International Trade and Income Inequality*

Taiji Furusawa† Hideo Konishi‡
Hitotsubashi University Boston College

October 12, 2012

Abstract

We propose a theory that explains why international trade can widen a wage gap between top income-earners and others. We consider two symmetric countries in which individuals with different abilities work either as knowledge workers, who develop products produced in a differentiated-good sector, or as production workers, who engage in actual production processes. In equilibrium, *ex ante* symmetric firms post different wages for knowledge workers and hence attract workers with different abilities from other firms, creating the difference in quality of their products. International trade will benefit firms that produce high-quality products and harm firms that produce low-quality products. The relative wage gap between individuals with high ability and those with low ability expands as a result. Indeed, we show that international trade increases the real wages for those with lowest and highest abilities but decreases the real wages for those with intermediate abilities. We also extend the basic model to the one with asymmetric countries and show that the wage gap created by international trade is severer in the smaller country than the larger country.

Preliminary and incomplete.

* **Work in progress.** We are grateful to James Anderson, Pol Antras, Arnaud Costinot, Elhanan Helpman,Oleg Itskhoki, Tomoya Mori, and Esteban Rossi-Hansberg for helpful comments. We also thank Laura Bonacorsi for her research assitance.

† Graduate School of Economics, Hitotsubashi University. (E-mail) furusawa@econ.hit-u.ac.jp

‡ Department of Economics, Boston College. Email: hideo.konishi@bc.edu

1 Introduction

It has been widely documented that wage gap increases significantly in many countries, especially in the developed countries. In Britain, for example, "chief executives can expect to receive average compensation in excess of £4.5m ($ 6.9m) this year. Pay at the top grew by over 300% between 1998 and 2010. At the same time, the median British worker's real wage has been pretty stagnant. These trends mean the ratio of executive to average pay at FTSE 100 firms jumped from 47 to 120 times in 12 years." (*The Economist*, January 14th-20th, 2012, p. 11) It has also been documented that job polarization has occurred in many developed countries, including the United States and some European countries, such that the shares of employment in high-skilled occupations and low-skilled occupations grow while that of middle-wage occupation declines (Acemoglu and Autor, 2011).

Technological changes and globalization are often argued to be the causes of the job polarization and expansion of income inequality. Machines have replaced middle-skilled workers who engage in routine tasks. Offshoring routine tasks to low-wage countries reduces demands for middle-skilled workers in high-wage developed countries. Offshoring is unambiguously an important aspect of globalization. But we argue here that international trade in goods, which is more fundamental part of globalization, alone can cause some sort of job polarization and expand income inequality among different skill groups of workers. In particular, we relate the recent trend of market concentration in manufacture industries, i.e., the winner-take-all market trend, to the job polarization and the expansion of the income inequality. Thanks to globalization (i.e., an increase in the market size and wider information transmission), a difference in quality becomes an important source of differential profitability within industries: firms that sells high-quality products command disproportional market shares. This, in turn, gives workers who work as knowledge workers in the winning firms disproportional shares of income. Knowledge workers in such firms are winners because they are the sources of the firms' success. But that implies that the war for talent arises and large portion of the firms' operating profits go to the knowledge workers as the rents for their talents. What determines the product quality is the talent of knowledge workers such as managers and

R&D workers hired in firms. Firms that hire talented knowledge workers enjoy benefits from the employment, which in turn benefits such knowledge workers in the form of high wages.

To show this phenomenon, which becomes more and more important in recent years, we build a two-country model in which *ex ante* symmetric firms in the differentiated good (manufacture) sector hire knowledge workers with different abilities, thereby producing products with different qualities. In equilibrium, knowledge workers are assortatively allocated in different firms: firms that hire a group of highly-talented knowledge workers produce high-quality products, while those hire mediocre knowledge workers produce relatively low-quality products. Firms also hire production workers to produce their products. We assume that workers are homogeneous in their productivities when hired as production workers despite the difference in their abilities. The wage gap may arise between knowledge workers and production workers even within a firm; the wage gap within a firm is particularly serious in profitable firms that produce high-quality products.

International trade affects firms' profitability differently across firms. Top-tier firms that produce high-quality products are the winners of opening to trade; getting access to an additional market give them large benefits. Medium-tier and Lowest-tier firms, on the other hand, loose from opening to trade. They suffer from foreign top-tier firms' penetration into their own market. Although the medium-tier firms sell their products to the foreign market as well as their own market, the additional export profits after the subtraction of fixed export costs are not enough to offset the loss that they incur in their domestic market. Consequently, top income earners who are best talented and work in the top-tier exporting firms benefit from opening to trade; they are the winners of globalization. On the contrary, workers in the middle-income class, who (used to) work as knowledge workers in the middle-tier firms, are likely to suffer from opening to trade. Their nominal incomes drop because the profits for the firms in which they are working fall after trade liberalization. Although the real wages may still rise thanks to the increased varieties of products in their consumption after trade liberalization, we show that under a relatively mild condition their real wages decline by opening to trade. Indeed, workers in the middle-income class are the only losers from trade liberalization. Thanks to the increased varieties in consumption, the real wages for the least talented workers, who work as production workers, increase by opening to trade.

We also extend the basic model to the one with two countries of asymmetric size. We find that production workers earn a higher wage in the larger country than the smaller one. The export cutoff quality is higher in the larger country due to the higher wage than in the smaller country, while the entry cutoff quality is lower in the large country. The wage gap created by international trade can be said to be smaller in the larger country since the wages for production workers increase while the range of qualities for products sold only domestically expands in the larger country and contracts in the smaller country by opening to trade. We also find that as the variable trade costs increase, the wage rate for production workers in the larger country relative to the one in the smaller country increases.

We are not the first to theoretically predict that international trade widens wage gap across different income groups. Manasse and Turrini (2001), Yeaple (2005), Costinot and Vogel (2010), Helpman *et al.* (2010), Helpman *et al.* (2011), Helpman *et al.* (forthcoming), and Blanchard and Willmann (2011), among others, show in their models that international trade in goods widens wage gap at least in some trading countries.

Among these studies, Manasse and Turrini (2001) and Yeaple (2005) are the closest to our paper. Manasse and Turrini (2001) employ the same basic model structure as ours; *ex ante* symmetric firms produce products of different qualities because they are run by entrepreneurs with different abilities. In their analysis, however, the number of firms in the differentiated good industry is fixed and it is not affected by opening to trade. As a consequence, they cannot analyze how international trade affects profitability of firms and hence the wage distribution through a change in the number of operating firms. This channel is important because the winner-take-all phenomenon is prevalent in the global markets and this reduces the number of firms in each industry, thereby reducing the jobs for workers in the middle-income class. Yeaple (2005) derives similar predictions to ours in a similar model environment. In his model, firms choose both their individual production technologies and types of workers. A distinguishing feature of his model is the complementarity between the technology and skills of labor; high productivity technology is matched with high-skilled workers. Beside the fact that the average talent of knowledge workers is the only source of firm heterogeneity in our model, our model is different from his in that we separate labor into two endogenously allocated categories: knowledge workers and production workers. This distinction is a key to our analysis of the impact of trade on income redistribution. First, we can discuss differential effects of trade on workers within firms, which is an important wage gap as well as the wage gap within sectors and within occupations. Second and more broadly, separating knowledge workers from production workers is important to understand the effect of globalization on labor market. Knowledge can be embedded into products so that it is duplicated unboundedly with the help of capital and production workers to possibly earn a fortune in the global market. Globalization does not necessarily increase the demands for knowledge workers. It only increases the demands for talent. Knowledge created by a limited number of knowledge workers is embedded in the products and travels over the world.

2 The Model

We consider two countries, 1 and 2, in which a differentiated good with many varieties is produced and consumed. In each country $i = 1, 2$, there is a continuum of workers with the mass L_i. The differentiated good consists of a continuum of varieties each of which (denoted by $\omega \in \Omega$) is produced under monopolistic competition by a firm in a continuum pool of firms. Let $x(\omega)$ denote the consumption level of a variety ω of the differentiated good. Quality of the differentiated good may vary across varieties, which is represented by $\alpha(\omega)$. Then, a representative consumer's preferences are represented by the utility function:

$$u = \left[\int_{\Omega} \alpha(\omega)^{\frac{1}{\sigma}} x(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}}. \tag{1}$$

The utility levels derived from the consumption of the varieties of the good are aggregated with the use of CES aggregator where $\sigma > 1$ denotes the elasticity of substitution. The higher the $\alpha(\omega)$, the higher the utility a consumer derives from the consumption of variety ω.

Labor is the only production factor in this economy. But workers are employed either as knowledge workers in the differentiated good sector to develop a product or as production workers to produce the differentiated goods. We choose labor provided by production workers as the numeraire. Workers are heterogeneous in their abilities (or abilities) that only matter when they are hired as knowledge workers. Thus, they are heterogeneous as knowledge workers, but homogenous as production workers. In the basic model, ability is measured by $a \in \mathbb{R}_+$, the distribution of which in country i is described by the cumulative probability distribution G_i with probability density function g_i; the mass of workers with their abilities less than or equal to a is, therefore, given by $LG_i(a)$.

