The Impact of Trade Liberalization
on Industrial Productivity

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First version: June 22, 2012

Current version: November 19, 2012

Abstract: An empirical finding by Trefler (2004, AER) and others that industrial productivity increases more strongly in liberalized industries than in non-liberalized industries has been widely accepted as evidence for the Melitz (2003, Econometrica) model. This paper shows that a multi-industry version of the Melitz model does not predict this relationship. Instead, it predicts the opposite relationship that industrial productivity increases more strongly in non-liberalized industries than in liberalized industries.

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1 Introduction

A central question in the study of international trade is how trade liberalization improves resource allocation in the liberalizing country. While traditional studies have emphasized reallocation across industries, recent studies have discovered that reallocation occurs even within industries. In the last decade, the empirical literature has established that trade liberalization improves productivity by shifting resources from less productive to more productive firms within industries.

By comparing industries that experienced different degrees of trade liberalization (e.g. tariff cuts), several studies found that intra-industry reallocation improves industrial productivity more strongly in liberalized industries than in non-liberalized industries.[1] For instance, by investigating the long run impact of the Canada-USA free trade agreement on Canadian manufacturing industries, Trefler (2004) found that industrial productivity increased more strongly in liberalized industries than in non-liberalized industries, and that the rise in industrial productivity was mainly due to the shift of resources from less productive to more productive firms. Other studies also found that the exit of low productive firms from an industry, which contributes to a rise in industrial productivity, is positively associated with the decrease in tariffs and other import costs in the industry (Bernard, Jensen, and Schott, 2006, for the US; Lilieva, 2008, for Canada; Nataraj, 2011, for India; and Eslava, Haltiwanger, Kugler, and Kugler, 2009, for Colombia).[2]

The seminal model by Melitz (2003) has been accepted as the central model of intra-industry reallocation due to trade liberalization. By combining the Hopenhayn (1992) model of the entry and exit of heterogeneous firms and the Krugman (1979, 1980) model with fixed trade costs, Melitz (2003) theoretically demonstrated that trade liberalization improves the aggregate productivity of economies through resource reallocation toward more productive firms.

It is widely believed that the Melitz model explains the fact that industrial productivity increases more strongly in liberalized industries than in non-liberalized industries. Virtually all survey papers recently written by leading scholars cite Trefler (2004) and Bernard, Jensen, and Schott (2006) as

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[1]Early studies use firm size as a proxy for firm productivity (Head and Ries, 1999; Baggs, 2005) and/or trade volume as a proxy for trade policy meaures (Pavcnik, 2002). Recent studies use more direct productivity measures and trade policy measures. Tybout (2003) surveys early studies in this literature.

[2]Other studies found positive but statistically insignificant associations between productivity improvement due to intra-industry reallocation and the extent of liberalization (Tybout and Westbrook, 1995, for Mexico; Fernandes, 2007, for Colombia; Harrison, Martin, and Nataraj, 2011 and Sivadasan, 2009, for India).

evidence in support of the Melitz model (Bernard, Jensen, Redding, and Schott, 2007, 2012; Helpman, 2011; Redding, 2011; Melitz and Trefler, 2012). For instance, Helpman (2011, p.107) writes in his recent book: "In other words, the Canadian experience conforms to the theoretical analysis." With this empirical support, the reallocation mechanism of Melitz (2003) has been applied to various issues and is central to the theoretical trade research of the last decade.[3]

The purpose of our paper is to revisit the correspondence between empirical studies and the Melitz model. The type of trade liberalization that typical empirical studies investigated is different from the type of trade liberalization that Melitz (2003) theoretically analyzed. The liberalization in the one industry Melitz model is multilateral and uniform liberalization, in which all countries reduce tariffs on all goods by the same amount. On the other hand, the above mentioned empirical studies investigate unilateral and non-uniform liberalization, in which one country reduces tariffs and the extent of tariff reductions vary across industries.[4]

The gap between theory and evidence has probably been overlooked because the analysis of unilateral and non-uniform liberalization in a multi-industry Melitz model appears to be complicated and intractable. To overcome this theoretical difficulty, we develop a brand new way of solving the Melitz model using simple and intuitive diagrams. We show that these new techniques can be used to solve a multi-industry version of the Melitz model and study what happens when one country reduces tariffs in some industries but not others.

Our surprising result is that the multi-industry Melitz model does not predict that industrial productivity rises more strongly in liberalized industries than in non-liberalized industries. Instead, it predicts the opposite relationship that industrial productivity rises more strongly in non-liberalized industries than in liberalized industries. This result forces us to re-think the match between theory and evidence: an empirical fact that has been widely cited as evidence for the Melitz model is actually evidence against the Melitz model. Furthermore, when the size of a liberalized industry is small compared to the size of the liberalizing country, we find a more striking prediction: industrial productivity decreases in the liberalized industry, while it increases in the non-liberalized industries. This result calls for re-

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[3]Applications of the Melitz (2003) mechanism of intra-industry reallocation include too many papers to cite in this limited space. We encourage readers to consult with the above-mentioned survey papers for applications.

[4]Trefler (2004) and Lileeva (2008) controlled for the US tariff changes when they estimate the impact of the Canadian tariff cuts introduced by the Canada-USA free trade agreement (CUFTA). Therefore, although the CUFTA is a bilateral trade agreement, the effects of Canadian tariff cuts estimated by these two papers should be interpreted as the effects of unilateral liberalization.

thinking the model's implications for industrial promotion policy. If the government of a country is interested in raising the productivity of a small "target" industry through a resource reallocation from less productive to more productive firms, the theoretically correct advice based on the Melitz model is to protect the target industry, not trade liberalization. This is obviously the opposite of what Trefler (2004) and other empirical studies suggest.

As illustrated in Table 1, no previous paper has analyzed unilateral and non-uniform trade liberalization in a Melitz model with multiple Melitz industries. This is required to compare the model with the empirical facts from cross-industry regressions. Melitz and Ottaviano (2008), Demidova (2008), Demidova and Rodriguez-Clare (2009, 2011), and Felbermayr, Jung, and Larch (2012) analyze unilateral trade liberalization in models with just one Melitz industry. Bernard, Redding, and Schott (2007) and Okubo (2009) develop models with multiple Melitz industries but only analyze multilateral trade liberalization.

	One Melitz Industry	Multiple Melitz Industries
Multilateral Liberalization	Melitz (2003)	Bernard, Redding & Schott(07) Okubo (09)
Unilateral Liberalization	Melitz & Ottaviano(08) Demidova(08) Demidova & Rodriguez-Clare (09, 11) Felbermayr, Jung & Larch (12)	Our paper

Table 1: Related Literature

The rest of the paper is organized as follows. In section 2, we present a multi-industry version of the Melitz (2003) model. In section 3, we solve the model analytically for the effects of unilateral and non-uniform trade liberalization. We also present an intuitive explanation for our results in subsection 3.2 and solve the model numerically to illustrate the intuition. In section 4, we offer some concluding comments and there is an Appendix where calculations that we did to solve the model are presented in more detail.

2   The Model

This section presents a multi-industry version of the Melitz (2003) model. Our model differs from the original model in five points: (1) our model has two industries and two countries; (2) industries and

countries are asymmetric so that wages differ between countries; (3) trade costs are asymmetric and depend on the direction of trade; (4) the utility function of consumers has two tiers, the Cobb-Douglas upper tier and the CES lower tier; and (5) firms draw their productivities from Pareto distributions. The last two specifications are commonly used in applications of the Melitz model.

2.1 Setting

Consider two countries, 1 and 2, with two differentiated goods sectors (or industries), A and B. Throughout the paper, subscripts i and j denote countries ($i, j \in \{1, 2\}$) and subscript s denotes sectors ($s \in \{A, B\}$). Though the model has infinitely many periods, there is no means for saving over periods. We focus on a stationary steady state equilibrium where aggregate variables do not change over time and omit notation for time periods.

The representative consumers in both countries have an identical two-tier (Cobb-Douglas plus CES) utility function:

$$U \equiv C_A^{\alpha_A} C_B^{\alpha_B} \qquad \text{where} \qquad C_s \equiv \left[\int_{\omega \in \Omega_s} q_s(\omega)^{\rho_s} \, d\omega \right]^{1/\rho_s}. \tag{1}$$

In equation (1), $q_s(\omega)$ is the consumer's quantity consumed of a product variety ω produced in sector s, Ω_s is the set of available varieties in sector s and ρ_s measures the degree of product differentiation in sector s. We assume that products within a sector are closer substitutes than products across sectors, which implies that a within-sector elasticity of substitution $\sigma_s \equiv 1/(1 - \rho_s)$ satisfies $\sigma_s > 1$. Given that $\alpha_A + \alpha_B = 1$, α_s represents the share of consumer expenditure on sector s products.

