginal revenues, and hence forces firms to decrease wages so as to satisfy the optimal employment condition. In the case where there is no search friction, firms set their wages as $w = \left(\frac{2(\sigma-1)}{2\sigma-1}\right)\tilde{\varphi}\mathcal{P}$ for any θ.

It is also worth noting that these two equations do not depend on respective firm's productivity φ but only on average productivity $\tilde{\varphi}$. Hence, by the intersection of these two curves, labor market equilibrium (w^*, θ^*) is determined given $\tilde{\varphi}$, as shown in Figure2.1.

Lemma 1 : *Given the average productivity $\tilde{\varphi}$, the labor market equilibrium (w^*, θ^*) uniequely exists.*

Proof. See appendix. □

Now, we show the effect of the change in average productivity $\tilde{\varphi}$ on the labor market equilibrium. First, while an increase in average productivity directly leads to an increase in the marginal revenue of hiring a worker, it also indirectly causes a decrease in the marginal revenue through a decrease of the price index $\mathcal{P}$. This latter effect captures fiercer competition in the labor market caused by an increase in average productivity

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in the product market.[10] These two effects are cancelled out in our model because of a property in the Melitz model, so the intercept of job creation curve does not shift.[11] Second, an increase of average productivity leads to a fall of total hiring costs to fill a vacancy, because it is paid in the unit of final good. This flattens both wage curve and job creation curve.

With these respective effects, the net effect of average productivity on the labor market equilibrium is summarized in the following proposition.

Proposition 1 : *An increase of average productivity $\tilde{\varphi}$ leads to an increase of the labor market tightness θ. Moreover, if the elasticity of firm's macthing rate with respect to the labor market tightness is negative but greater than -1, wages also increases.*

Proof. See appendix. □

As shown in the appendix, the restriction concerning the elasticity of firm's matching rate is satisfied in the case of the matching function being Cobb-Douglas form.

Even though Felbermayr et al. (2008, 2011) obtain the same results as ours, their results rely on the assumption that $\mathcal{P} = 1$. By assuming it, their job creation curves move upward, slopes of both job creation curve and wage curve being remained, so wages and the labor market tightness are raised. On the other hand, in our model, an increase of average productivity leads to a decrease in hiring costs, resulting in increases of wages and the labor market tightness. Therefore, our mechanism is different from Felbermayr et al. (2008, 2011).

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[10]Felbermayr et al. (2008, 2011) set $\mathcal{P} = 1$, and hence they do not capture this fiercer competition effect.

[11]In the Melitz model, the price index can be written as $\mathcal{P} = constant \times \frac{1}{\tilde{\varphi}}$.

$$\frac{2(\sigma-1)}{2\sigma-1}\tilde{\varphi}P$$

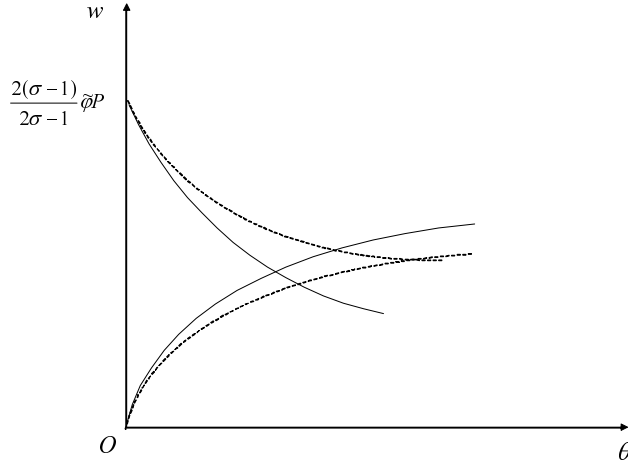Figure 2.2: The impact of price index decreased

3   Sector Y, occupational choice, and the labor market equilibrium

As mentioned in the above, sector Y is assumed to have a perfectly competitive product market and labor market, and each firm produces homogeneous good Y directly using only labor. This production process is done according to constant returns to scale and normalized as unit labor requirement being one. These simplifications make it appropriate to set home's good Y as numeraire, and therefore wage rate for one unit of labor as a sector Y worker in home country is also one. In addition, as good Y is freely traded across countries, the price of good Y and wage rate is equalized across countries.

This economy is composed of L families, and each familiy has a continuum of workers. Each worker within a family has the same one unit of labor as a sector X worker and differentiated α unit of labor as a sector Y worker. The latter skill is distributed with a cummulative distribution function $F(\alpha)$ ($\alpha \in [\alpha_{min}, \infty)$). Within these heterogeneous workers, while a worker with low α chooses to engage in sector X, one with high α chooses sector Y, which reflects a kind of a comparative advantage.[12] In other words, a worker

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[12]Gibbson et al.(2005) empirically shows that the occupational choice process depends on the compar-

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optimally chooses his occupation by comparing his expected values and then the cutoff skill level is determined at such a point that makes discounted lifetime values indifferent between occupations.

On the one hand, the discounted value for a worker in sector Y is,

$$\frac{1+r}{r}\alpha, \tag{14}$$

where α is per period wages for a worker with α unit of labor. On the other hand, the discounted expected value for a sector X worker is

$$U = \frac{1}{r}\left[\frac{\theta q(\theta)w}{r+s+\theta q(\theta)}\right] = \frac{\theta}{r}\left[\frac{c_v \mathcal{P}}{1-\delta}\right]. \tag{15}$$

This equation explicitly shows that the expected value for unemployed workers is increasing in wages and the probability of matching. Second equality is obtained by substituting the wage curve, in which the value of a worker depends on average productivity and the labor market tightness. While an increase of average productivity has a positive effect itself on the worker's value, it also has an indirect effect through the change of the labor market tightness.