In the basic model, we assume that countries 1 and 2 are symmetric: that is, $L_1 = L_2 = L$ and $G_1(a) = G_2(a) = G(a)$ for all $a \in \mathbb{R}_+$. Until Section 5, we do not put subscripts on L and G to simplify the notations.

In the production of the differentiated good, each firm needs to hire l^s knowledge workers to develop a product. The average ability level of these workers determines the quality of the product; we simply assume that the quality of the product $\alpha(\omega)$ is equal to the average talent level of the knowledge workers employed in the firm. The differentiated good market is under the monopolistic competition with free entry and exit. To enter the market, firms need to develop the product by hiring l^s knowledge workers, which serves as entry costs. Production of the product itself requires only production workers; 1 unit of labor produces 1 unit of the good.

Because the average ability level of knowledge workers determines the product quality, firms in the differentiated good sector compete for talent. They post wages for knowledge workers, and workers apply for those positions. Then each firm chooses l^s workers from those who have applied for the firm's position. There is no friction in the labor markets, both for knowledge workers and production workers, nor does there exist information asymmetry between workers and firms on individual workers' ability levels.

In equilibrium, firms post different wages for knowledge workers attracting workers with different ability levels. Workers with highest abilities are hired by the firms that post highest wages, such that workers are sorted according to their ability levels; knowledge workers with the same or similar abilities work together in equilibrium. Thanks to the competition among firms, the entire operating profits for each firm are given as the rents to the knowledge workers. We assume that all knowledge workers, possibly with different ability level, hired in the same firm obtain the same wage. Letting $w(\omega)$ denote the knowledge worker's wage in the firm that produces the variety ω and $\tilde{\pi}(\omega)$ denote the firm's operating profits, we therefore have

$$w(\omega)l^s = \tilde{\pi}(\omega). \tag{2}$$

Firms produce varieties of different quality. Since the average ability level of knowledge workers determines the product quality, the firm distribution with respect to their product quality is determined from the ability distribution. Let $f(\alpha)$ be the density of the firms that produce a variety of quality α and F be the corresponding cumulative (non-probabilistic) function such that $F(\alpha) = \int_0^\alpha f(\alpha')d\alpha'$. Since the firms that produce varieties of quality that is greater or equal to α hire all workers whose ability levels are greater or equal to α (i.e., $a \geq \alpha$), we have

$$l^s \int_\alpha^\infty f(\alpha')d\alpha' = L[1 - G(\alpha)].$$

Taking the derivative of this identity with respect to α, we obtain the density of firms that produce a variety of quality α as

$$f(\alpha) = \frac{Lg(\alpha)}{l^s}. \tag{3}$$

As this relationship shows, each firm hires l^s knowledge workers of the same ability level; the equilibrium allocation of knowledge workers are perfectly assortative.

Workers who are not hired as knowledge workers will work as production workers. In equilibrium, there will be a cutoff ability level α^* such that all workers with $a \geq \alpha^*$ work as knowledge workers, while all workers with $a < \alpha^*$ work as production workers.

Once α^* is given, together with (3), the distribution of operating firms is completely determined.

3 Autarkic Equilibrium

This section derives the autarkic equilibrium and shows that knowledge workers obtain higher wages than production workers and that knowledge workers' wages increase proportionately with their ability levels.

Thanks to the symmetry assumed in the basic model, we need only consider a country to derive an autarkic equilibrium. Let I denote the aggregate income of a country, and let w be the wage rate of production worker. Since the wage of production workers can be normalized to 1 in one country model, each firm optimally selects the price of $p(\alpha) = \sigma/(\sigma-1)$, the constant mark-up price over the marginal cost of 1, regardless of its product quality α, so the firm that produces a variety of quality α sells

$$x(\alpha) = \frac{\alpha p(\alpha)^{-\sigma}}{\int_{\alpha^*}^{\infty} \alpha p(\alpha')^{1-\sigma} dF(\alpha')} I = \frac{\alpha}{\int_{\alpha^*}^{\infty} \alpha' dF(\alpha')} \frac{(\sigma-1)I}{\sigma} \tag{4}$$

units of the good. A firm's production level is higher if the quality of its product is higher and if the quality index, $\int_{\alpha^*}^{\infty} \alpha' dF(\alpha')$, is lower. The operating profits for the firm that produce a product with quality α are given by

$$\tilde{\pi}(\alpha) = \frac{\alpha}{\int_{\alpha^*}^{\infty} \alpha' dF(\alpha')} \frac{I}{\sigma}. \tag{5}$$

Letting $w(\alpha)$ denote the wage for a knowledge worker hired by the firm that produces the product with quality α with slight abuse of notation, we can write the profits for the firm as

$$\pi(\alpha) = \tilde{\pi}(\alpha) - w(\alpha) l^s.$$

If $\pi(\alpha)$ is greater than 0 for some firm with α, an entrant posts a slightly higher wage than $w(\alpha)$ and get all the knowledge workers from such a firm to profitably operate. Therefore, $\pi(\alpha) = 0$ in equilibrium, so that the knowledge workers' wage schedule is given by $w(\alpha) = \tilde{\pi}(\alpha)/l^s$ as (2) indicates.

The equilibrium is characterized by the two conditions: free entry (FE) condition and labor market clearing (LM) condition. Free entry condition expresses that the cutoff firm with α^* earns zero profits. The knowledge workers in the cutoff firm earn the wage of 1, i.e.,

$w(\alpha^*) = 1$, in equilibrium, since if $w(\alpha^*) > 1$ profitable entry by a firm that posts the wage of 1, for example, would occur. Thus, the free entry condition can be written as

$$\frac{\alpha^*}{\int_{\alpha^*}^{\infty} \alpha dF(\alpha)} \frac{I}{\sigma} = l^s. \tag{6}$$

The labor market clearing condition, on the other hand, expresses that total labor demands, demands for knowledge workers and those for production workers, must equal the labor supply L. Total demands for knowledge workers are given by $l^s \int_{\alpha^*}^{\infty} f(\alpha)d\alpha$. Total demands for production workers equal $(\sigma - 1)I/\sigma$. Thus, the labor market clearing condition can be written as

$$l^s \int_{\alpha^*}^{\infty} f(\alpha)d\alpha + \frac{\sigma - 1}{\sigma} I = L. \tag{7}$$

Figure 1 depicts the relationships between α^* and I that express the free entry and labor market clearing conditions. The free entry condition can be expressed by a negatively-sloped schedule FE, since the left-hand side of (6) increases with both α^* and I. The labor market clearing condition, on the other hand, can be expressed by a positively-sloped schedule LM, since the left-hand side of (7) decreases with α^* but increases with I. The intersection of these two schedules gives us the equilibrium values of α^* and I, which we call α_A^* and I_A.

Once the equilibrium threshold of α^* is determined, the equilibrium wage schedule readily obtains. As Figure 2 shows, wages are flat at 1 for all workers with their ability levels smaller than α_A^*. Their wages are 1 because they work as production workers. Those who have abilities greater than α_A^*, on the other hand, work as knowledge workers. Their wages are the rents for their abilities and are greater than 1 except for the knowledge workers whose ability levels are exactly equal to α_A^*. Note that the ratio of wages for knowledge workers with any two different levels of abilities is equal to the ratio of their abilities itself, as we can see from (5):

$$\frac{w_A(\alpha)}{w_A(\alpha_A^*)} = \frac{\tilde{\pi}(\alpha)}{\tilde{\pi}(\alpha_A^*)} = \frac{\alpha}{\alpha_A^*}.$$

Since $w_A(\alpha_A^*) = 1$, we have $w_A(\alpha) = \alpha/\alpha_A^*$ for all $\alpha \in [\alpha_A^*, \infty)$, and $w_A(\alpha) = 1$ for all $\alpha \in [0, \alpha_A^*]$.

The equilibrium real wages can also be readily derived. It is easy to infer from (4) that the worker with ability level of α consumes $\frac{\alpha'}{\int_{\alpha_A^*}^{\infty}\alpha'' dF(\alpha'')}\frac{(\sigma-1)w(\alpha)}{\sigma}$ units of the variety of quality α' for any $\alpha' \geq \alpha_A^*$. Then we substitute them into (1) to obtain the worker's indirect utility:

$$u(\alpha) = \frac{w(\alpha)}{\left(\frac{\sigma}{\sigma-1}\right)\left[\int_{\alpha_A^*}^{\infty}\alpha' dF(\alpha')\right]^{\frac{1}{1-\sigma}}}.$$

To see the impact of international trade on each worker's well-being, we define the real wage for workers with ability level α by

$$v(\alpha) = \frac{w(\alpha)}{\left[\int_{\alpha^*}^{\infty}\alpha' dF(\alpha')\right]^{\frac{1}{1-\sigma}}}, \tag{8}$$

which is proportional to the indirect utility $u(\alpha)$. The real wage in the autarkic equilibrium is given by $v_A(\alpha)$, which is nothing but $v(\alpha)$ defined in (8) with α^* being equal to α_A^*.