Country i is endowed with L_i unit of labor as the only factor of production. Labor is inelastically supplied and workers in country i earn the competitive wage rate w_i. We measure all prices relative to the price of labor in country 2 by setting $w_2 = 1$.

Firms are risk neutral and maximize expected profits. In each time period, let M_{ise} denote the measure of firms that choose to enter in country i and sector s. Each firm uses F_{is} units of labor to enter and incurs the fixed entry cost $w_i F_{is}$. Each firm then independently draws its productivity φ from a Pareto distribution. The cumulative distribution function $G_{is}(\varphi)$ and the density function $g_{is}(\varphi)$ are

given by

$$G_{is}(\varphi) = 1 - \left(\frac{b_{is}}{\varphi}\right)^{\theta_s} \quad \text{and} \quad g_{is}(\varphi) = \frac{\theta_s b_{is}^{\theta_s}}{\varphi^{\theta_s+1}} \quad \text{for} \quad \varphi \in [b_{is}, \infty), \tag{2}$$

where θ_s and b_{is} are the shape and scale parameters of the distribution for country i and sector s. We assume that $\theta_s > \sigma_s - 1$ to guarantee that expected profits are finite. A firm with productivity φ uses $1/\varphi$ units of labor to produce one unit of output and has constant marginal cost w_i/φ in country i. This firm must use f_{ijs} units of labor and incur the fixed "marketing" cost $w_i f_{ijs}$ to sell in country j. There are also iceberg trade costs associated with shipping products across countries: a firm that exports from country i to country $j \neq i$ in sector s needs to ship $\tau_{ijs} > 1$ units of a product in order for one unit to arrive at the foreign destination (if $j = i$, then $\tau_{iis} = 1$).

Because of the fixed marketing costs, there exist productivity cut-off levels φ_{ijs}^* such that only firms with $\varphi \geq \varphi_{ijs}^*$ sell products from country i to country j in sector s. In each country and sector, we assume that exporting requires higher fixed costs than local selling ($f_{ijs} > f_{iis}$). We solve the model for an equilibrium where both countries produces both goods A and B, and the more productive firms export ($\varphi_{iis}^* < \varphi_{ijs}^*$). In each period, there is an exogenous probability δ_{is} with which actively operating firms in country i and sector s die and exit. In a stationary steady state equilibrium, the mass of actively operating firms M_{is} and the mass of entrants M_{ise} in country i and sector s satisfy

$$[1 - G_{is}(\varphi_{iis}^*)] M_{ise} = \delta_{is} M_{is}, \tag{3}$$

that is, firm entry in each time period is matched by firm exit.

Let $p_{ijs}(\varphi)$ denote the price charged in country j by a firm with productivity φ from country i in sector s. Let $q_{ijs}(\varphi)$ denote the quantity that consumers in country j buy from this firm and let $r_{ijs}(\varphi) \equiv p_{ijs}(\varphi) q_{ijs}(\varphi)$ denote the corresponding firm revenue. Also, let P_{js} denote the index of consumer prices in country j and sector s. Since free entry implies that aggregate profit income is zero, in each time period, consumers in country j spend exactly what they earn in wage income $w_j L_j$. Consumer optimization calculations imply that consumer demand and the corresponding firm revenue are

$$q_{ijs}(\varphi) = \frac{p_{ijs}(\varphi)^{-\sigma_s} \alpha_s w_j L_j}{P_{js}^{1-\sigma_s}} \quad \text{and} \quad r_{ijs}(\varphi) = \frac{p_{ijs}(\varphi)^{1-\sigma_s} \alpha_s w_j L_j}{P_{js}^{1-\sigma_s}}. \tag{4}$$

A firm with productivity φ from country i earns variable profit $\pi_{ijs}(\varphi) = r_{ijs}(\varphi) - \frac{w_i \tau_{ijs}}{\varphi} q_{ijs}(\varphi)$ from

selling to country j in sector s. Solving for the profit-maximizing price, we obtain that

$$p_{ijs}(\varphi) = \frac{w_i \tau_{ijs}}{\rho_s \varphi}, \tag{5}$$

that is, each firm charges a fixed markup over its marginal cost $w_i \tau_{ijs}/\varphi$. Substituting this price back into the variable profit function yields $\pi_{ijs}(\varphi) = r_{ijs}(\varphi)/\sigma_s$.

2.2 Sector Equilibrium

We first derive equilibrium conditions for each sector, following the steps in Melitz (2003) and other previous studies. Since a firm with cut-off productivity φ^*_{ijs} just breaks even from selling to country j, it follows that φ^*_{ijs} is determined by the cut-off productivity condition

$$\frac{r_{ijs}\left(\varphi^*_{ijs}\right)}{\sigma_s} = w_i f_{ijs}. \tag{6}$$

A firm from country i needs to have productivity $\varphi \geq \varphi^*_{ijs}$ to justify paying the fixed marketing cost $w_i f_{ijs}$ of serving the country j market in sector s.

From (4), (5) and (6), the cut-off productivity levels of domestic and foreign firms in country j are related by trade costs and labor costs as follows:

$$\varphi^*_{ijs} = T_{ijs} \left(\frac{w_i}{w_j}\right)^{1/\rho_s} \varphi^*_{jjs}, \tag{7}$$

where $T_{ijs} \equiv \tau_{ijs} \left(f_{ijs}/f_{jjs}\right)^{1/(\sigma_s - 1)}$ captures both variable and fixed trade costs from country i to country j relative to the fixed trade cost within country j.

Let $\mu_{is}(\varphi)$ denote the equilibrium productivity density function for country i and sector s. Since only firms with productivity $\varphi \geq \varphi^*_{iis}$ produce in equilibrium and firm exit is uncorrelated with productivity, the equilibrium productivity density function is given by

$$\mu_{is}(\varphi) \equiv \begin{cases} \frac{g_{is}(\varphi)}{1 - G_{is}(\varphi^*_{iis})} & \text{if } \varphi \geq \varphi^*_{iis} \\ 0 & \text{otherwise.} \end{cases} \tag{8}$$

7

Given (3) and (8), the price index P_{js} satisfies

$$P_{js}^{1-\sigma_s} = \sum_{k=1,2} \frac{M_{kse}}{\delta_{ks}} \int_{\varphi_{kjs}^*}^{\infty} p_{kjs}(\varphi)^{1-\sigma_s} \, dG_{ks}(\varphi). \tag{9}$$

In each time period, there is free entry by firms in each sector s and country i. Let $\bar{\pi}_{is}$ denote the average profits across all domestic firms in country i and sector s (including the fixed marketing costs). Let $\bar{v}_{is} \equiv \sum_{t=0}^{\infty}(1-\delta_{is})^t \bar{\pi}_{is} = \bar{\pi}_{is}/\delta_{is}$ denote the present value of average profit flows in country i and sector s, taking into account the rate δ_{is} at which firms exit in each time period. Free entry implies that the probability of successful entry times the expected profits earned from successful entry must equal the cost of entry, that is, $[1 - G_{is}(\varphi_{iis}^*)]\bar{\pi}_{is}/\delta_{is} = w_i F_{is}$. Calculating the average profits across all domestic firms (exporters and non-exporters), we obtain

$$\frac{1}{\delta_{is}} \sum_{j=1,2} \int_{\varphi_{ijs}^*}^{\infty} \left[\frac{r_{ijs}(\varphi)}{\sigma_s} - w_i f_{ijs} \right] dG_{is}(\varphi) = w_i F_{is}, \tag{10}$$

that is, the expected lifetime profit from entry must be equal to the entry costs. Following Melitz (2003) and Demidova (2008), equation (10) can be rewritten as an expression of the cut-off productivity levels using (2), (5), and (6). Doing so yields the free entry condition

$$\sum_{j=1,2} \gamma_{is} f_{ijs} \varphi_{ijs}^{*-\theta_s} = F_{is} \tag{11}$$

where $\gamma_{1s} \equiv b_{1s}^{\theta_s}(\sigma_s - 1)/\left[\delta_{1s}(\theta_s - \sigma_s + 1)\right]$.