In equilibrium, the indifference condition between two values determines the cutoff skill level denoted by $\tilde{A}(\theta, \mathcal{P})$,

$$\tilde{A}(\theta, \mathcal{P}) = \frac{\theta}{1+r}\left[\frac{c_v \mathcal{P}}{1-\delta}\right]. \tag{16}$$

As the skill as a sector Y worker α is distributed with c.d.f. $F(\alpha)$, the fraction of $F(\tilde{A})$ engages in sector X, and $1 - F(\tilde{A})$ in sector Y.

The following lemma shows the movement of this cutoff skill level with respect to average productivity $\tilde{\varphi}$.

Lemma 2 : *The cutoff skill level $\tilde{A}(\theta, \mathcal{P})$ increases as a result of an increase of average*

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ative advantage.

productivity $\tilde{\varphi}$.

Proof. See appendix. □

Since an increase in average productivity leads to a rise of wages and the labor market tightness in sector X, an expected value of engaging in that sector goes up, resulting in inflow of some workers into the sector.

In addition to the division of workers into each sector, workers sorted into sector X have a possibility to be unemployed because of search and matching friction in sector X labor market. In other words, all workers sorted into sector X are divided into unemployed or employed workers, and some of whom in each category move toward the other category by a given probability in every period. While a fraction of unemployed workers $L_U \theta q(\theta)$ in a family changes their status into employed workers by matching with firms, that of employed workers $[F(\tilde{A}) - L_U]s$ becomes unemployed workers, where L_U denotes the number of unemployed workers in each family.[13] In stationary equilibrium, the inflow into unemployed pool has to be equalized to the outflow from that pool,

$$L_U(\theta, \tilde{A}) = \frac{F(\tilde{A})s}{\theta q(\theta) + s}. \tag{17}$$

We summarize the determination of occupational choice equilibrium as,

Definition 1 : *Given average productivity $\tilde{\varphi}$, the occupational choice equilibrium ($\tilde{A}^*$, L_U^*) is pinned down by equation (16), and (17).*

4 Product market equilibrium

Now we move on to the firm's entry and exit process, and the product market equilibrium in sector X, both of which are similar to Melitz (2003).

The entry process of a firm is mainly divided into two steps. First, among an infinitely large number of potential entrants, firms which incur the entry sunk cost F_E in the unit

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[13] L_U can be also interpreted as an economy-wide unemployment rate.

of homogeneous good, is allowed to enter the market. Second, after firms having known their productivity φ which is distributed with c.d.f. $G(\varphi)$, and p.d.f. $g(\varphi)$ ($\varphi \in (0, \infty)$), only firms having sufficiently high productivity to earn positive profits, can actually start the production by incurring per period fixed costs f_H in the unit of homogeneous good, or otherwise exit from the market without starting the production. This cutoff point is denoted by φ_H^*, pinned down by the product market equilibrium conditions we will see later. In addition, a highest productivity fraction of firms can engage in exporting activity by incurring additional per period export fixed costs f_F in the unit of homogeneous good. The cutoff productivity between a firm engaging only in domestic activity and a firm engaging both domestic and exporting activities, is denoted by φ_F^*, which is also determined by conditions defined later. The rate of firms engaging in exporting activity conditioned by that engaging in domestic activity is defined by $\rho = \frac{1 - G(\varphi_F^*)}{1 - G(\varphi_H^*)}$.

Following these entry processes, the average productivity of active firms $\tilde{\varphi}_T$ in home country is defined by the following weighted average,

$$\tilde{\varphi}_T = \left[\frac{1}{M_H + \rho M_H} \left(M_H \tilde{\varphi}_H^{\sigma-1} + \rho M_H (\tau^{-1} \tilde{\varphi}_F)^{\sigma-1} \right) \right]^{\frac{1}{\sigma-1}}, \tag{18}$$

where M_H is mass of domestic firms active in home market. Taking into account that countries are symmetric ($M_H = M_F$), $M_H + \rho M_H$ can be interpreted as the total mass of competing firms in home country, and $\tilde{\varphi}_T$ also can be regarded as the average productivity of all firms engaging in home country (all domestic firms plus some foreign ones).

In equation (18), two average productivities $\tilde{\varphi}_H(\varphi_H^*)$ and $\tilde{\varphi}_F(\varphi_F^*)$ are defined as follows:

$$\tilde{\varphi}_H = \left[\frac{1}{1 - G(\varphi_H^*)} \int_{\varphi_H^*}^{\infty} \varphi^{\sigma-1} g(\varphi) d\varphi \right]^{\frac{1}{\sigma-1}}, \text{ and } \quad \tilde{\varphi}_F = \left[\frac{1}{1 - G(\varphi_F^*)} \int_{\varphi_F^*}^{\infty} \varphi^{\sigma-1} g(\varphi) d\varphi \right]^{\frac{1}{\sigma-1}}, \tag{19}$$

each of which is strictly increasing function of each cutoff productivity. Therefore to determine the average productivity $\tilde{\varphi}_T$, the two cutoff productivities have to be pinned down by product market equilibrium conditions.