4 Trade equilibrium

Let us turn to the analysis of the impact of international trade on the firm distribution and wages. We suppose that firms in the differentiated good sector need to incur the fixed cost of exporting in terms of f_X units of labor. Exporting firms also incur iceberg trade cost such that they need to ship τ units of the good to supply 1 unit in the foreign market.

We consider the realistic case in which only a fraction of the firms export their products. Let α^X denote the threshold quality such that the products are exported (as well as supplied domestically) if and only if their individual qualities are higher or equal to α^X. It is easy to see that the operating profits from the domestic sales and foreign sales are given by

$$\tilde{\pi}_d(\alpha) = \frac{\alpha}{\int_{\alpha^*}^{\infty}\alpha' dF(\alpha') + \tau^{1-\sigma}\int_{\alpha^X}^{\infty}\alpha' dF(\alpha')}\frac{I}{\sigma}, \tag{9}$$

$$\tilde{\pi}_X(\alpha) = \frac{\alpha\tau^{1-\sigma}}{\int_{\alpha^*}^{\infty}\alpha' dF(\alpha') + \tau^{1-\sigma}\int_{\alpha^X}^{\infty}\alpha' dF(\alpha')}\frac{I}{\sigma}, \tag{10}$$

respectively. Then, the profits for the firm that produces a product of quality α are equal to

$$\pi(\alpha) = \begin{cases} \tilde{\pi}_d(\alpha) - w(\alpha)l^s & \text{if } \alpha^* \leq \alpha < \alpha^X \\ \tilde{\pi}_d(\alpha) + \tilde{\pi}_X(\alpha) - w(\alpha)l^s - f_X & \text{if } \alpha \geq \alpha^X \end{cases} \tag{11}$$

Since the values of the quality index are the same between the two countries due to the symmetry, it is easy to compare the operating profits from exporting and those from domestic sales. In particular, we compare the operating profits from exporting for the threshold firm with α^X, i.e., $\tilde{\pi}_X(\alpha^X) = f_X$, and those for the other threshold firm with α^*, i.e., $\tilde{\pi}_d(\alpha^*)$. Then, it follows from (9) and (10) that the relationship between α^X and α^* can be expressed by the function:

$$\alpha^X(\alpha^*) = \frac{\tau^{\sigma-1} f_X}{l^s} \alpha^*.$$

We assume that $\tau^{\sigma-1} f_X > l^s$ so that only a fraction of the firms export their products.

The free-entry condition and the labor market clearing condition can be written as

$$\frac{\alpha^*}{\int_{\alpha^*}^{\infty} \alpha dF(\alpha) + \tau^{1-\sigma} \int_{\alpha^X(\alpha^*)}^{\infty} \alpha dF(\alpha)} \frac{I}{\sigma} = l^s, \tag{12}$$

$$l^s \int_{\alpha^*}^{\infty} f(\alpha) d\alpha + f_X \int_{\alpha^X(\alpha^*)}^{\infty} f(\alpha) d\alpha + \frac{\sigma - 1}{\sigma} I = L. \tag{13}$$

By comparing the free-entry condition (12) with the autarky counterpart (6), we find that the FE schedule shifts up as Figure 3 indicates. International trade intensifies the domestic competition. In order for the same threshold producer to be break-even, the total income must increase. Similarly, by comparing (13) with (7), we see that the LM schedule shifts down. The labor market becomes tighter because demands for labor used for exporting are created by opening to trade. Thus, the income must decrease so that these increased demands are offset by the decreased demands for production workers. Trade equilibrium income denoted by I_T may be greater or smaller than I_A, i.e., the impact of trade on nominal income measured by the numeraire good is ambiguous. However, trade will unambiguously raise the threshold quality α_T^*. International trade intensifies competition in individual domestic markets, which lowers the profitability of firms that only serve their individual domestic markets. In addition, the labor market becomes tighter due to the demands for labor for exporting. These two effects work as factors to increase the bar for entry into the differentiated good sector.

Proposition 1 *International trade raises the threshold quality of the differentiated good $\alpha_A^* < \alpha_T^*$ and hence raises the average quality of the good.*

The equilibrium wage schedule is described by a piecewise linear function: (i) for $\alpha \in [0, \alpha_T^*]$,

$$w_T(\alpha) = 1$$

(ii) for $\alpha \in [\alpha_T^*, \alpha^X]$,

$$w_T(\alpha) = \frac{\alpha}{\alpha_T^*}$$

and (iii) for $\alpha \in [\alpha^X, \infty)$,

$$w_T(\alpha) = \frac{\alpha}{\alpha_T^*} + \frac{\tau^{1-\sigma} I_T}{\sigma \left[\int_{\alpha_T^*}^{\infty} \alpha' dF(\alpha') + \tau^{1-\sigma} \int_{\alpha^X}^{\infty} \alpha' dF(\alpha') \right] \ell^S} \times (\alpha - \alpha^X).$$

Figure 4 shows how wage schedule changes as a result of opening to trade. The wage schedule in trade equilibrium shows that there are more production workers who obtain the wage of 1 than before the opening to trade due to the reduction of the mass of firms in the differentiated good sector. Note in particular that workers whose ability α is between α_A^* and α_T^* used to work as knowledge workers but now work as production workers after opening to trade. In trade equilibrium, workers whose ability is between α_T^* and α^X work as knowledge workers in the firms that serve only their own domestic market. Their wages are lower in trade equilibrium than in autarky because their firms suffer from an increased competition in their domestic market. Workers whose ability is greater than α^X are the knowledge workers who work in firms that export their products. Profits for a firm that barely meets the criterion for exporting are smaller than those in autarky. This is because profits from exporting after paying the fixed costs of exporting are not sufficient to offset the losses in the domestic sales caused by the import penetration. Wages for knowledge workers who work in such firms decline as a result of opening to trade. However, the wage schedule beyond α^X is steeper than that in autarky between α_T^* and α^X due to the existence of the fixed cost of exporting paid by the numeraire good, as (11) shows. Consequently, the wages for knowledge workers whose ability is higher than a certain threshold increase as a result of opening trade. Trade increases nominal wages (measured by labor provided by the production workers) only for those who are highly talented.

Proposition 2 *International trade raises the nominal income (measured by labor provided by the production workers) only for those who are most talented.*

What is the impact of trade on real wages? It is easy to see that workers who work as production workers both before and after trade benefit from trade. Their real wages increase because the price index falls due to the increased varieties in consumption while the nominal wages are unaffected. What about the impact on the middle income class? To derive a clear-cut answer to this question, we assume now that the ability is distributed according to the Pareto distribution with its cumulative distribution function $G(a) = 1 - (a_0/a)^k$, where $a_0 > 0$ and $k > 1$. Then, it can be shown that under a mild restriction, international trade decreases the real wages for the middle income class as Figure 5 shows.

Proposition 3 *Suppose that there are two symmetric countries in which workers' ability distribution follows Pareto distribution. Then, the lowest income earners who work as production workers as well as the highest income earners who work as knowledge workers are better off by international trade. Those who belong to the middle income class, however, experience a decrease in real wages by opening to trade, if and only if $\sigma > 2$ holds. All knowledge workers who work in firms that only serve their individual domestic markets belong to such middle income class.*

We use two measures of social welfare to evaluate the effect of international trade below. If the ability is distributed according to the Pareto distribution, we also have an unambiguous result regarding the impact of trade on social welfare that is a simple aggregation of individuals' utilities (or real wages).

Proposition 4 *Suppose that there are two symmetric countries in which workers' ability distribution follows a Pareto distribution. Then, international trade unambiguously improves utilitarian social welfare for individual countries.*

The other measure of social welfare is the Lorenze domination, which is a measure to evaluate the equality of income distribution. We define the Lorenz function by

$$\mathcal{L}(a) = \frac{\int_0^a w(a')dG(a')}{\int_0^\infty w(a')dG(a')},$$

the fraction of total income earned by those who have the ability a or less. We say that the income distribution characterized by the Lorenz function $\mathcal{L}_{\mathcal{A}}$ is **Lorenz-dominated** by the one characterized by $\mathcal{L}_{\mathcal{B}}$, if $\mathcal{L}_A(a) \leq \mathcal{L}_B(a)$ for any a with strict inequality for some a.

Proposition 5 *Suppose that there are two symmetric countries in which workers' ability distribution follows a Pareto distribution. Then, the income distribution under international trade is Lorenz-dominated by the one under autarky.*

5 Asymmetric Countries

In this section, we extend the basic model to the one in which countries are asymmetric in terms of population. We will show that the impact of trade on income inequality is severer in a smaller country than a larger country.