For each sector s, four equations [(7) for $ij = 12, 21$ and (11) for $i = 1, 2$] determine four cut-off productivity levels [φ_{ijs}^* for $ij = 11, 12, 21, 22$] as functions of w_1 and trade costs (T_{12s}, T_{21s}). This simple observation highlights a general equilibrium effect of trade liberalization on industrial productivities: liberalization in one sector affects the cut-off productivity levels in other sectors through the factor market.

2.3 General Equilibrium

To analyze the general equilibrium effect linking the two sectors, we solve for the country 1 equilibrium wage rate w_1 directly from the country 1 labor market clearing condition. We are able to do so thanks

to two convenient properties of the current model with the Cobb-Douglas upper tier utility (1) and the Pareto distribution (2).

The first convenient property is that labor demand L_{is} by all firms in country i and sector s is proportional to the mass of entrants M_{ise}. We show this in three steps. First, equation (11) implies that the fixed costs (the entry costs plus the marketing costs) are proportional to the mass of entrants in each country i and sector s:

$$w_i \left(M_{ise} F_{is} + \sum_{j=1,2} \int_{\varphi^*_{ijs}}^{\infty} f_{ijs} M_{is} \mu_{is}(\varphi) \, d\varphi \right) = w_i M_{ise} \left(\frac{\theta_s F_{is}}{\sigma_s - 1} \right). \tag{12}$$

Second, equation (10) implies that the fixed costs are equal to the gross profits in each country i and sector s, that is, $w_i M_{ise} \left(\frac{\theta_s F_{is}}{\sigma_s - 1} \right) = \sigma_s^{-1} \sum_{j=1,2} R_{ijs}$ where $R_{ijs} \equiv \int_{\varphi^*_{ijs}}^{\infty} r_{ijs}(\varphi) M_{is} \mu_{is}(\varphi) d\varphi$ is the total revenue associated with shipments from country i to country j in sector s. Third, free entry also implies that wage payments to labor equal total revenue in each country i and sector s, that is, $w_i L_{is} = \sum_{j=1,2} R_{ijs}$. These three steps lead immediately to:

$$L_{is} = \frac{1}{w_i} \sum_{j=1,2} R_{ijs} = M_{ise} X_{is}, \tag{13}$$

where $X_{is} \equiv \theta_s F_{is}/\rho_s$ is the labor demand per entrant in country i and sector s. Notice that the industrial labor demand L_{is} depends only on the mass of entrants M_{ise} and not on any cut-off productivity levels φ^*_{ijs}. We will exploit this remarkable simple property to solve the model.

The second convenient property of the model is that we can solve for the mass of entrants M_{1se} as a function of the wage w_1 and trade costs T_{12s} and T_{21s}. Let ϕ_{ijs} denote the ratio of the expected profit of an entrant in country i from selling to country j in sector s to that captured by an entrant in country j from selling to country j:

$$\phi_{ijs} \equiv \frac{\delta_{is}^{-1} \int_{\varphi^*_{ijs}}^{\infty} \left[\frac{r_{ijs}(\varphi)}{\sigma_s} - w_i f_{ijs} \right] dG_{is}(\varphi)}{\delta_{js}^{-1} \int_{\varphi^*_{jjs}}^{\infty} \left[\frac{r_{jjs}(\varphi)}{\sigma_s} - w_j f_{jjs} \right] dG_{js}(\varphi)}.$$

Using (2), (5), (6) and (7), this relative expected profit simplifies to

$$\phi_{ijs} = \frac{\delta_{js} f_{ijs}}{\delta_{is} f_{jjs}} \left(\frac{b_{is}}{b_{js}}\right)^{\theta_s} T_{ijs}^{-\theta_s} \left(\frac{w_i}{w_j}\right)^{1-\theta_s/\rho_s}, \tag{14}$$

so ϕ_{ijs} is a function of T_{ijs} and w_1. From (2), (3), (4), (5), (6), (7), (8), (9), and (14), total revenue R_{ijs} can be written as

$$R_{ijs} = \alpha_s w_j L_j \frac{M_{ise} \phi_{ijs}}{\sum_{k=1,2} M_{kse} \phi_{kjs}}. \tag{15}$$

From (13) and (15), we obtain

$$\sum_{j=1,2} \alpha_s w_j L_j \frac{\phi_{ijs}}{\sum_{k=1,2} M_{kse} \phi_{kjs}} = w_i X_{is}. \tag{16}$$

For each sector s, (16) represents a system of linear equations that can be solved using Cramer's Rule for M_{ise}. We find that the mass of entrants in country 1 and sector s is

$$M_{1se} = \alpha_s \left(\frac{w_1 L_1}{w_1 X_{1s} - \phi_{12s} X_{2s}} - \frac{\phi_{21s} L_2}{X_{2s} - \phi_{21s} w_1 X_{1s}}\right). \tag{17}$$

Given (14), equation (17) defines M_{1se} as a function of w_1, T_{12s} and T_{21s}, and can be written in function form as $M_{1se}(w_1, T_{12s}, T_{21s})$. It is straightforward to verify that it has the properties $\partial M_{1se}/\partial w_1 < 0$, $\partial M_{1se}/\partial T_{12s} < 0$ and $\partial M_{1se}/\partial T_{21s} > 0$. These effects are quite intuitive. Increases in the wage ($w_1 \uparrow$) and export barriers ($T_{12s} \uparrow$) discourages entry ($M_{1se} \downarrow$), while an increase in import barriers ($T_{21s} \uparrow$) encourages entry ($M_{1se} \uparrow$).

Having already established that labor demand in country 1 is proportional to the mass of entrants ($L_{1s} = M_{1se} X_{1s}$), it follows that labor demand in country 1 is a function of w_1, T_{12s} and T_{21s}. This function can be written in function form as $L_{1s}(w_1, T_{12s}, T_{21s})$ and it has the same properties as the $M_{1se}(w_1, T_{12s}, T_{21s})$ function: $\partial L_{1s}/\partial w_1 < 0$, $\partial L_{1s}/\partial T_{12s} < 0$ and $\partial L_{1s}/\partial T_{21s} > 0$. In particular, we obtain the nice property that country 1 labor demand in each sector ($s = A$ and $s = B$) is downward sloping in the country 1 wage rate w_1. The country 1 labor supply is given by L_1 so the requirement

that labor supply equals labor demand

$$L_1 = \sum_{s=A,B} L_{1s}\left(w_1, T_{12s}, T_{21s}\right) \tag{18}$$

uniquely determines the equilibrium wage rate w_1 given the trade costs (T_{12s}, T_{21s}).

Figure 1 describes the determination of the equilibrium wage from (18) by using a graphical technique commonly used for the specific factors model. The vertical axis represents country 1's wage rate w_1 and the width of the box is set equal to country 1's labor endowment L_1. The left bottom corner represents the origin for sector A, while the right bottom corner represents the origin for sector B. The labor demand of each sector is drawn as a downward sloping curve relative to its corresponding origin. The intersection of the two curves determines the equilibrium wage and the allocation of labor across sectors.
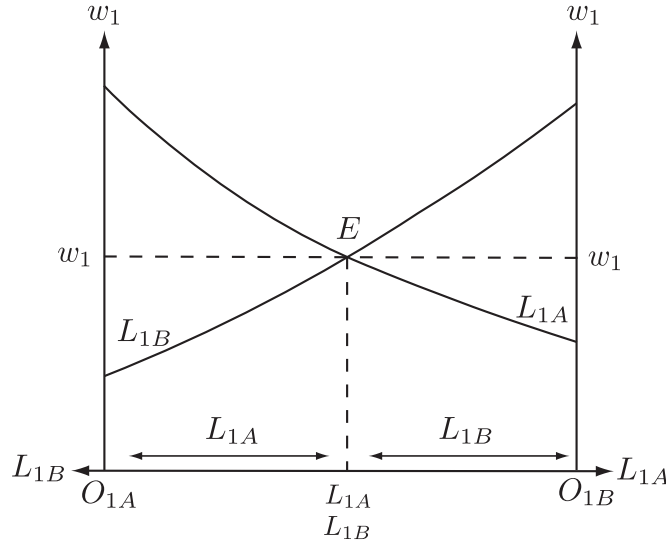