The expected operating profit of a domestic firm is,

$$\Pi_H(\varphi) = (1 - \delta) \sum_{t=0}^{\infty} (1 - r - \delta)^t \pi_H(\varphi) - \frac{c_v \mathcal{P}}{q(\theta)} h_H(\varphi) - f_H, \tag{20}$$

where the flow profit in each period is

$$\pi_H(\varphi) = p_H(\varphi) x_H(\varphi) - w h_H(\varphi) - \frac{c_v \mathcal{P}}{q(\theta)} \chi h_H(\varphi) - f_H. \tag{21}$$

The equation (20) suggests that in the initial period, an entering firm only pays fixed costs and hiring costs for the profit from the next period and some firms are forced to exit by facing constant shocks δ without reaping any profit. Moreover, these two equations imply that after a firm employs the optimal number of workers h_H at the initial period, it recruits, from the next period, the exact number of workers it lost by the match specific job destruction χ in order to keep their hiring levels being optimal.

Let us consider the determination of domestic cutoff productivity φ_H^*. At this productivity level, the operating expected profit has to be equalized to zero in order to admit only profitable firms in the market. This feature leads to,

$$\pi_H(\tilde{\varphi}_H) = h_H(\tilde{\varphi}_H) \frac{r + \delta}{1 - \delta} \frac{c_v \mathcal{P}}{q(\theta)} + \left[\left(\frac{\tilde{\varphi}_H}{\varphi_H^*} \right)^{\sigma - 1} \frac{1 + r}{1 - \delta} - 1 \right] f_H, \tag{22}$$

which is called as *the domestic zero cutoff profit condition*, whose derivation is given in the appendix. This condition is almost the same as that in Melitz (2003) except for the first term in the right hand side, which exists due to search frictions in the labor market. In the same manner, at the exporting cutoff productivity φ_F^*, we can derive *the exporting zero cutoff profit condition* as follows,

$$\pi_F(\tilde{\varphi}_F) = h_F(\tilde{\varphi}_F) \frac{r + \delta}{1 - \delta} \frac{c_v \mathcal{P}}{q(\theta)} + \left[\left(\frac{\tilde{\varphi}_F}{\varphi_F^*} \right)^{\sigma - 1} \frac{1 + r}{1 - \delta} - 1 \right] f_F, \tag{23}$$

which means that a firm with this productivity is indifferent between engaging in exporting activity or not.

In addition to these ex-post entry equilibrium conditions, ex-ante entry process is controlled by *the free entry condition*,

$$\frac{F_E}{1 - G(\varphi_H^*)} = \frac{1-\delta}{r+\delta}\pi_H(\tilde{\varphi}_H) - \frac{c_v\mathcal{P}}{q(\theta)}h_H(\tilde{\varphi}_H) - f_H + \rho\left[\frac{1-\delta}{r+\delta}\pi_F(\tilde{\varphi}_F) - \frac{c_v\mathcal{P}}{q(\theta)}h_F(\tilde{\varphi}_F) - f_F\right].$$

(24)

That is, in equilibrium, entry sunk costs F_E is equalized to the sum of the expected profit of engaging only in domestic market (first term in the right hand side) plus that of exporting activity (second term). In other words, if the sum of the expected profits is greater than the entry sunk costs, more firms are willing to enter the market until the equality is achieved.

By combining these three conditions, we obtain an equation concerning the relation between φ_H^* and φ_F^*,

$$F_E = (1 - G(\varphi_H^*))\left[\left(\frac{\tilde{\varphi}_H}{\varphi_H^*}\right)^{\sigma-1} - 1\right]\frac{1+r}{r+\delta}f_H + (1 - G(\varphi_F^*))\left[\left(\frac{\tilde{\varphi}_F}{\varphi_F^*}\right)^{\sigma-1} - 1\right]\frac{1+r}{r+\delta}f_F.$$

(25)

With the other equation on the relation between φ_H^* and φ_F^*,

$$\tau^{1-\sigma}\left(\frac{\varphi_F^*}{\varphi_H^*}\right)^{\sigma-1}f_H = f_F,$$

(26)

the equilibrium in the product market is determined.[14]

In the sector X product market, the mass of firms is determined so as to clear the labor market,

$$\frac{M}{1+\rho}[h_H(\tilde{\varphi}_H) + \rho h_F(\tilde{\varphi}_F)] = L(F(\tilde{A}) - L_U),$$

(27)

where $M = M_H + \rho M_H$ is the total mass of effective intermediate firms in this country. Even though in the usual heterogeneous firm model, the mass of firms plays an important role in determining aggregate variables, it is not the case in the current setting because of the modeling of the production function.

Definition 2 : *The product market equilibrium* (φ_H^*, φ_F^*, M_H^*) *is determined by equation*

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[14]Derivation of this equation is given in appendix.

(25), (26), and (27).

Here, cutoff productivities φ_H^* and φ_F^* are not affected by labor market variables. This property is called *separavility* in Felbermayr et al. (2011), and enables us to analyze the effect of trade liberalization on economy-wide unemployment in a simple manner. It is obtained because revenue, wage payments, and hiring costs for each firm can be written in linear form in terms of the amount of employment for each firm.

5   The effect of trade liberalization

Here, we show the effect of trade liberalization on the labor market outcomes and the occupational choice outcomes. The scenario of trade liberalization supposes to be the decrease of iceberg transport costs τ. By Proposition 1 and Lemma 2, it suffices to consider the effect of decreasing τ on the average productivity $\tilde{\varphi}_T$.[15]

To identify the direction of equilibrium average productivity movement after trade liberalization, it is rewritten as follows,

$$\tilde{\varphi}_T = \varphi_H^* \left[\frac{1}{1+\rho} \left(\frac{F_E/f_H}{1-G(\varphi_H^*)} \frac{r+\delta}{1+r} + 1 + \rho \frac{f_F}{f_H} \right) \right]. \tag{28}$$

Then, the following proposition can be obtained.