Let us suppose that country 1 is larger than country 2, i.e., $L_1 > L_2$. We continue to assume that the ability distributions are the same between the two countries, so the firm distribution in country i is given by the density function similar to the one in (3):

$$f_i(\alpha) = \frac{L_i g(\alpha)}{l^s}. \tag{14}$$

We choose the labor provided by the production workers in country 2 as a numeraire, so the wage rate for the production workers in country 2 equals 1. We let w denote the wage rate for the production workers in country 1.

To derive equilibrium conditions, we first note that the total income for each country equals the total revenue for all the domestic firms. We can write the total income for country 1, for example, as

$$I_1 = \frac{w^{1-\sigma} \int_{\alpha_1^*}^{\infty} \alpha dF_1(\alpha)}{w^{1-\sigma} \int_{\alpha_1^*}^{\infty} \alpha dF_1(\alpha) + \tau^{1-\sigma} \int_{\alpha_2^X}^{\infty} \alpha dF_2(\alpha)} I_1 + \frac{w^{1-\sigma}\tau^{1-\sigma} \int_{\alpha_1^X}^{\infty} \alpha dF_1(\alpha)}{\int_{\alpha_2^*}^{\infty} \alpha dF_2(\alpha) + w^{1-\sigma}\tau^{1-\sigma} \int_{\alpha_1^X}^{\infty} \alpha dF_1(\alpha)} I_2.$$

Using (14), we can rewrite this equality as

$$I_1 = \frac{w^{1-\sigma} L_1 \int_{\alpha_1^*}^{\infty} \alpha dG(\alpha)}{w^{1-\sigma} L_1 \int_{\alpha_1^*}^{\infty} \alpha dG(\alpha) + \tau^{1-\sigma} L_2 \int_{\alpha_2^X}^{\infty} \alpha dG(\alpha)} I_1 + \frac{w^{1-\sigma}\tau^{1-\sigma} L_1 \int_{\alpha_1^X}^{\infty} \alpha dG(\alpha)}{L_2 \int_{\alpha_2^*}^{\infty} \alpha dG(\alpha) + w^{1-\sigma}\tau^{1-\sigma} L_1 \int_{\alpha_1^X}^{\infty} \alpha dG(\alpha)} I_2.$$

Similarly, we have

$$I_2 = \frac{L_2 \int_{\alpha_2^*}^{\infty} \alpha dG(\alpha)}{L_2 \int_{\alpha_2^*}^{\infty} \alpha dG(\alpha) + w^{1-\sigma}\tau^{1-\sigma} L_1 \int_{\alpha_1^X}^{\infty} \alpha dG(\alpha)} I_2 + \frac{\tau^{1-\sigma} L_2 \int_{\alpha_2^X}^{\infty} \alpha dG(\alpha)}{w^{1-\sigma} L_1 \int_{\alpha_1^*}^{\infty} \alpha dG(\alpha) + \tau^{1-\sigma} L_2 \int_{\alpha_2^X}^{\infty} \alpha dG(\alpha)} I_1,$$

for country 2. Although they look different, it is easy to show that these two equations are equivalent and can be written as

$$\frac{w^{1-\sigma}\tau^{1-\sigma} L_1 \int_{\alpha_1^X}^{\infty} \alpha dG(\alpha)}{L_2 \int_{\alpha_2^*}^{\infty} \alpha dG(\alpha) + w^{1-\sigma}\tau^{1-\sigma} L_1 \int_{\alpha_1^X}^{\infty} \alpha dG(\alpha)} I_2 = \frac{\tau^{1-\sigma} L_2 \int_{\alpha_2^X}^{\infty} \alpha dG(\alpha)}{w^{1-\sigma} L_1 \int_{\alpha_1^*}^{\infty} \alpha dG(\alpha) + \tau^{1-\sigma} L_2 \int_{\alpha_2^X}^{\infty} \alpha dG(\alpha)} I_1. \tag{15}$$

The left-hand side of this equation represents country 1's export volume while the right-hand side represents country 2's export volume. Thus, this equation shows that the trade should balance between the two countries, which is the first equilibrium condition of this extended model.

The second set of equilibrium conditions are the free-entry conditions for countries 1 and 2. Using (14), we can write the conditions as

$$\frac{w^{1-\sigma}\alpha_1^* I_1}{\sigma\left[w^{1-\sigma} L_1 \int_{\alpha_1^*}^{\infty} \alpha dG(\alpha) + \tau^{1-\sigma} L_2 \int_{\alpha_2^X}^{\infty} \alpha dG(\alpha)\right]} = w, \tag{16}$$

$$\frac{\alpha_2^* I_2}{\sigma\left[L_2 \int_{\alpha_2^*}^{\infty} \alpha dG(\alpha) + w^{1-\sigma}\tau^{1-\sigma} L_1 \int_{\alpha_1^X}^{\infty} \alpha dG(\alpha)\right]} = 1. \tag{17}$$

Whereas these conditions determine the entry cutoffs for the two countries, the next two conditions determine the cutoff qualities for exporting firms:

$$\frac{l^s w^{-1\sigma}\tau^{1-\sigma}\alpha_1^X I_2}{\sigma\left[L_2 \int_{\alpha_2^*}^{\infty} \alpha dG(\alpha) + w^{1-\sigma}\tau^{1-\sigma} L_1 \int_{\alpha_1^X}^{\infty} \alpha dG(\alpha)\right]} = w f_X, \tag{18}$$

$$\frac{l^s \tau^{1-\sigma}\alpha_2^X I_1}{\sigma\left[w^{1-\sigma} L_1 \int_{\alpha_1^*}^{\infty} \alpha dG(\alpha) + \tau^{1-\sigma} L_2 \int_{\alpha_2^X}^{\infty} \alpha dG(\alpha)\right]} = f_X, \tag{19}$$

for countries 1 and 2, respectively.

The last two conditions are the labor market clearing conditions:

$$\begin{aligned} L_1 \int_{\alpha_1^*}^{\infty} dG(\alpha) \quad &+ \quad \frac{f_X L_1}{l^s} \int_{\alpha_1^X}^{\infty} dG(\alpha) + \frac{w^{1-\sigma} L_1 \int_{\alpha_1^*}^{\infty} \alpha dG(\alpha)}{w^{1-\sigma} L_1 \int_{\alpha_1^*}^{\infty} \alpha dG(\alpha) + \tau^{1-\sigma} L_2 \int_{\alpha_2^X}^{\infty} \alpha dG(\alpha)} \frac{\sigma-1}{\sigma} I_1 \\ &+ \quad \frac{w^{1-\sigma}\tau^{1-\sigma} L_1 \int_{\alpha_1^X}^{\infty} \alpha dG(\alpha)}{L_2 \int_{\alpha_2^*}^{\infty} \alpha dG(\alpha) + w^{1-\sigma}\tau^{1-\sigma} L_1 \int_{\alpha_1^X}^{\infty} \alpha dG(\alpha)} \frac{\sigma-1}{\sigma} I_2 = L_1, \end{aligned} \tag{20}$$

$$L_2 \int_{\alpha_2^*}^{\infty} dG(\alpha) \quad + \quad \frac{f_X L_2}{l^s} \int_{\alpha_2^X}^{\infty} dG(\alpha) + \frac{L_2 \int_{\alpha_2^*}^{\infty} \alpha dG(\alpha)}{L_2 \int_{\alpha_2^*}^{\infty} \alpha dG(\alpha) + w^{1-\sigma}\tau^{1-\sigma} L_1 \int_{\alpha_1^X}^{\infty} \alpha dG(\alpha)} \frac{\sigma-1}{\sigma} I_2$$
$$+ \quad \frac{\tau^{1-\sigma} L_2 \int_{\alpha_2^X}^{\infty} \alpha dG(\alpha)}{w^{1-\sigma} L_1 \int_{\alpha_1^*}^{\infty} \alpha dG(\alpha) + \tau^{1-\sigma} L_2 \int_{\alpha_2^X}^{\infty} \alpha dG(\alpha)} \frac{\sigma-1}{\sigma} I_1 = L_2, \tag{21}$$

for countries 1 and 2, respectively.

We have 7 equations, (15)–(21), and 7 unknowns (α_1^*, α_1^X, α_2^*, α_2^X, w, I_1, I_2). Naturally, it is rather difficult to analytically solve this simultaneous equation system. Thus, we conduct some simulations to derive the effects of international trade on income inequality when the two countries are asymmetric. In the first simulation, we fix the trade cost, τ, and change the population size of country 2. Then we fix the population of the two individual countries and change the trade cost to see the impact of trade on income inequality within each country. In both simulations, we select $a_0 = 1$ and $k = 5$ as the Pareto-distribution parameter values for both countries. We also select $\sigma = 4$, $l^s = 1$, $f_X = 3$, and $L_1 = 100$.