Figure 1: The labor market equilibrium

Having found the equilibrium wage rate w_1, we can now solve for the equilibrium cut-off productivity levels. From (7) and (11), we obtain the export productivity cut-off φ^*_{12s} for country 1 in sector s as a function of the country 1 wage rate w_1:

$$\varphi^*_{12s} = \left[\frac{\gamma_{1s} f_{12s}(1 - \phi_{12s}\phi_{21s})}{F_{2s}(\phi_{12s}/w_1) - \phi_{12s}\phi_{21s}F_{1s}}\right]^{1/\theta_s}, \tag{19}$$

where $\phi_{12s}\phi_{21s} < 1$ from $f_{ijs} > f_{iis}$. Given (14), the export productivity cut-off φ^*_{12s} is an unambiguously increasing function of w_1. When the wage rate goes up, firms have higher labor costs and need to be more productive to justify exporting their products. Following Demidova and Rodriguez-Clare (2011), we refer to equation (19) as the "competitiveness curve" for country 1 and sector s. Figure 2 illustrates the competitiveness curves (C_{1s} curves) for both sectors s in country 1 together with the labor market diagram.
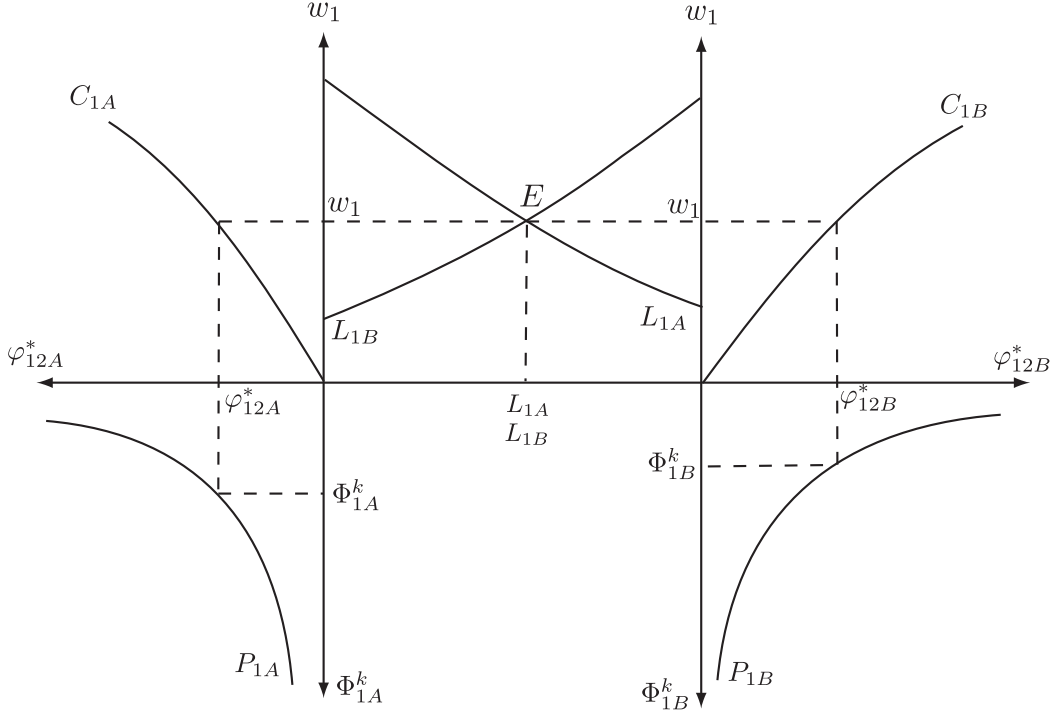

Figure 2: Determination of the equilibrium

Finally, we show that industrial productivity only depends on the export productivity cut-off level. We consider three measures of industrial productivity. The first measure of industrial productivity Φ^R_{1s} is defined as the industrial average of firm productivity weighted by each firm's revenue share in the industry:

$$\Phi^R_{1s} \equiv \int_0^\infty \varphi v_{1s}(\varphi) \, d\varphi \quad \text{where} \quad v_{1s}(\varphi) \equiv \frac{\sum_{j=1,2} I(\varphi \geq \varphi^*_{1js}) r_{1js}(\varphi) M_{1s} \mu_{1s}(\varphi)}{\sum_{k=1,2} R_{1ks}}. \tag{20}$$

In this definition, $I(\varphi \geq \varphi^*_{1js})$ is an indicator function that takes on the value 1 if $\varphi \geq \varphi^*_{1js}$

and 0 otherwise. The function $v_{1s}(\varphi)$ is a revenue-weighted density function for φ and satisfies $\int_0^\infty v_{1s}(\varphi)d\varphi = 1$. We need to assume $\theta_s > \sigma_s$ so that Φ_{1s}^R takes a finite value. This measure is widely used in empirical studies (e.g. Olley and Pakes, 1996) and is a simpler version of the measure that Melitz (2003) used. The second measure of industrial productivity Φ_{1s}^L is industrial labor productivity defined as the real industrial output per unit of labor:

$$\Phi_{1s}^L = \frac{\left(\sum_{j=1,2} R_{1js}\right)/\tilde{P}_{1s}}{L_{1s}}. \tag{21}$$

In this definition, the price deflator $\tilde{P}_{1s} \equiv \int_{\varphi_{11s}^*}^\infty p_{11s}(\varphi)\mu_{1s}(\varphi)d\varphi$ is the simple average of prices set by domestic firms at the factory gate and aims to resemble the industrial product price index, which is used for the calculation of the real industrial output. This measure is also widely used in empirical studies (e.g. Trefler, 2004). The third measure of industrial productivity Φ_{1s}^W is industrial labor productivity calculated using the theoretically consistent "exact" price index P_{1s} that we derived earlier in equation (9):

$$\Phi_{1s}^W = \frac{\left(\sum_{j=1,2} R_{1js}\right)/P_{1s}}{L_{1s}}. \tag{22}$$

This measure is motivated by thinking about consumer welfare. Consider the representative consumer in country 1 who supplies one unit of labor. Since her utility U_1 satisfies

$$U_1 = \left(\alpha_A \Phi_{1A}^W\right)^{\alpha_A} \left(\alpha_B \Phi_{1B}^W\right)^{\alpha_B}, \tag{23}$$

Φ_{1A}^W and Φ_{1B}^W are the productivity measures for industries A and B that are directly relevant for calculating consumer welfare U_1.

The next lemma shows that, regardless of which measure of industrial productivity we use, we can draw a negative-sloped curve between industrial productivity and the export productivity cut-off, and this curve does not shift as a result of changes in variable trade costs.

Lemma 1 *All three measures of industrial productivity Φ_{1s}^k ($k = R, L, W$) can be expressed as decreasing functions of the export productivity cut-off φ_{12s}^* and these functions $\Phi_{1s}^k(\varphi_{12s}^*)$ do not contain any other endogenous variables or variable trade costs.*

Proof. The first step is to rewrite $\Phi_{1s}^k(k = R, L, W)$ as functions of φ_{11s}^* and φ_{12s}^*. From (2), (3), (4), (5), (6) and (8), Φ_{1s}^R is simplified as

$$\Phi_{1s}^R = \xi_s \left[\left(\frac{1}{1 + z_s} \right) \varphi_{11s}^* + \left(\frac{z_s}{1 + z_s} \right) \varphi_{12s}^* \right], \tag{24}$$

where $\xi_s \equiv (\theta_s - \sigma_s + 1) / (\theta_s - \sigma_s) > 0$ and $z_s \equiv \frac{f_{12s}}{f_{11s}} \left(\frac{\varphi_{11s}^*}{\varphi_{12s}^*} \right)^{\theta_s}$. From (13), it holds that $\Phi_{1s}^L = w_1 / \tilde{P}_{1s}$ and $\Phi_{1s}^W = w_1 / P_{1s}$. By calculating $\tilde{P}_{1s}$ from (2), (5) and (8), and P_{1s} from (4), (5) and (6), we obtain

$$\Phi_{1s}^L = \left(\frac{\theta_s + 1}{\theta_s} \right) \rho_s \varphi_{11s}^* \quad \text{and} \quad \Phi_{1s}^W = \left(\frac{\alpha_s L_1}{\sigma_s f_{11s}} \right)^{1/(\sigma_s - 1)} \rho_s \varphi_{11s}^*. \tag{25}$$

Second, a total differentiation of the free entry condition (11) gives

$$\frac{d\varphi_{11s}^*}{d\varphi_{12s}^*} = -z_s \left(\frac{\varphi_{11s}^*}{\varphi_{12s}^*} \right) < 0. \tag{26}$$