Proposition 2 : *A decrease of iceberg transport cost τ leads to an increase of the average productivity $\tilde{\varphi}_T$, under the pertition condition $\tau^{\sigma-1} f_F \geq f_H$ being satisfied.*

Proof. See appendix. □

The rise of average productivity due to trade liberalization reflects exits of low productivity firms out of the product market and a change of exporting strategies for high

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[15]Because of the modeling of production function (Blanchard and Giavazzi type), the price index does not depend on the mass of firms, but only on the average productivity. This characteristic is shown in the appendix.

productive firms. Then as seen in the proposition 1, this change caused by trade liberalization leads to an increase in wages and the labor market tightness. This means that even though both hiring expansion by high productivity exporting firms and hiring shrinkage by low productive domestic firms and exiting firms occur, the former effect dominates as a result of hiring costs reduction ($\frac{c_v \mathcal{P}}{q(\theta)} \downarrow$). In other words, labor is successfully reallocated from low productivity firms to high producitivty ones.

Even though the within-sector labor market reaction mentioned above is intriguing, it then also leads to a change in the occupational choice decision in the current two-sector economy. As can be seen in Lamma 2, a rise of wages and the labor market tightness causes an increase in the cutoff skill level $\tilde{A}(\theta, \mathcal{P})$, which means that more workers inflow into the sector X due to higher wages and higher probability of matching with a firm. Thus, the implication on unemployment after trade liberalization is clearly different from what one-sector analysis suggests. In other words, while one sector models only capture the change of unemployment rates due to job creations or losses within a sector, two sector models can additionally take into account the change of labor composition across sectors.

As defined in the previous section, the rate of unemployment in this economy depends on how many workers shift to sector X, as determined by the cutoff skill level $\tilde{A}(\theta, \mathcal{P})$ as well as on the extent of job creation captured by the labor market tightness θ. As a result, in some cases higher labor market tightness *ceteris paribus* decreases unemployment in sector X (*job creation effect*), induced expansion of job creation and higher wages lead to some workers to moving from sector Y to sector X (*labor inflow effect*), resulting in the net unemployment outcome depending on the scale of these two effects. This can be seen in a following equation:

$$\frac{\partial L_U}{\partial \tilde{\varphi}} = \frac{s}{\theta q(\theta) + s} \left[\underbrace{\frac{\partial F(\tilde{A})}{\partial \tilde{A}} \frac{\partial \tilde{A}}{\partial \tilde{\varphi}}}_{\text{labor inflow effect}} - \underbrace{\frac{F(\tilde{A})}{\theta q(\theta) + s} \frac{\partial \theta q(\theta)}{\partial \tilde{\varphi}}}_{\text{job creation effect}} \right], \tag{29}$$

where the first term in the bracket indicates labor inflow effect which leads to an increase
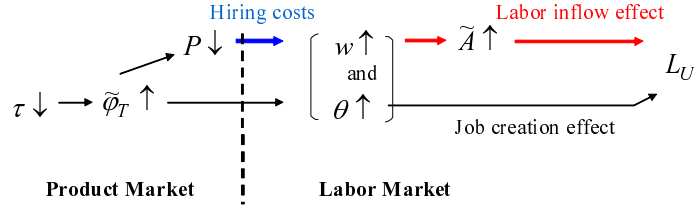
Figure 5.1: The effect of trade liberalization on unemployment

of unemployment and the second term in the bracket is job creation effect causing a reduction of unemployment. In addition, we notice from the equation that matching quality captured by the function $q(\cdot)$ and the skill distribution $F(\cdot)$ should affect the net effect of trade liberalization on unemployment. For instance, if the workers' skill distribution around the cutoff skill level is thick in a country, workers' movement from sector Y to sector X is large, and so this labor inflow effect should dominate job creation effect in sector X, causing a rise of unemployment rates.

To analyze the net impact of trade liberalization on unemployment, the skill distribution of workers is assumed to be uniformly distributed and the matching function is Cobb-Douglas form: $q(\theta) = \bar{m}\theta^{-\alpha_1}$.

The effect of trade liberalization on unemployment is summarized in the following proposition, and Figure5.1 illustrates the influence path for each case.

Proposition 3 : *a) Trade liberalization has an impact on economy-wide unemployment through two channels: (1) labor inflow effect, and (2) job creation effect, whose effects operate opposite directions. Net effect depends on the countries characteristics such as matching quality and worker distribuction.*

b) Under the assumption that the matching function is Cobb-Douglas and the skill distribution is Pareto, economy-wide unemployment decreases if

$1 + \frac{s}{\bar{m}\theta^{1-\alpha_1}} - \frac{1}{k}\left(\left(\frac{\tilde{A}}{\alpha_{min}}\right)^k - 1\right)\left(1 + \frac{\bar{m}\theta^{1-\alpha_1}}{2(r+s)}\right)$ *is negative, and vice versa.*

Proof. See appendix. $\square$

Our distinctive result is caused by introduction of the outside sector and worker heterogeneiry. Despite the job creation after trade liberalization, a rise of matching

probability for a worker leads to the inflow of workers into sector X, and thus both effects attenuate each other. In the case of the Cobb-Douglas matching function and the Pareto skill distribution, the net impact of trade liberalization on economy-wide unemployment depends on the efficiency of the matching parameter $\bar{m}$ and α_1, job destruction rate s and r, skill distribution parameter k and α_{min}, and endogenous variable $\tilde{A}$ and θ. In other words, if the quality of matching process and the labor market tightness is relatively high, trade liberalization reduces an economy-wide unemployment rate, whereas if job destruction rates are high, it increases unemployment rates.