The first simulation examines the effect of the difference in population between the two countries. We fix $\tau = 1.2$ and change L_2 from 0 to 100. We find that as L_2 decreases from $L_1 = 100$, α_1^* decreases while α_1^X increases in country 1, and α_2^* increases while α_2^X decreases in country 2. This is intuitive. As the size of country 2 decreases relative to country 1, the per-capita mass of exporting firms in country 1 decreases and that in country 2 increases. But this implies that country 2's market becomes more competitive relative to country 1's, so the per-capita mass of firms in country 1 increases while that in country 2 decreases. We also observe that the wage rate for production workers in country 1 relative to that in country 2 increases as the difference in country size expands. The average income in country 1, I_1/L_1, increases while that in country 2, I_2/L_2, remains the same as L_2 decreases. We confirm a typical result in the field of the new economic geography that the larger country has a higher per-capita income in the existence of trade costs.

In the next simulation, we fix $L_2 = 50$ and change τ from 1 to 2. We find that as trade costs decreases from $\tau = 2$ to $\tau = 1$, both α_1^* and α_2^* increase while α_1^X and α_2^X decrease as expected. In addition, both w and I_1/L_1 decrease while I_2/L_2 remains the same as τ decreases. Although the larger country still enjoys a higher per-capita income than the smaller country, the difference diminishes as the trade costs falls (when the trade costs are sufficiently small).

Figure 6 shows the (nominal) wage schedule of the two countries of asymmetric size when they trade goods with trade costs. As illustrated, country 2, the smaller country, has a smaller range of middle-income class. But the top income earners are richer in country 2. Trade benefits top income earners disproportionately especially in country 2 that exports goods to the larger country. Income inequality caused by trade is greater in the small country than the large country.

6 Conclusion

We have built a two-country trade model in which the average ability of knowledge workers hired in a firm determines the quality of the product that the firm produces, in order to examine the impact of international trade on income inequality across workers with different abilities. Knowledge workers are assortatively allocated across firms in equilibrium, which entails firm heterogeneity in terms of product quality. We find that international trade will benefit the firms that produce the highest qualities, while trade decreases the profits for those that barely export their products and those that serve only their individual domestic markets. Consequently, income inequality within knowledge workers, who earn higher income than production workers, expands. International trade increases the real wages for top income earners while decreases those for the middle-income class. The real wages for those in the low-income class who work as production workers increase thanks to the reduced price index as a result of increased varieties of products consumed.

In the basic model, we focus on a single dimensional ability (or talent). In reality, however, there is a variety of abilities–the mathematical ability and artistic ability for example–and workers' abilities in these different dimensions are not perfectly correlated. In the presence of multiple dimensional abilities, globalization can affect different workers differently. Trade liberalization raises the relative price for the export good. If a worker has a great talent that is valued highly in the comparative disadvantage sector, she is likely to be hired in the comparative disadvantage sector and receive relatively wages. But trade liberalization reduces her wages, while she would obtain much higher wages if she migrate to the foreign country that has a comparative advantage in that sector. Thus, globalization can change industrial structure significantly, and affect workers quite differently based on their ability profiles.

Appendix

This appendix collects the proofs of Propositions 3, 4 and 5.

Proposition 3. *Suppose that there are two symmetric countries in which workers' ability distribution follows a Pareto distribution. Then, the lowest income earners who work as production workers as well as the highest income earners who work as knowledge workers are better off by international trade. Those who belong to the middle income class, however, experience a decrease in real wages by opening to trade, if and only if $\sigma > 2$ holds. All knowledge workers who work in firms that only serve their individual domestic markets belong to such middle income class.*

Proof. Before trade, product quality index (8) is written as

$$\int_{\alpha_A^*}^{\infty} \alpha' dF(\alpha') = \frac{Lk\alpha_0^k}{\ell^S} \int_{\alpha_A^*}^{\infty} \alpha^{-k} d\alpha = \frac{Lk\alpha_0^k}{(k-1)\ell^S \left(\alpha_A^*\right)^{k-1}},$$

and after trade product quality index is written as

$$\begin{aligned} &\int_{\alpha_T^*}^{\infty} \alpha' dF(\alpha') + \tau^{1-\sigma} \int_{\frac{\tau^{\sigma-1} f^X}{\ell^S} \alpha_T^*}^{\infty} \alpha' dF(\alpha') \\ = \; & \frac{Lk\alpha_0^k}{(k-1)\ell^S \left(\alpha_T^*\right)^{k-1}} \left[1 + \tau^{k(1-\sigma)} \left(\frac{f^X}{\ell^S} \right)^{1-k} \right]. \end{aligned}$$

Now, we will calculate α_A^* and α_T^* by utilizing FE and LM equations, (6) and (7). First, we start with autarkic equilibrium. Substituting autarkic index into condition FE, we obtain

$$\begin{aligned}
I_A &= \frac{\sigma \ell^S}{\alpha_A^*} \int_{\alpha_A^*}^{\infty} \alpha' dF(\alpha') \\
&= \frac{\sigma \ell^S}{\alpha_A^*} \times \frac{Lk\alpha_0^k}{(k-1)\ell^S \left(\alpha_A^*\right)^{k-1}} \\
&= \frac{\sigma L k}{(k-1)} \left(\frac{\alpha_0}{\alpha_A^*}\right)^k .
\end{aligned}$$

With Pareto distribution, condition LM is written as:

$$\begin{aligned}
L &= \ell^S \int_{\alpha_A^*}^{\infty} dF(\alpha') + \left(\frac{\sigma - 1}{\sigma}\right) I_A \\
&= \ell^S \times \frac{L}{\ell^S} \left(\frac{\alpha_0}{\alpha_A^*}\right)^k + \left(\frac{\sigma - 1}{\sigma}\right) I_A \\
&= \ell^S \times \frac{L}{\ell^S} \left(\frac{\alpha_0}{\alpha_A^*}\right)^k + \left(\frac{\sigma - 1}{\sigma}\right) \frac{\sigma L k}{k-1} \left(\frac{\alpha_0}{\alpha_A^*}\right)^k \\
&= L \left(\frac{\alpha_0}{\alpha_A^*}\right)^k \left[1 + \left(\frac{\sigma - 1}{\sigma}\right) \frac{\sigma k}{k-1}\right] \\
&= L \left(\frac{\alpha_0}{\alpha_A^*}\right)^k \left[1 + (\sigma - 1) \frac{k}{k-1}\right] \\
&= L \left(\frac{\alpha_0}{\alpha_A^*}\right)^k \left(\frac{\sigma k - 1}{k-1}\right).
\end{aligned}$$

Thus, we have

$$\alpha_A^* = \alpha_0 \left(\frac{\sigma k - 1}{k - 1}\right)^{\frac{1}{k}} .$$

This implies that $\alpha_A^* > \alpha_0$ holds, since the contents of the parenthesis is positive by noting $\sigma > 1$.

Let us turn to trade equilibrium. Substituting the after trade index into condition FE

(12), we obtain

$$
\begin{aligned}
I_T &= \frac{\sigma \ell^S}{\alpha_T^*}\left[\int_{\alpha_T^*}^{\infty} \alpha' dF(\alpha') + \tau^{1-\sigma}\int_{\alpha^X}^{\infty} \alpha' dF(\alpha')\right] \\
&= \frac{\sigma \ell^S}{\alpha_T^*} \times \frac{Lk\alpha_0^k}{(k-1)\ell^S \left(\alpha_T^*\right)^{k-1}}\left[1+\tau^{k(1-\sigma)}\left(\frac{f^X}{\ell^S}\right)^{1-k}\right] \\
&= \frac{\sigma Lk}{(k-1)}\left(\frac{\alpha_0}{\alpha_T^*}\right)^k \left[1+\tau^{k(1-\sigma)}\left(\frac{f^X}{\ell^S}\right)^{1-k}\right] \\
&= \frac{\sigma Lk}{(k-1)}\left(\frac{\alpha_0}{\alpha_T^*}\right)^k \left[1+\tau^{k(1-\sigma)}\left(\frac{\ell^S}{f^X}\right)^{k-1}\right].
\end{aligned}
$$