From (25) and (26), we immediately see that both Φ_{1s}^L and Φ_{1s}^W are decreasing functions of φ_{12s}^*. We obtain the total derivative $d\Phi_{1s}^R / d\varphi_{12s}^* = \partial\Phi_{1s}^R / \partial\varphi_{12s}^* + \left(\partial\Phi_{1s}^R / \partial\varphi_{11s}^* \right) \left(d\varphi_{11s}^* / d\varphi_{12s}^* \right)$ from (24) and (26) as

$$\frac{d\Phi_{1s}^R}{d\varphi_{12s}^*} = -\xi_s \left(\frac{z_s}{1 + z_s} \right) (\theta_s - 1) \left(1 - \frac{\varphi_{11s}^*}{\varphi_{12s}^*} \right) < 0.$$

Therefore, Φ_{1s}^R is a decreasing function of φ_{12s}^*. Finally, note that equations (11), (24), and (25) do not contain any other endogenous variables or variable trade costs. ∎

The intuition behind Lemma 1 is as follows. Suppose the export productivity cut-off φ_{12s}^* falls. This means that exporting becomes more profitable for some firms in country 1 that could not previously afford to pay the exporting fixed cost $w_1 f_{12s}$. Since all exporters face the same demand function and the same level of trade barriers, exporting must become more profitable for existing exporters also. As a result, a potential entrant in country 1 sees an increase in the expected profits from entry and more firms enter the industry in country 1. Some of these new entrants draw sufficiently high productivity to survive. This means that the industry becomes more populated with firms and local consumer demand for each individual firm's product decreases.[5] Thus, all firms earn lower profits from domestic sales

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[5]The decrease in local consumer demand can be confirmed as follows. By using $\Phi_{1s}^W = w_1 / P_{1s}$ in the proof of Lemma 1, (4) and (5), local consumer demand for an individual firm can be written as $q_{11s}(\varphi) = (\rho_s \varphi)^{\sigma_s} \left(\Phi_{1s}^W \right)^{1 - \sigma_s} \alpha_s L_1$. Therefore, local demand $q_{11s}(\varphi)$ falls if and only if productivity Φ_{1s}^W rises.

and the lowest productivity non-exporting firms exit, that is, the domestic productivity cut-off φ_{11s}^* increases. Since expanding exporters are more productive than the exiting non-exporters, resources are reallocated from less to more productive firms, increasing industrial productivity Φ_{1s}^k.[6]

Using Lemma 1, we draw the negative relationship between φ_{12s}^* and Φ_{1s}^k for each sector s in the bottom two diagrams in Figure 2. We refer to the $\Phi_{1s}^k(\varphi_{12s}^*)$ functions as "productivity curves" and label them as P_{1s} curves in Figure 2. Factor market clearing determines w_1, then the competitiveness curves determine φ_{12s}^* and then the productivity curves determine Φ_{1s}^k. This figure clearly shows that the productivity of each sector s is connected though the factor market.

Lemma 1 implies that the source of a rise in industrial productivity is higher profits from exporting. Since the productivity curves $\Phi_{1s}^k(\varphi_{12s}^*)$ do not include any variable trade costs, trade liberalization (multilateral or unilateral) improves industrial productivity Φ_{1s}^k only when exporting becomes more profitable and the export productivity cut-off φ_{12s}^* falls.

3 Trade Liberalization

We are now ready to analyze the impact of trade liberalization on industrial productivity. While Melitz (2003) considered only multilateral and uniform liberalization, in which all countries reduce variable trade costs on all products in a uniform way, we consider unilateral and non-uniform liberalization: country 1 liberalizes tariffs only for sector A. Following Melitz (2003), import tariffs take the form of iceberg trade costs. So trade liberalization for us means decreasing τ_{21A} while holding τ_{12A}, τ_{12B} and τ_{21B} fixed. We call sector A the liberalized industry and sector B the non-liberalized industry.

3.1 Structurally Symmetric Industries

We focus on the impact of trade liberalization when the two industries are structurally symmetric except for their consumption share in GDP (α_A is allowed to differ from α_B).

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[6]The weighted average productivity measure in Melitz (2003), $\tilde{\varphi}_{1s} \equiv \left[\int_{\varphi_{11s}^*}^{\infty} \varphi^{\sigma_s-1} \mu_{1s}(\varphi)d\varphi\right]^{1/(\sigma_s-1)} = \left[\theta_s/(\theta_s - \sigma_s + 1)\right]^{1/(\sigma_s-1)} \varphi_{11s}^*$ also satisfies Lemma 1. Since φ_{11s}^* and φ_{12s}^* move in the opposite direction from (11), productivity $\tilde{\varphi}_{1s}$ rises if and only if φ_{12s}^* falls. Since w_1 and τ_{ijs} do not show up in either $\tilde{\varphi}_{1s}$ or (11), they affect $\tilde{\varphi}_{1s}$ only through φ_{12s}^*.

Definition 1 *The two industries are structurally symmetric if $\rho_A = \rho_B$, $\theta_A = \theta_B$, $\delta_{iA} = \delta_{iB}$, $b_{iA} = b_{iB}$, $f_{ijA} = f_{ijB}$, $F_{iA} = F_{iB}$, and $\tau_{ijA} = \tau_{ijB}$.*

This is a natural benchmark case for the analysis of unilateral and non-uniform trade liberalization. The Melitz (2003) model only has one industry but requires balanced trade and labor market clearing as in general equilibrium models. Thus, it is natural to think of the one industry in the Melitz model as a representative industry. Note that Definition 1 requires symmetry only across industries. Countries can differ in their factor endowments, technologies and trade costs.
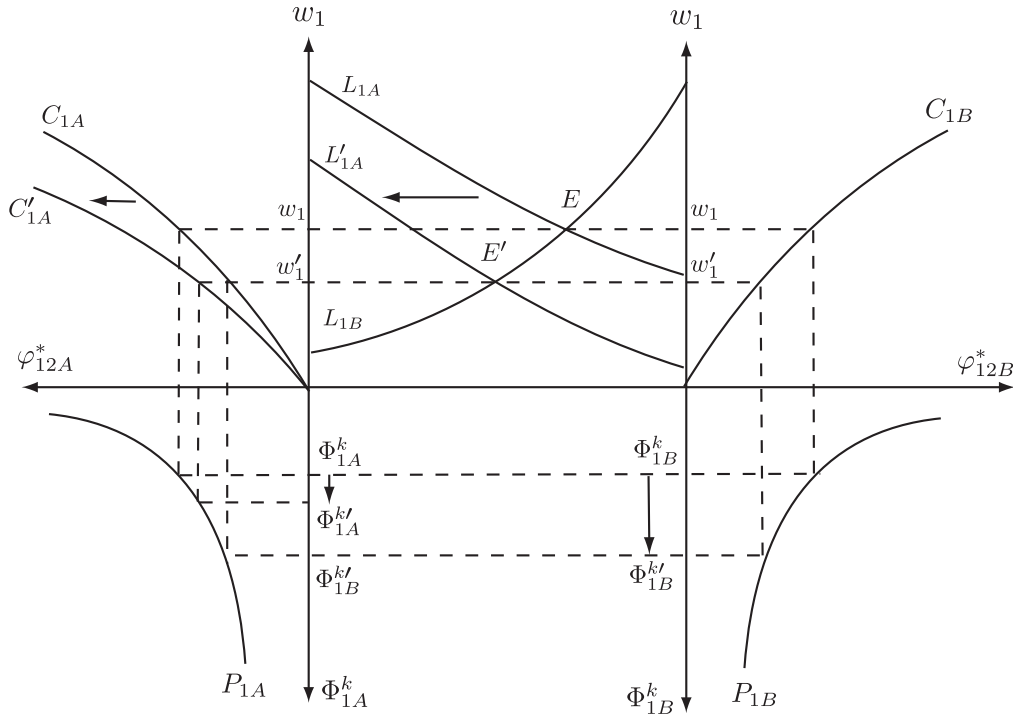

Figure 3: Productivity rises more strongly in the non-liberalized industry

The diagrams developed in the previous section greatly simplifies the analysis. Figure 3 shows the same diagrams we used in Figure 2 for the structurally symmetric industries case. Before trade liberalization, both industries have symmetric competitiveness curves (the C_{1A} and C_{1B} curves) and symmetric productivity curves (the P_{1A} and P_{1B} curves), which implies that both industries have the same productivity $\Phi_{1A}^k = \Phi_{1B}^k$ ($k = R, L, W$).[7]

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[7]Before liberalization, country 1 always produce positive outputs in both sectors since $L_{1A}/\alpha_A = L_{1B}/\alpha_B$ holds from (13) and (17).