This result is in contrast with Helpman and Itkhoki (2010) where an economy-wide unemployment rate is determined only by labor inflow effect. In their model, a sectoral unemployment rate depend only on the labor market parameters which do not change even after trade liberalzation and hence an economy-wide unemployment rate is determined by how many workers are in the more frictional differentiated-good sector. Trade liberalization in a differentiated-good sector motivates workers to move from low frictional outside sector to a differentiated-good sector, resulting in an increase in an economy-wide unemployment rate. In contrast, the economy-wide unemployment rate in our model takes into account both labor movements and the change of the sectoral unemployment rate, resulting in ambiguous effect. Moreover, the net outcome is influenced by shape of skill distribution in our heterogenous workers setting.

6   Conclusions

The purpose of this paper is to examine the impact of trade liberalization on unemployment in the two-sector situation where one sector has the Melitz type product market with the search-matching frictional labor market, and the other has perfectly competitive markets. The results demonstrate that trade liberalization has generally ambiguous effect on the rate of economy-wide unemployment, for job creation effect and labor inflow effect operate in the opposite direction. In the case of the Cobb-Douglas matching function and Pareto skill distribution of workers, we show a sufficient condition under which trade

liberalization decreases the economy-wide unemployment rate. That condition is mainly composed of labor market parameters and skill distributional parameters.

Although the within-sectoral labor market outcomes are similar to Felbermayr et al. (2008, 2011), our results are obtained by the different mechanism. That is, trade liberalization does not lead to a change of the marginal revenue of hiring, but leads to a reduction of the marginal costs of hiring, resulting in an increase in both wages and the labor market tightness. In addition, because of our two-sector setting, economy-wide outcome is affected by labor inflow effect. Moreover, our result is also different from Helpman and Itskhoki (2010) because our model takes into account both labor inflow effect and within-sectoral job creation effect in equilibrium.

Despite these results, there are some limitations to our study. First, our results depend on the assumption that hiring costs are paid in the unit of final good X. Even though it is partly justified by an explanation, we have to endogenize this mechanism. Second, an economy in our model is composed of two extreme types of sectors in that only one sector has labor market frictions and the other has no friction. This artificial situation can be justified by the situation of developing countries. Another justification is that the model considers the extreme case of two arbitrary sectors where one sector has a relatively high frictional labor market and the other a low frictional labor market.

Finally, even though within-sectoral frictions (search and matching friction) in sector X are highlighted in this model, between-sectoral frictions are not considered. In reality, the latter friction is greater than the former: a worker must encounter uncertainty and other costs resulting from sector changes which is greater than these in the case of occupational changes within a sector. Therefore in the future work, the effect of trade liberalization on labor movement across sectors might be considered with friction.

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Appendix

- **Derivation of wage curve and job creation curve**

  By substituting the modified value function of an employed worker and the firm's envelop condition into Stole and Zweibel bargaining equation, we obtain

  $$\frac{\partial w}{\partial h} = -2\frac{w}{h} + \frac{rU}{h} + \frac{1}{h}\frac{\partial R}{\partial h}.$$

  Now, we use the method of variation of constants. After solving the homogeneous equation, we assume the constant term C as $C(h)$:

  $$w = C(h)h^{-2}.$$

  Substituting this into the former equation,

  $$C'(h) = rUh^{-1} + \frac{\partial R}{\partial h}h^{-1}.$$

  Noticing that $\frac{\partial R}{\partial h} = \frac{\sigma-1}{\sigma}D(\varphi)\varphi^{\frac{\sigma-1}{\sigma}}h(\varphi)^{-\frac{1}{\sigma}}$, $C(h)$, which is obtained by the integrateion of $C'(h)$, is substituted into w:

  $$w = \frac{1}{2}rU + \frac{\sigma}{2\sigma - 1}\frac{\partial R}{\partial h}. \qquad (A1)$$

  Based on the above, we first derive the wage equation. By $(A1)$ and firm's optimal employment condition (9), some calculations leads to

  $$w = rU + \Delta\frac{c_v\mathcal{P}}{q(\theta)}.$$

  By the value function of both unemployed and employed workers, we obtain the wage equation:

  $$w = \Delta\frac{c_v\mathcal{P}}{q(\theta)} + \theta\frac{c_v\mathcal{P}}{1 - \delta}.$$

As for the derivation of job creation curve, substitutions of $\frac{\partial R}{\partial h}$, a bargaining condition, and $p_H(\tilde{\varphi}) = \mathcal{P}$ into the equation $(A1)$ lead to

$$w = \frac{2(\sigma - 1)}{2\sigma - 1}\tilde{\varphi}\mathcal{P} - \Delta\frac{c_v\mathcal{P}}{q(\theta)}.$$

This is the job creation curve.

- **Proof for Lemma 1**

  As be mentioned, the wage curve is increasing and the job creation curve decreasing in θ:

  $$\frac{\partial w}{\partial \theta}\bigg|_W = \frac{c_v\mathcal{P}}{1-\delta} - \Delta\frac{c_v\mathcal{P}}{[q(\theta)]^2}q'(\theta) > 0,$$
  $$\frac{\partial w}{\partial \theta}\bigg|_{JC} = \Delta\frac{c_v\mathcal{P}}{[q(\theta)]^2}q'(\theta) < 0.$$

  In order to obtain the intersection of these curves, the intercept of wage curve has to lower than that of job creation curve. While the former is zero, the latter $\frac{2(\sigma-1)}{2\sigma-1}\tilde{\varphi}\mathcal{P} > 0$, desirable result being obtained.