With Pareto distribution, condition LM (13) is written as:

$$
\begin{aligned}
L &= \ell^S \int_{\alpha_T^*}^{\infty} dF(\alpha') + f^X \int_{\frac{\tau^{\sigma-1} f^X}{\ell^S}\alpha_T^*}^{\infty} dF(\alpha') + \left(\frac{\sigma-1}{\sigma}\right) I_T \\
&= \ell^S \times \frac{L}{\ell^S}\left(\frac{\alpha_0}{\alpha_T^*}\right)^k + f^X \times \frac{L}{\ell^S}\left(\frac{\alpha_0}{\alpha_T^*}\right)^k \left(\frac{\ell^S}{\tau^{\sigma-1} f^X}\right)^k + \left(\frac{\sigma-1}{\sigma}\right) I_T \\
&= \ell^S \times \frac{L}{\ell^S}\left(\frac{\alpha_0}{\alpha_T^*}\right)^k + L\left(\frac{\alpha_0}{\alpha_T^*}\right)^k \tau^{k(1-\sigma)}\left(\frac{\ell^S}{f^X}\right)^{k-1} \\
&\quad + \left(\frac{\sigma-1}{\sigma}\right)\frac{\sigma Lk}{(k-1)}\left(\frac{\alpha_0}{\alpha_T^*}\right)^k \left[1+\tau^{k(1-\sigma)}\left(\frac{\ell^S}{f^X}\right)^{k-1}\right] \\
&= L\left(\frac{\alpha_0}{\alpha_T^*}\right)^k \left(1+\tau^{k(1-\sigma)}\left(\frac{\ell^S}{f^X}\right)^{k-1}\right)\left[1+\left(\frac{\sigma-1}{\sigma}\right)\frac{\sigma k}{(k-1)}\right] \\
&= L\left(\frac{\alpha_0}{\alpha_T^*}\right)^k \left(1+\tau^{k(1-\sigma)}\left(\frac{\ell^S}{f^X}\right)^{k-1}\right)\left(\frac{\sigma k-1}{k-1}\right).
\end{aligned}
$$

Thus, we have

$$
\alpha_T^* = \alpha_0 \left(1+\tau^{k(1-\sigma)}\left(\frac{\ell^S}{f^X}\right)^{k-1}\right)^{\frac{1}{k}} \left(\frac{\sigma k-1}{k-1}\right)^{\frac{1}{k}}.
$$

The above calculations show:

$$
\begin{aligned}
\int_{\alpha_A^*}^{\infty} \alpha' dF(\alpha') &= \frac{Lk\alpha_0^k}{(k-1)\ell^S \left(\alpha_A^*\right)^{k-1}} \\
&= \frac{Lk\alpha_0^k}{(k-1)\ell^S \alpha_0^{k-1}\left(\frac{\sigma k-1}{k-1}\right)^{\frac{k-1}{k}}}
\end{aligned}
$$

and

$$\begin{aligned}
&\int_{\alpha_T^*}^{\infty} \alpha' dF(\alpha') + \tau^{1-\sigma} \int_{\frac{\tau^{\sigma-1} f^X}{\ell^S}\alpha_T^*}^{\infty} \alpha' dF(\alpha') \\
= \;& \frac{Lk\alpha_0^k}{(k-1)\ell^S \left(\alpha_T^*\right)^{k-1}} \left[1 + \tau^{k(1-\sigma)} \left(\frac{\ell^S}{f^X}\right)^{k-1}\right] \\
= \;& \frac{Lk\alpha_0^k}{(k-1)\ell^S \alpha_0^{k-1} \left(1 + \tau^{k(1-\sigma)} \left(\frac{\ell^S}{f^X}\right)^{k-1}\right)^{\frac{k-1}{k}} \left(\frac{\sigma k-1}{k-1}\right)^{\frac{k-1}{k}}} \left[1 + \tau^{k(1-\sigma)} \left(\frac{\ell^S}{f^X}\right)^{k-1}\right] \\
= \;& \frac{Lk\alpha_0^k}{(k-1)\ell^S \alpha_0^{k-1} \left(\frac{\sigma k-1}{k-1}\right)^{\frac{k-1}{k}}} \left[1 + \tau^{k(1-\sigma)} \left(\frac{\ell^S}{f^X}\right)^{k-1}\right]^{\frac{1}{k}} \\
= \;& \int_{\alpha_T^*}^{\infty} \alpha' dF(\alpha') \times \left[1 + \tau^{k(1-\sigma)} \left(\frac{\ell^S}{f^X}\right)^{k-1}\right]^{\frac{1}{k}}.
\end{aligned}$$

By these explicit formulae, we can first conclude

$$\int_{\alpha_T^*}^{\infty} \alpha' dF(\alpha') + \tau^{1-\sigma} \int_{\frac{\tau^{\sigma-1} f^X}{\ell^S}\alpha_T^*}^{\infty} \alpha' dF(\alpha') > \int_{\alpha_A^*}^{\infty} \alpha' dF(\alpha').$$

This implies that international trade increases the real income of consumers whose gross wage rates stay the same. Since unskilled workers' wages stay the same, the poorest workers are better off by international trade.

Finally we can check whether or not the condition for the existence of suffering middle class by trade holds. This condition

$$\frac{\alpha_T^*}{\alpha_A^*} > \left[\frac{\int_{\alpha_T^*}^{\infty} \alpha' dF(\alpha') + \tau^{1-\sigma} \int_{\frac{\tau^{\sigma-1} f^X}{\ell^S}\alpha_T^*}^{\infty} \alpha' dF(\alpha')}{\int_{\alpha_A^*}^{\infty} \alpha' dF(\alpha')}\right]^{\frac{1}{\sigma-1}}$$

is satisfied if and only if we have

$$\left(1 + \tau^{k(1-\sigma)} \left(\frac{\ell^S}{f^X}\right)^{k-1}\right)^{\frac{1}{k}} > \left[\left(1 + \tau^{k(1-\sigma)} \left(\frac{\ell^S}{f^X}\right)^{k-1}\right)^{\frac{1}{k}}\right]^{\frac{1}{\sigma-1}}.$$

The last inequality is satisfied if and only if $\frac{1}{\sigma-1} < 1$, or $\sigma > 2$.□

Proposition 4. *Suppose that there are two symmetric countries in which workers' ability distribution follows a Pareto distribution. Then, international trade unambiguously improves utilitarian social welfare for individual countries.*

Proof. Since our utility function is homogeneous of degree one, aggregate income adjusted by quality index describes the country's utilitarian social welfare:

$$SW = \frac{I}{\left(\frac{\sigma}{\sigma-1}\right)\left[\int \alpha' dF(\alpha')\right]^{\frac{1}{1-\sigma}}} \propto \frac{I}{\left[\int \alpha' dF(\alpha')\right]^{\frac{1}{1-\sigma}}}.$$

Thus, trade improves utilitarian social welfare if and only if

$$\frac{I_A}{\left[\int_{\alpha_A^*}^{\infty} \alpha' dF(\alpha')\right]^{\frac{1}{\sigma-1}}} < \frac{I_T}{\left[\int_{\alpha_T^*}^{\infty} \alpha' dF(\alpha') + \tau^{1-\sigma}\int_{\frac{\tau^{\sigma-1}f^X}{\ell^S}\alpha_T^*}^{\infty} \alpha' dF(\alpha')\right]^{\frac{1}{\sigma-1}}},$$

or

$$\frac{I_T}{I_A} > \left[\frac{\int_{\alpha_T^*}^{\infty} \alpha' dF(\alpha') + \tau^{1-\sigma}\int_{\frac{\tau^{\sigma-1}f^X}{\ell^S}\alpha_T^*}^{\infty} \alpha' dF(\alpha')}{\int_{\alpha_A^*}^{\infty} \alpha' dF(\alpha')}\right]^{\frac{1}{\sigma-1}}$$

We know

$$\begin{aligned}
\frac{I_T}{I_A} &= \frac{\frac{\sigma Lk}{k-1}\left(\frac{\alpha_0}{\alpha_T^*}\right)^k\left[1+\tau^{k(1-\sigma)}\left(\frac{\ell^S}{f^X}\right)^{k-1}\right]}{\frac{\sigma Lk}{k-1}\left(\frac{\alpha_0}{\alpha_A^*}\right)^k} \\
&= \left(\frac{\alpha_A^*}{\alpha_T^*}\right)^k\left[1+\tau^{k(1-\sigma)}\left(\frac{\ell^S}{f^X}\right)^{k-1}\right] \\
&= \left(1+\tau^{k(1-\sigma)}\left(\frac{\ell^S}{f^X}\right)^{k-1}\right)^{-1}\left[1+\tau^{k(1-\sigma)}\left(\frac{\ell^S}{f^X}\right)^{k-1}\right] = 1
\end{aligned}$$

and

$$\left[\frac{\int_{\alpha_T^*}^{\infty} \alpha' dF(\alpha') + \tau^{1-\sigma}\int_{\frac{\tau^{\sigma-1}f^X}{\ell^S}\alpha_T^*}^{\infty} \alpha' dF(\alpha')}{\int_{\alpha_A^*}^{\infty} \alpha' dF(\alpha')}\right]^{\frac{1}{\sigma-1}} = \left[1+\tau^{k(1-\sigma)}\left(\frac{\ell^S}{f^X}\right)^{k-1}\right]^{\frac{1}{k}\times\frac{1}{\sigma-1}}.$$

Since $\frac{1}{k}\times\frac{1}{\sigma-1} > 0$, the above inequality is always satisfied.□

Proposition 5. Suppose that there are two symmetric countries in which workers' ability distribution follows a Pareto distribution. Then, the income distribution under international trade is Lorenz-dominated by the one under autarky.