The top-center diagram in Figure 3 describes the impact of liberalization on the labor market of the liberalizing country 1. The labor demand curve of the liberalized industry A shifts leftward since the mass of entrants drops for a given wage level w_1 from (17). Workers move from the liberalized to the non-liberalized industry and the wage drops in the liberalizing country. The top-right and bottom-right diagrams in Figure 3 describe how the general equilibrium effect operating through the labor market affects the productivity of the non-liberalized industry B. The competitiveness curve in industry B does not shift in response to liberalization since φ^*_{12B} in (19) includes no trade costs for industry A. As country 1's wage w_1 declines, less productive firms start exporting and the export productivity cut-off φ^*_{12B} unambiguously falls in the non-liberalized industry B. Then, from the bottom-right diagram, industrial productivity Φ^k_{1B} unambiguously rises in the non-liberalized industry B.

The liberalized industry A responds to trade liberalization in a more complex way since the competitiveness curve depends on trade costs in the industry. From (14) and (19), the competitiveness curve of the liberalized industry A shifts leftward (curve C_{1A} shifts to C'_{1A}) for a given wage level w_1.[8] This leftward shift of the C_{1A} curve tends to increase the export productivity cut-off φ^*_{12A}, while the declining wage w_1 tends to decrease it. Therefore, the net effect on the export productivity cut-off φ^*_{12A} in the liberalized industry is ambiguous. Lemma 1 then implies that the net effect on the productivity of the liberalizing industry is also ambiguous. We have established

Theorem 1 *In the multi-industry Melitz model with structurally symmetric industries, unilateral trade liberalization by country 1 in industry A ($\tau_{21A} \downarrow$) leads to a decrease in the country 1 wage rate ($w_1 \downarrow$) and an increase in the productivity of the non-liberalized industry ($\Phi^k_{1B} \uparrow$). However, whether productivity rises or falls in the liberalized industry is in general ambiguous ($\Phi^k_{1A} \uparrow$ or $\downarrow$).*

Although trade liberalization has an ambiguous effect on productivity in the liberalized industry, we can make an unambiguous statement about the difference in the productivity change between the liberalized and the non-liberalized industries. If the C_{1A} curve remained at the pre-liberalization position, then productivity would rise by the same amount in each industry due to the decrease in the equilibrium wage rate w_1. However, since the C_{1A} curve shifts leftward (to C'_{1A}), productivity increases more strongly in industry B than in industry A. As shown in Figure 3, productivity increases from

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[8]The Appendix spells out the derivation in detail. Demidova and Rodriguez-Clare (2011) show that this leftward shift occurs in a one industry Melitz model.

Φ_{1B}^k to $\Phi_{1B}^{k\prime}$ in industry B, while the productivity increase is smaller in industry A, from $\Phi_{1A}^k = \Phi_{1B}^k$ to $\Phi_{1A}^{k\prime} < \Phi_{1B}^{k\prime}$. Thus, productivity rises more strongly in the non-liberalized industry than in the liberalized industry, i.e. $\Delta\Phi_{1B}^k - \Delta\Phi_{1A}^k > 0$, $k = R, L, W$. This "difference-in-difference" prediction is sufficient for our purpose of matching the model with empirical studies. Because typical empirical studies estimate cross-industry regressions with time fixed effects and industry fixed effects (e.g. Trefler, 2004), their estimates only tell us whether trade liberalization increases productivity in liberalized industries *relative to* non-liberalized industries. We have established

Theorem 2 *In the multi-industry Melitz model with structurally symmetric industries, unilateral trade liberalization by country 1 in industry A ($\tau_{21A} \downarrow$) leads to productivity rising more strongly in the non-liberalized industry than in the liberalized industry ($\Delta\Phi_{1B}^k > \Delta\Phi_{1A}^k$ for $k = R, L, W$).*

Theorem 2 is our central result. An empirical finding by Trefler (2004) and others that industrial productivity increases more strongly in liberalized industries than in non-liberalized industries has been widely accepted as evidence for the Melitz (2003) model. Theorem 2 shows that a multi-industry version of the Melitz model does not predict this relationship. Instead, it predicts the opposite relationship that industrial productivity increases more strongly in non-liberalized industries than in liberalized industries. Theorem 2 forces us to re-think the match between theory and evidence: an empirical fact that has been widely cited as evidence for the Melitz model is actually evidence against the Melitz model.

Next, we study whether the effects of trade liberalization depend on the size of the industry that opens up to trade. Does trade liberalization have different effects, depending on whether the liberalized industry is small or large? Since the parameter α_s determines the size of industry s, we analyze how the response of industrial productivity to trade liberalization depends on α_A, the size of the liberalized industry.

Holding all other parameter values fixed, a change in α_A has no effect on the equilibrium wage w_1. Since employment in the two industries satisfies $L_{1A}/\alpha_A = L_{1B}/\alpha_B$ from (13) and (17), the labor market clearing condition (18) can be rewritten as

$$L_1 = L_{1A} + L_{1B} = L_{1A}\left(\frac{\alpha_A + \alpha_B}{\alpha_A}\right) = \frac{L_{1A}}{\alpha_A}.$$

18

Now $L_1 = L_{1A}(w_1, T_{12s}, T_{21s})/\alpha_A$ uniquely determines the equilibrium wage w_1 and L_{1A}/α_A does not depend on α_A from (17). Thus, the equilibrium wage w_1 does not depend on α_A.
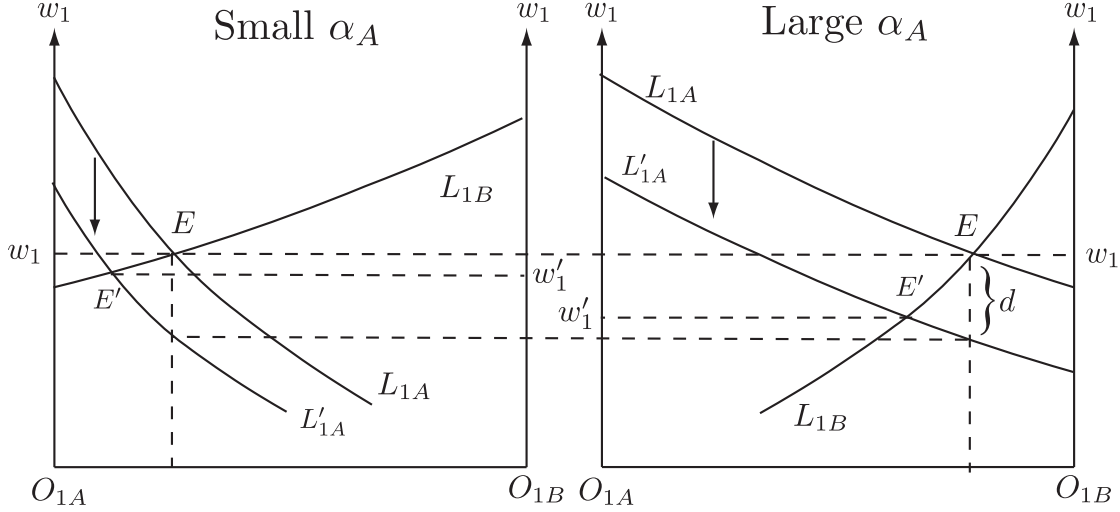

Figure 4: How much the wage declines depends on the size of the liberalized industry

As illustrated in Figure 4, the pre-liberalization wage w_1 is the same whether α_A is small or large. Trade liberalization causes the labor demand curve L_{1A} to shift leftward, or equivalently, to shift down. Equation (17) implies that the size of the downward shift in the labor demand curve L_{1A} ("d" in Figure 4) does not depends on α_A. Equation (17) also implies that as α_A increases, the slope of the labor demand curve L_{1A} becomes flatter because the number of entrants in industry A increases in α_A. Similarly, as α_A increases, which means that $\alpha_B = 1 - \alpha_A$ decreases, the slope of the labor demand curve L_{1B} becomes steeper. Thus, as illustrated in Figure 4, the wage drop due to trade liberalization is larger when α_A is larger.