  □

- **Proof for Proposition 1**

  Tottaly differentiation of the wage curve and the job creation curve generates the following two equation:

  $$\frac{dw}{d\tilde{\varphi}} = \left[\Delta\frac{c_v}{q(\theta)} + \theta\frac{c_v}{1-\delta}\right]\frac{d\mathcal{P}}{d\tilde{\varphi}} - \left[\Delta\frac{c_v\mathcal{P}}{[q(\theta)]^2}q'(\theta) + \frac{c_v\mathcal{P}}{1-\delta}\right]\frac{d\theta}{d\tilde{\varphi}},$$
  $$\frac{dw}{d\tilde{\varphi}} = -\Delta\frac{c_v}{q(\theta)}\frac{d\mathcal{P}}{d\tilde{\varphi}} + \Delta\frac{c_v\mathcal{P}}{[q(\theta)]^2}q'(\theta)\frac{d\theta}{d\tilde{\varphi}}.$$

  From these equation, we can obtain,

  $$\frac{d\theta}{d\tilde{\varphi}} = \frac{\left[2\Delta\frac{c_v}{q(\theta)} + \frac{c_v\theta}{1-\delta}\right]\frac{d\mathcal{P}}{d\tilde{\varphi}}}{2\Delta\frac{c_v\mathcal{P}}{[q(\theta)]^2}q'(\theta) - \frac{c_v\mathcal{P}}{1-\delta}} > 0. \qquad (A2)$$

29

where we use $\frac{\tilde{\varphi}}{\mathcal{P}}\frac{d\mathcal{P}}{d\tilde{\varphi}} = -1$. Since the second order differential of matching function is negative and $\frac{d\mathcal{P}}{d\tilde{\varphi}}$ is also negative, the sign of $\frac{d\theta}{d\tilde{\varphi}}$ is positive.

Similarly, as for $\frac{dw}{d\tilde{\varphi}}$, we conduct total differentiation,

$$\frac{dw}{d\tilde{\varphi}} = \frac{\theta}{2 - q(\theta)^2/[(r+s)q'(\theta)]}\left[1 + \frac{1}{\epsilon_{q(\theta)}}\right]\frac{c_v}{1-\delta}\frac{d\mathcal{P}}{d\tilde{\varphi}}.$$

where,

$$\epsilon_{q(\theta)} = \frac{\theta}{q(\theta)}\frac{dq(\theta)}{d\theta}.$$

Therefore, if $-1 < \epsilon_{q(\theta)} < 0$, then $\frac{\partial w}{\partial \tilde{\varphi}}$ is positive. In the case of Cobb-Douglas matching function where $q(\theta) = \bar{m}\theta^{-\alpha_1}$, this inequality is satisfied.

□

- **Proof for Lemma 2**

  Differentiation of $\tilde{A}(\theta, \tilde{\varphi})$ with respect to $\tilde{\varphi}$ generates the following equation,

  $$\frac{\partial \tilde{A}(\theta, \tilde{\varphi})}{\partial \tilde{\varphi}} = \frac{\tilde{A}(\theta, \tilde{\varphi})}{\tilde{\varphi}}\left[\frac{\tilde{\varphi}}{\theta}\frac{\partial \theta}{\partial \tilde{\varphi}} - 1\right].$$

  So, the sign of $\frac{\partial \tilde{A}}{\partial \tilde{\varphi}}$ depends on whether $\frac{\tilde{\varphi}}{\theta}\frac{\partial \theta}{\partial \tilde{\varphi}}$ is greater than one or not.

  Now, by equation $(A2)$, and the assumption that the matching function is Cobb-Douglas form,

  $$\epsilon_\theta \equiv \frac{\tilde{\varphi}}{\theta}\frac{\partial \theta}{\partial \tilde{\varphi}} = \frac{\left[2\Delta\frac{c_v\mathcal{P}}{q(\theta)} + \theta\frac{c_v\mathcal{P}}{1-\delta}\right]\frac{\tilde{\varphi}}{\mathcal{P}}\frac{d\mathcal{P}}{d\tilde{\varphi}}}{2\Delta\frac{c_v\mathcal{P}}{[q(\theta)]^2}\theta q'(\theta) - \theta\frac{c_v\mathcal{P}}{1-\delta}},$$
  $$= \frac{2\Delta\frac{c_v\mathcal{P}}{q(\theta)} + \theta\frac{c_v\mathcal{P}}{1-\delta}}{\theta\frac{c_v\mathcal{P}}{1-\delta} - 2\Delta\frac{c_v\mathcal{P}}{[q(\theta)]^2}\theta q'(\theta)}. \qquad (A3)$$

  The second equality uses $\frac{\tilde{\varphi}}{\mathcal{P}}\frac{d\mathcal{P}}{d\tilde{\varphi}} = -1$. Since the matching function is concave $(-\frac{\theta}{q(\theta)}\frac{dq(\theta)}{d\theta} \le 1)$, the numerator is greater than the denominator, so $\frac{\tilde{\varphi}}{\theta}\frac{\partial \theta}{\partial \tilde{\varphi}}$ is greater than one.