Proof. In autarkic equilibrium, the wage schedule satisfies

1. $w_A(\alpha) = 1$ for all $\alpha \in [0, \alpha_A^*]$

2. $w_A(\alpha) = \frac{\alpha}{\alpha_A^*}$ for all $\alpha \in [\alpha_A^*, \infty)$.

In contrast, the after-trade wage schedule satisfies

1. $w_T(\alpha) = 1$ for $\alpha \in [0, \alpha_T^*]$,

2. $w_T(\alpha) = \frac{\alpha}{\alpha_T^*}$ for $\alpha \in [\alpha_T^*, \alpha^X]$,

3. $w_T(\alpha) = \frac{\alpha}{\alpha_T^*} + \frac{\tau^{1-\sigma} I_T}{\sigma\left[\int_{\alpha_T^*}^{\infty} \alpha' dF(\alpha') + \tau^{1-\sigma} \int_{\alpha^X}^{\infty} \alpha' dF(\alpha')\right] \ell^S} \times (\alpha - \alpha^X)$ for $\alpha \in [\alpha^X, \infty)$.

From Proposition 1 we have $\alpha_A^* < \alpha_T^* < \alpha^X$, there must be $\bar{\alpha} > \alpha^X$ such that (i) $w_T(\bar{\alpha}) = w_A(\bar{\alpha})$, (ii) $w_T(\alpha) \leq w_A(\alpha)$ for all $\alpha \leq \bar{\alpha}$, and (iii) $w_T(\alpha) > w_A(\alpha)$ for all $\alpha > \bar{\alpha}$. This implies that the Lorenz function $\mathcal{L}_A(a) = \mathcal{L}_T(a)$ for all $a \leq \alpha_A^*$, and $\mathcal{L}_A(a) > \mathcal{L}_T(a)$ for all $a > \alpha_A^*$, since $\int_0^\infty w_A(a)dG(a) = \int_0^\infty w_T(a)dG(a)$ due to $I_T = I_A$ from the proof of Proposition 4. Thus, the income distribution under international trade is Lorenz-dominated by the one under autarky.□

References

[1] Acemoglu, Daron and David Autor (2011), "Skills, Tasks and Technologies: Implications for Employment and Earnings," *Handbook of Labor Economics* 4(B), 1043-1171, Elsevier.

[2] Azevedo, E., and J. Leshno, 2011, A Supply and Demand Framework for Two-Sided Matching Markets, Working Paper, Harvard University.

[3] Blanchard, Emily and Gerald Willmann (2011), "Trade, Education, and the Shrinking Middle Class," unpublished manuscript, Dartmouth College.

[4] Burnstein, A.T., and A. Monge-Naranjo, 2009, Foreign Know-How, Firm Control, and the Income of Developing Countries, Quarterly Journal of Economics 124, 149-195.

[5] Caliendo, L., and E. Rossi-Hansberg, 2011, The Impact of Trade on Organization and Productivity, forthcoming in Quarterly Journal of Economics.

[6] Costinot, Arnaud and Jonathan Vogel (2010), “Matching and Inequality in the World Economy,” *Journal of Political Economy*, 118, 747-786.

[7] The Economist, January 14th-20th 2012.

[8] Helpman, Elehana, Oleg Itskhoki, and Stephen Redding (2010), “Inequality and Unemployment in a Global Economy,” *Econometrica*, 78, 1239-1283.

[9] Helpman, Elehana, Oleg Itskhoki, Marc-Andreas Muendler, and Stephen Redding (2011), “Trade and Inequality: From Theory to Estimation,” unpublished manuscript, Harvard University.

[10] Helpman, Elehana, Oleg Itskhoki, and Stephen Redding (forthcoming), “Unequal Effects of Trade on Workers with Different Abilities,” *Journal of European Economic Association, Papers and Proceedings.*

[11] Krussell, P., L. Ohanian, J.-V. Rios-Rull, and G. Violante, 2000, Capital-Skill Complementarity and Inequality, Econometrica 68, 1026-1053.

[12] Lucas, R.E. Jr., 1978, On the Size Distribution of Business Firms, Bell Journal of Economics 9, 508-523.

[13] Mas-Colell, A., 1984, ”A Theorem of Schmeidler”, Journal of Mathematical Economics 13, 201-206.

[14] Manasse, Paolo and Alessandro Turrini (2001), “Trade, Wages, and ‘Superstars’,” *Journal of International Economics*, 54, 97-117.

[15] Melitz, M., 2003, “The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity”, *Econometrica*, 71, 1695-1725.

[16] Yeaple, Stephen Ross (2005), “A Simple Model of Firm Heterogeneity, International Trade, and Wages”, *Journal of International Economics*, 65, 1-20.