Figure 3 shows that whether productivity increases in the liberalized industry A depends on the net effect of two opposite changes: the leftward shift of the competitiveness curve C_{1A} ("the competitiveness effect") and the drop in the wage rate w_1 ("the wage effect"). The competitiveness effect does not depend on α_A since equation (19) does not include α_A. However, as we have just shown, the wage effect is larger when α_A is larger. If α_A is sufficiently small and the wage effect is sufficiently small, then the competitiveness effect must dominate the wage effect. Figure 5 illustrates this case.

The export productivity cut-off φ_{12A}^* rises and productivity Φ_{1A}^k unambiguously falls in the liberalized industry. If α_A is sufficiently large and satisfies $\alpha_A = 1$, then the model reduces to a one industry
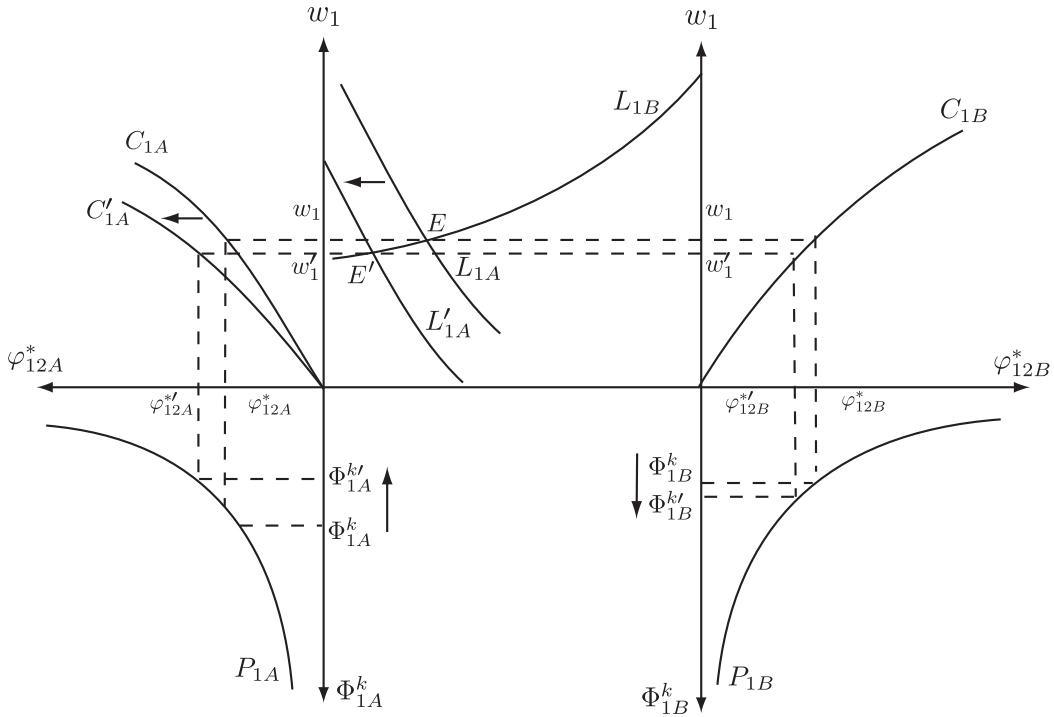
Figure 5: Productivity falls in the liberalized industry if the liberalized industry is small

Melitz model where Demidova and Rodriguez-Clare (2011) proved that unilateral liberalization raises industrial productivity. Since the model's properties are continuous in parameter α_A, we obtain the following theorem:

Theorem 3 *In the multi-industry Melitz model with structurally symmetric industries, suppose that there is unilateral trade liberalization by country 1 in industry A ($\tau_{21A} \downarrow$). Then there exists a threshold $\bar{\alpha}_A \in (0, 1)$ such that productivity Φ_{1A}^k falls in the liberalized industry if $\alpha_A < \bar{\alpha}_A$ and rises if $\alpha_A > \bar{\alpha}_A$.*

Theorem 3 helps us to understand contrasting results in previous studies on unilateral trade liberalization using versions of the Melitz model with one Melitz industry. Melitz and Ottaviano (2008) and Demidova (2008) find that the productivity of the liberalized industry falls, while Demidova and Rodriguez-Clare (2011) and Felbermayr, Jung, and Larch (2012) find that the productivity of the liberalized industry rises. The main difference between these papers is that Melitz and Ottaviano (2008) and Demidova (2008) consider a class of models with a homogeneous numeraire good that is pro-

duced with constant returns to scale technology and is freely traded under perfect competition, while Demidova and Rodriguez-Clare (2011) and Felbermayr, Jung, and Larch (2012) consider models with a single Melitz industry. The former with the numeraire good that fixes the wage rate corresponds to our model with a negligible α_A, while the latter corresponds to our model with $\alpha_A = 1$.

By combining the results in Theorems 1 and 3, we obtain one more theorem:

Theorem 4 *In the multi-industry Melitz model with structurally symmetric industries, suppose that there is unilateral trade liberalization by country 1 in industry A ($\tau_{21A} \downarrow$). If α_A is sufficiently small, then productivity falls in the liberalized industry A and rises in the non-liberalized industry B ($\Phi_{1A}^k \downarrow$ and $\Phi_{1B}^k \uparrow$).*

Theorem 4 provides a surprising policy implication. If the government of a country is interested in raising the productivity of a small "target" industry through a resource reallocation from less productive to more productive firms, the theoretically correct advice based on the Melitz model is to protect the target industry, not trade liberalization. This is obviously the opposite of the policy recommendation that is suggested by Trefler (2004) and other empirical studies.

3.2   Intuition

Trade liberalization (decreasing τ_{21A}) causes two curves to shift in Figure 3. The shift in the competitiveness curve of the liberalized industry C_{1A} expresses the competitiveness effect of trade liberalization, a partial equilibrium effect that decreases productivity in the liberalized industry. The shift in the labor demand curve of the liberalized industry L_{1A} expresses the wage effect of trade liberalization, a general equilibrium effect that increases productivity in both industries. To understand the overall effect of trade liberalization on industrial productivity, we need to understand both the competitiveness effect and the wage effect.

To understand the competitiveness effect, suppose that country 1 opens up to trade in industry A ($\tau_{21A} \downarrow$) and the wage rate w_1 is held fixed. Then firms in country 2 earn higher profits from exporting to country 1 in industry A. Since exports increase not only for existing exporters (the intensive margin) but also by the entry of less productive firms into exporting (the extensive margin), the export productivity cut-off φ_{21A}^* falls. Because exporting becomes more profitable in country 2, more firms enter

and industry A in country 2 becomes more populated with firms, so local consumer demand for each individual firm's product decreases in country 2. Thus all firms earn lower profits from domestic sales and the lowest productivity non-exporting firms exit. Since expanding exporters are more productive than exiting non-exporters, resources are reallocated from less to more productive firms, increasing industry A productivity Φ_{2A}^k in country 2. This increase in country 2 productivity sets in motion the opposite chain of events in country 1. Now industry A in country 2 is occupied by more firms and more productive firms. This means that it becomes less profitable for country 1 firms to export to country 2. The export productivity cut-off φ_{12A}^* increases and industry A productivity Φ_{1A}^k decreases in country 1.

To understand the wage effect, it is helpful to think about the balanced trade condition. Let E_{ijs} be the expenditure of country i on country j goods in sector s. Then the exports in sector s by country 1 is $\sum_{j=1,2} R_{1js} - E_{11s}$ and the imports in sector s by country 1 is E_{12s}. The balanced trade condition can be written as

$$\sum_{s=A,B} \left[\left(\sum_{j=1,2} R_{1js} - E_{11s} \right) - E_{12s} \right] = 0. \tag{27}$$

From $\sum_{j=1,2} R_{1js} = w_1 L_{1s}$ and $\sum_{j=1,2} E_{1js} = \alpha_s w_1 L_1$, the excess exports of sector s can be expressed as

$$\left(\sum_{j=1,2} R_{1js} - E_{11s} \right) - E_{12s} = w_1 \alpha_s \left(\frac{L_{1s}(w_1, T_{12s}, T_{21s})}{\alpha_s} - L_1 \right). \tag{28}$$

By summing up (28) for both industries, we see that the balanced trade condition (27) is equivalent to the labor market clearing condition (18).