  □

- **Derivation of the zero cutoff profit conditions**

  By the per period flow profit, we can derive

$$\frac{\pi_H(\tilde{\varphi}_H) + f_H}{\pi_H(\varphi_H^*) + f_H} = \frac{h_H(\tilde{\varphi}_H)}{h_H(\varphi_H^*)}. \tag{A4}$$

  From the production function of an intermediate-good firm and the demand function,

$$\frac{h_H(\tilde{\varphi}_H)}{h_H(\varphi_H^*)} = \left(\frac{\tilde{\varphi}_H}{\varphi_H^*}\right)^{\sigma-1}. \tag{A5}$$

  Now, at the domestic cutoff productivity φ_H^*, the operating profit is,

$$\pi_H(\varphi_H^*) = \frac{r+\delta}{1-\delta}\left[\frac{c_v \mathcal{P}}{q_H(\theta)} h_H(\varphi_H^*) + f_H\right].$$

  Combining these equations,

$$\pi_H(\tilde{\varphi}_H) = h_H(\tilde{\varphi}_H)\frac{r+\delta}{1-\delta}\frac{c_v\mathcal{P}}{q(\theta)} + \left[\left(\frac{\tilde{\varphi}_H}{\varphi_H^*}\right)^{\sigma-1}\frac{1+r}{1-\delta} - 1\right]f_H.$$

  By the similar way, the exporting zero cutoff profit condition is also obtained.

  □

- **Derivation of equation** (26)

  By operating profit from domestic market,

$$\frac{\pi_H(\varphi_F^*) + f_H}{\pi_H(\varphi_H^*) + f_H} = \left(\frac{\varphi_F^*}{\varphi_H^*}\right)^{\sigma-1},$$

  and, this can be rewritten as follow,

$$\begin{aligned}
&\pi_H(\varphi_H^*) + f_H - \left(\frac{r+\delta}{1-\delta}\right)\frac{c_v\mathcal{P}}{q(\theta)}h_H(\varphi_F^*) \\
&= \left(\frac{\varphi_F^*}{\varphi_H^*}\right)^{\sigma-1}\left(\frac{r+\delta}{1-\delta}\right)\left[\frac{1-\delta}{r+\delta}\pi_H(\varphi_H^*) + \frac{1-\delta}{r+\delta}f_H - \frac{c_v\mathcal{P}}{q(\theta)}h_H(\varphi_H^*)\right].
\end{aligned} \tag{A6}$$

Then, substituting the domestic zero cutoff profit condition results in,

$$\pi_H(\varphi_H^*) + f_H - \left(\frac{r+\delta}{1-\delta}\right)\frac{c_v\mathcal{P}}{q(\theta)}h_H(\varphi_F^*) = \left(\frac{\varphi_F^*}{\varphi_H^*}\right)^{\sigma-1}\frac{1+r}{1-\delta}f_H.$$

Now, since $\frac{h_H(\varphi)}{h_F(\varphi)} = \tau^{\sigma-1}$, we can derive,

$$\pi_H(\varphi_F^*) + f_H - \left(\frac{r+\delta}{1-\delta}\right)\frac{c_v\mathcal{P}}{q(\theta)}h_H(\varphi_F^*) = \tau^{\sigma-1}\left[\pi_F(\varphi_F^*) + f_F - \left(\frac{r+\delta}{1-\delta}\right)\frac{c_v\mathcal{P}}{q(\theta)}h_F(\varphi_F^*)\right].$$

Then, by substituting this into equation $(A6)$, and some calculations lead to,

$$\tau^{\sigma-1}\left(\frac{\varphi_F^*}{\varphi_H^*}\right)^{\sigma-1}f_H = f_F.$$

□

- **The dependence of price index only on the average productivity**

  As defined in the above, the price index is,

  $$\mathcal{P} = \left[(M)^{-1}\int_{j\in J}p(j)^{1-\sigma}dj\right]^{\frac{1}{1-\sigma}},$$

  which can be rewritten as,

  $$\mathcal{P} = \left[(M)^{-1}\int_0^\infty p(\varphi)^{1-\sigma}\mu(\varphi)Md\varphi\right]^{\frac{1}{1-\sigma}},$$

  where μ is $\frac{g(\varphi)}{1-G(\varphi_H^*)}$ if $\varphi \geq \varphi_H^*$, and otherwise zero. By substituting $p(\varphi) = \frac{\sigma}{\sigma-1}\frac{1}{\varphi}MC$ where $MC = w + \frac{\partial w}{\partial h(\varphi)}h(\varphi) + \Delta\frac{c_v\mathcal{P}}{q(\theta)}$, we obtain,

  $$\mathcal{P} = \frac{\sigma}{\sigma-1}\frac{1}{\tilde{\varphi}}MC = p(\tilde{\varphi}),$$

  where $\tilde{\varphi} \equiv \left[\int_0^\infty \varphi^{\sigma-1}\mu(\varphi)d\varphi\right]^{\frac{1}{\sigma-1}}$. This clearly depends only on the average productivity.