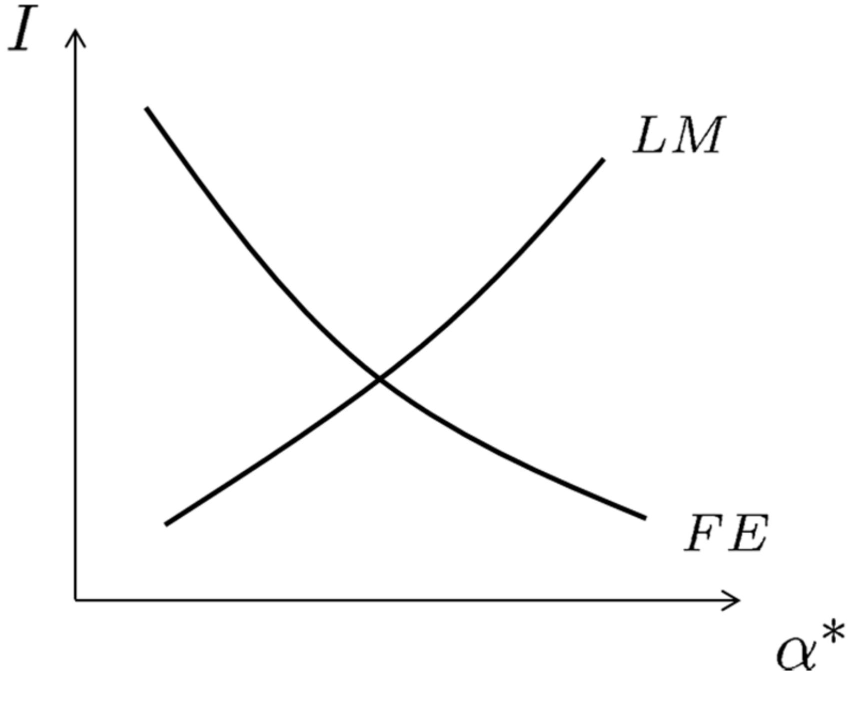

Figure 1. Autarkic equilibrium

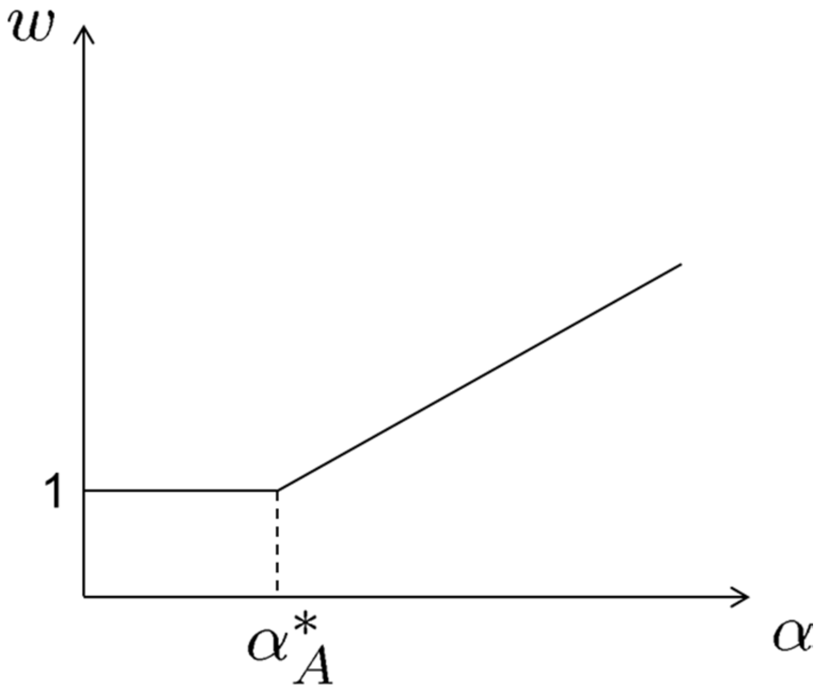

Figure 2. Autarkic equilibrium wage schedule

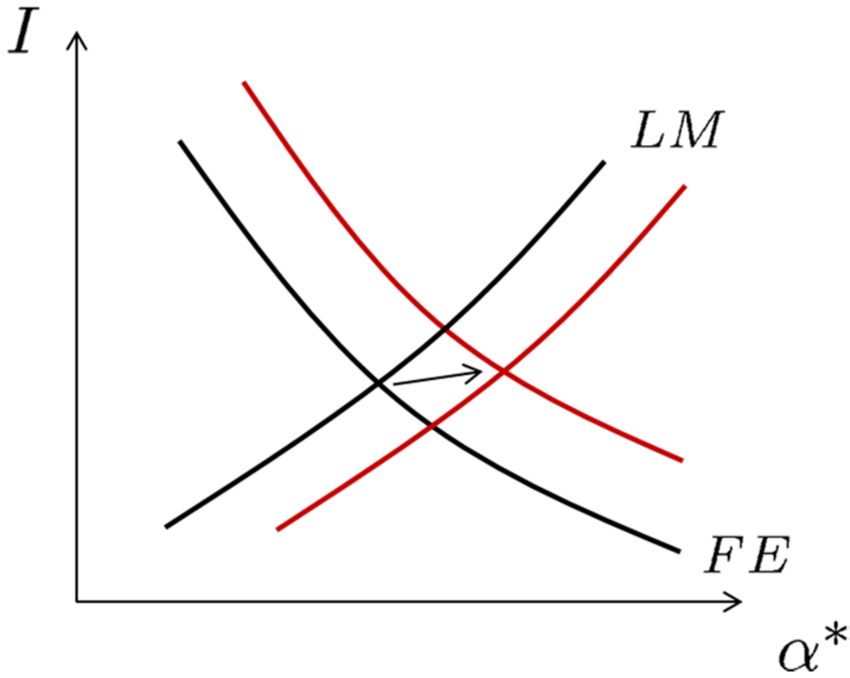

Figure 3. Trade equilibrium

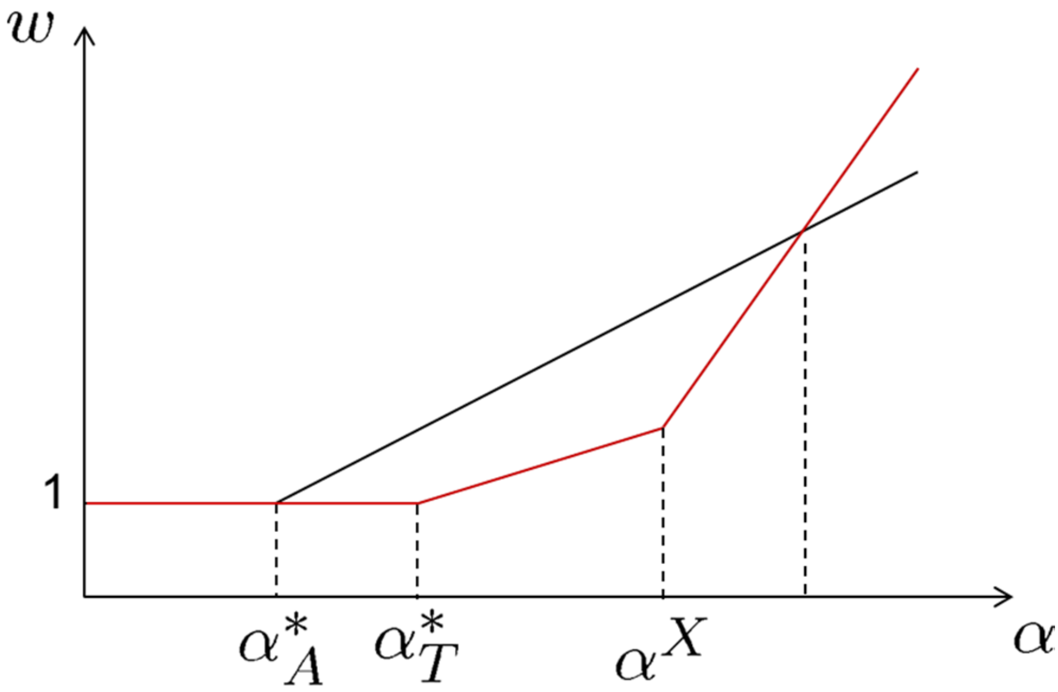

Figure 4. Impact of trade on nominal wages

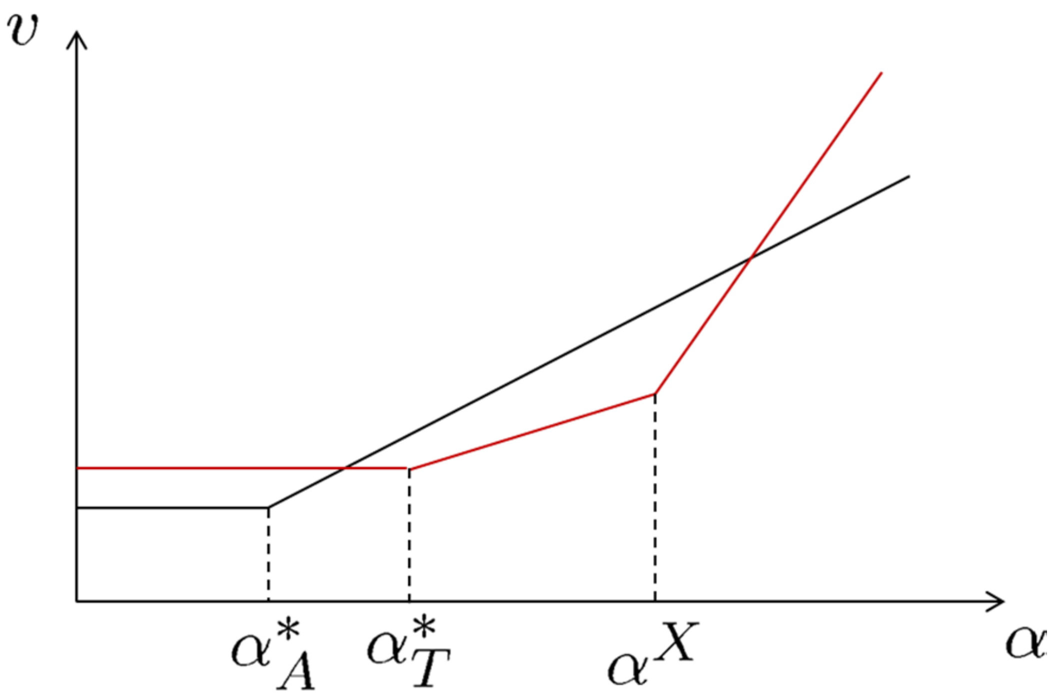

Figure 5. Impact of trade on real wages

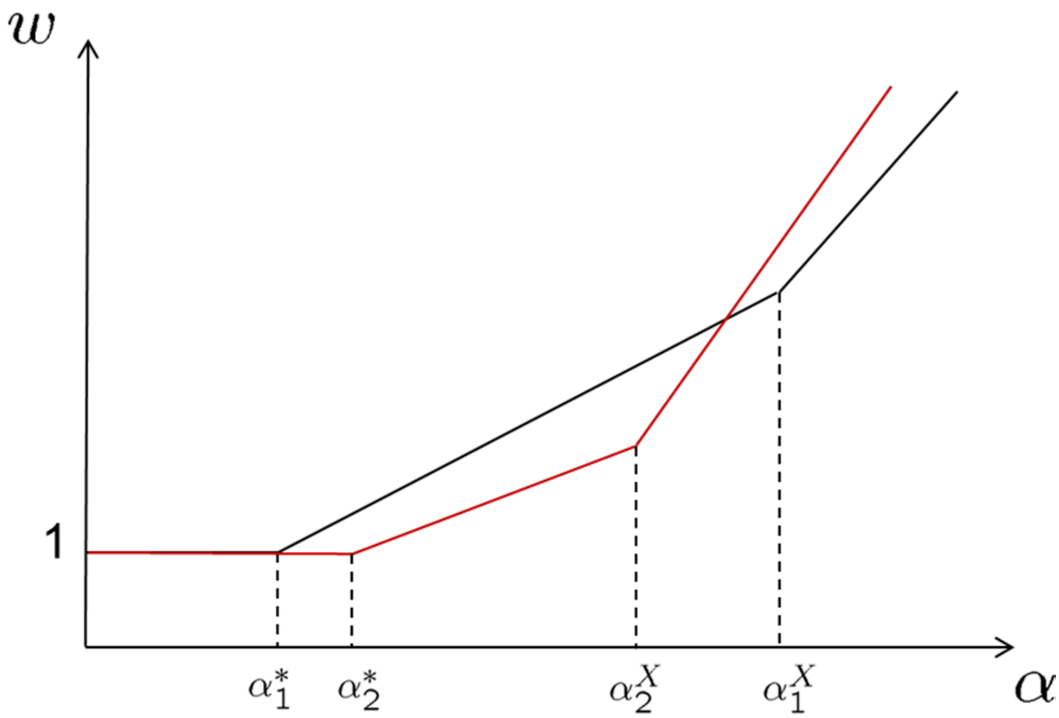

Figure 6. Impact of trade on nominal wages when countries are difference in size