  □

- **Proof for Proposition 2**

  First, we want to show how the cutoff productivities φ_H^* and φ_F^* are changed by the movement of τ. By differentiating equation (25) with respect to τ,

  $$
  \begin{aligned}
  0 = &-\frac{dG(\varphi_H^*)}{d\tau}k(\varphi_H^*)\frac{1+r}{r+\delta}f_H + (1-G(\varphi_H^*))\frac{dk(\varphi_H^*)}{d\tau}\frac{1+r}{r+\delta}f_H \\
  &-\frac{dG(\varphi_F^*)}{d\tau}k(\varphi_F^*)\frac{1+r}{r+\delta}f_F + (1-G(\varphi_F^*))\frac{dk(\varphi_F^*)}{d\tau}\frac{1+r}{r+\delta}f_F.
  \end{aligned}
  $$

  Now, since

  $$
  \begin{aligned}
  \frac{dG(\varphi_H^*)}{d\tau} &= g(\varphi_H^*)\frac{\varphi_H^*}{d\tau}, \\
  \frac{dG(\varphi_F^*)}{d\tau} &= g(\varphi_F^*)\left[\frac{\varphi_F^*}{\tau} + \left(\frac{\varphi_F^*}{\varphi_H^*}\right)\frac{d\varphi_H^*}{d\tau}\right], \\
  k(\varphi) &= \left[\left(\frac{\tilde{\varphi}}{\varphi}\right)^{\sigma-1} - 1\right], \\
  \frac{\partial k(\varphi)}{\partial \varphi} &= \frac{g(\varphi)k(\varphi)}{1-G(\varphi)} - \frac{(\sigma-1)[k(\varphi)+1]}{\varphi},
  \end{aligned}
  $$

  we obtain,

  $$
  \frac{\partial \varphi_H^*}{\partial \tau} = -\frac{f_F\frac{\varphi_F^*}{\tau}\Xi(\varphi_F^*)}{f_H\Xi(\varphi_H^*) + f_F\left(\frac{\varphi_F^*}{\varphi_H^*}\right)\Xi(\varphi_F^*)} < 0,
  $$

  where

  $$
  \Xi(\varphi_i^*) \equiv (1-G(\varphi_i^*))k'(\varphi_i^*) - g(\varphi_i^*)k(\varphi_i^*), \quad (i=H,F).
  $$

  Next, the derivative of φ_F^* with respect to τ is

  $$
  \frac{\partial \varphi_X^*}{\partial \tau} = \frac{\varphi_F^*}{\tau}\left[1 - \frac{1}{\left(\frac{\varphi_H}{\varphi_F^*}\right)\frac{f_H}{f_F}\frac{\Xi(\varphi_H^*)}{\Xi(\varphi_F^*)} + 1}\right] > 0.
  $$

  Given the above results, we can conclude that as long as $f_F \geq f_H$ is satisfied, $\tilde{\varphi}_T$ is decreasing in τ.

  $\square$

- **Proof for Proposition 3**

  We calculate $\frac{\partial L_U}{\partial \tilde{\varphi}}$ in the case that the skill distribution is Pareto and the matching

function is Cobb-Douglas: $F(\alpha) = 1 - \left(\frac{\alpha_{min}}{\alpha}\right)^k$ $(k > 1)$ and $q(\theta) = \bar{m}\theta^{-\alpha_1}$. Since $\frac{\partial \tilde{A}}{\partial \tilde{\varphi}} = \frac{c_v}{(1+r)(1-\delta)}(\mathcal{P}\frac{\partial \theta}{\partial \tilde{\varphi}} + \theta\frac{\mathcal{P}}{\tilde{\varphi}})$, $\frac{\tilde{\varphi}}{\mathcal{P}}\frac{\partial \mathcal{P}}{\partial \tilde{\varphi}} = -1$, and $\epsilon_\theta \equiv \frac{\tilde{\varphi}}{\theta}\frac{\partial \theta}{\partial \tilde{\varphi}}$,

$$\text{sign}\left\{\frac{\partial L_U}{\partial \tilde{\varphi}}\right\} = \text{sign}\left\{\epsilon_\theta \left(1 - \frac{1 - \alpha_{min}^k \tilde{A}^{-k}}{k\alpha_{min}^k \tilde{A}^{-k}}\frac{(1-\alpha_1)\bar{m}\theta^{1-\alpha_1}}{\bar{m}\theta^{1-\alpha_1}}\right) - 1\right\},$$

By using $(A3)$, we know that

$$\text{sign}\left\{\epsilon_\theta \left(1 - \frac{1 - \alpha_{min}^k \tilde{A}^{-k}}{k\alpha_{min}^k \tilde{A}^{-k}}\frac{(1-\alpha_1)\bar{m}\theta^{1-\alpha_1}}{\bar{m}\theta^{1-\alpha_1}}\right) - 1\right\}$$

$$= \text{sign}\left\{\frac{\epsilon_\theta - 1}{\epsilon_\theta} - \frac{1 - \alpha_{min}^k \tilde{A}^{-k}}{k\alpha_{min}^k \tilde{A}^{-k}}\frac{1-\alpha_1}{1 + s/(\bar{m}\theta^{1-\alpha_1})}\right\},$$

$$= \text{sign}\left\{1\Big/\left(1 + \frac{\bar{m}\theta^{1-\alpha_1}}{2(r+s)}\right) - \frac{1 - \alpha_{min}^k \tilde{A}^{-k}}{k\alpha_{min}^k \tilde{A}^{-k}}\Big/\left(1 + \frac{s}{\bar{m}\theta^{1-\alpha_1}}\right)\right\}.$$

Therefore,

$$\text{sign}\left\{\frac{\partial L_U}{\partial \tilde{\varphi}}\right\} = \text{sign}\left\{1 + \frac{s}{\bar{m}\theta^{1-\alpha_1}} - \frac{1}{k}\left(\left(\frac{\tilde{A}}{\alpha_{min}}\right)^k - 1\right)\left(1 + \frac{\bar{m}\theta^{1-\alpha_1}}{2(r+s)}\right)\right\}.$$

□