Starting from balanced trade and holding the wage w_1 fixed, trade liberalization leads to excess imports in industry A by the liberalizing country 1. Then (27) and (28) imply that the wage w_1 must drop to increase exports by *both* industries in the liberalizing country until trade balance is restored. Since exports increase not only for existing exporters (the intensive margin) but also by the entry of less productive firms into exporting (the extensive margin), the export productivity cut-offs φ_{12s}^* fall in both industries when w_1 falls. Because exporting becomes more profitable, more firms enter and both industries become more populated with firms, so local consumer demand for each individual firm's product decreases. Thus all firms earn lower profits from domestic sales and the lowest productivity

non-exporting firms exit. Since expanding exporters are more productive than exiting non-exporters, resources are reallocated from less to more productive firms, increasing industrial productivity. With structurally symmetric industries, the wage effect by itself contributes to increase productivity equally in both industries.

Balanced trade also explains why the size of the liberalized industry α_A determines the relative size of the wage effect and the competitiveness effect. Since $L_{1s}(\cdot)/\alpha_s$ does not depend on α_s from (17), equation (28) shows that the size of excess exports in sector s is proportional to α_s and that the rise in exports in sector s due to a wage drop increases in α_s. Therefore, if α_A is sufficiently large, trade balance after liberalization is mainly restored by an increase in the exports in the liberalized industry. Since the mass of firms in the liberalized industry falls, the exports increases by the entry of less productive firms into exporting (the extensive margin) as well as for existing exporters (the intensive margin). To achieve this, the wage must drop by a sufficiently large amount so that the wage effect dominates the competitiveness effect and the export productivity cutoff φ_{12s}^* falls. On the other hand, if α_A is small, then liberalization leads to a small trade deficit, so a small wage drop can restore trade balance by increasing exports in the non-liberalized industry. Therefore, the wage effect is too small to dominate the competitiveness effect if α_A is sufficiently small.

The effects of trade liberalization are summarized in Table 2. The wage effect of trade liberalization tends to increase productivity in both industries symmetrically, while the competitiveness effect tends to decrease productivity in the liberalized industry. As a consequence, industrial productivity unambiguously rises in the non-liberalized industry but it can rise or fall in the liberalized industry, depending on the relative size of the wage effect and the competitiveness effect. Figure 3 illustrates the case where the wage effect of trade liberalization dominates the competitiveness effect, with the consequence that productivity rises in the liberalized industry. Productivity rises even more in the non-liberalized industry because there is a positive wage effect and no negative competitiveness effect. Figure 5 illustrates what happens when a sufficiently small industry is opened up to trade. Then the positive wage effect of trade liberalization is sufficiently small so that the negative competitiveness effect dominates. Productivity falls in the liberalized industry and only rises in the non-liberalized industry.

	Impacts on Industrial Productivity		
	Liberalized (A) $\Delta\Phi_{1A}^k$	Non-liberalized (B) $\Delta\Phi_{1B}^k$	Difference $\Delta\Phi_{1A}^k - \Delta\Phi_{1B}^k$
Competitiveness Effect	$-$	0	$-$
Wage Effect	$+$	$+$	0
Total Effect	$+$ or $-$	$+$	$-$

Table 2: The effects of trade liberalization

3.3 Numerical Results

As a check that our analytically derived results are correct, we also solve the model numerically. Looking at a numerical example is helpful for understanding the intuition behind the results.[9]

For the numerical results reported in Table 3, we assume structurally symmetric industries and countries. Then there are only ten parameters that need to be chosen. We use the following benchmark parameter values: $\sigma_s = 3.8$, $\delta_{is} = .025$, $b_{is} = .2$, $\theta_s = 4.582$, $F_{is} = 2$, $f_{iis} = .043$, $L_i = 1$, $\alpha_A = .5$, $\tau_{ijs} = 1.3$ and $f_{ijs} = .0588$. The first six parameter values come from Balistreri, Hillbery and Rutherford (2011), where a version of the Melitz model is calibrated to fit trade data. $L_i = 1$ is a convenient normalization given that an increase in country size L_i has no effect on the key endogenous variables that we are solving for (the relative wage w_1/w_2, productivity cutoff levels φ_{ijs}^* and industry productivity levels Φ_{is}^R). $\alpha_A = .5$ means that both industries are equally large: consumers spend 50 percent of their income on industry A products and 50 percent of their income on industry B products. $\tau_{ijs} = 1.3$ corresponds to a 30 percent tax on all traded goods. Finally, we chose $f_{ijs} = .0588$ to guarantee that 18 percent of firms export in our benchmark equilibrium, consistent with evidence for the United States (Bernard et al., 2007).

The first column of numbers in Table 3 shows the benchmark equilibrium (when $\alpha_A = .5$ and $\tau_{21A} = 1.30$). The second column shows what happens when country 1 unilaterally opens up to trade in industry A (τ_{21A} is decreased from 1.30 to 1.15 holding $\tau_{21B} = \tau_{12A} = \tau_{12B} = 1.30$ fixed). This leads to productivity rising more strongly in the non-liberalized industry B (Φ_{1B}^R increases from .5564 to .5651) than in the liberalized industry A (Φ_{1A}^R increases from .5564 to .5590), consistent with Theorem 2. Since productivity rises in the liberalized industry, we are illustrating a case where the wage effect of trade liberalization dominates the competitiveness effect. The third and fourth columns

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[9]The MATLAB files used to solve the model can be obtained from the authors upon request.

	$\alpha_A = .5$ Case		$\alpha_A = .3$ Case	
	$\tau_{21A} = 1.30$	$\tau_{21A} = 1.15$	$\tau_{21A} = 1.30$	$\tau_{21A} = 1.15$
w_1/w_2	1.0000	.9707	1.0000	.9801
L_{1A}	.5000	.4221	.3000	.2275
L_{1B}	.5000	.5779	.7000	.7725
L_{2A}	.5000	.5757	.3000	.3711
L_{2B}	.5000	.4243	.7000	.6289
φ_{12A}^*	.3257	.3206	.3257	.3273
φ_{11A}^*	.2240	.2250	.2240	.2238
φ_{12B}^*	.3257	.3092	.3257	.3144
φ_{11B}^*	.2240	.2274	.2240	.2262
φ_{21A}^*	.3257	.3012	.3257	.2957
φ_{22A}^*	.2240	.2296	.2240	.2314
φ_{21B}^*	.3257	.3443	.3257	.3380
φ_{22B}^*	.2240	.2214	.2240	.2222
Φ_{1A}^R	.5564	.5590	.5564	.5556
Φ_{1B}^R	.5564	.5651	.5564	.5623
Φ_{2A}^R	.5564	.5694	.5564	.5724
Φ_{2B}^R	.5564	.5476	.5564	.5505
U_1	.1230	.1242	.1376	.1385
U_2	.1230	.1238	.1376	.1381

Table 3: Effects of Trade Liberalization

show the effects of the same trade liberalization when industry A is smaller ($\alpha_A = .3$, all other parameter values unchanged). Then the wage effect of trade liberalization is smaller and is dominated by the competitiveness effect. Productivity in the liberalized industry decreases (Φ_{1A}^R decreases from .5564 to .5556) and productivity in the non-liberalized industry increases (Φ_{1B}^R increases from .5564 to .5623), consistent with Theorem 4.

To see the intuition behind these results, consider the $\alpha_A = .3$ "small industry" case first and focus on what happens in industry A. When country 1 opens up to trade in industry A, country 2 firms earn higher profits from exporting. These higher export profits lead to more entry and greater industrial employment (L_{2A}, which is proportional to the mass of entrants and active firms, increases from .3000 to .3711). As the industry becomes more populated with firms, the country 2 demand for each individual firm's product decreases, so the least productive firms are forced to exit (φ_{22A}^* increases from .2240 to .2314). Even though the increase in labor demand bids up the wage rate in country 2 (w_1/w_2 decreases from 1.000 to .9801), the wage increase is not large enough to completely offset the tariff reduction by country 1 and more country 2 firms become exporters (φ_{21A}^* decreases from .3257 to .2957). Since expanding exporters are more productive than exiting non-exporters, productivity rises for country 2 in industry A (Φ_{2A}^R increases from .5564 to .5724). For firms in country 1, the picture is very different. Now they are competing against more productive firms in their export market, they earn lower profits from exporting and this sets into motion the opposite effects. Fewer country 1 firms become exporters (φ_{12A}^* increases from .3257 to .3273), entry is discouraged and the mass of firms in the industry falls (L_{1A} decreases from .3000 to 2275) until the expected profits from domestic sales increase to offset the loss of expected profits from exporting. The increase in domestic profits allows less productive firms to survive in the domestic market (φ_{11A}^* decreases from .2240 to .2238). Thus, we get a reallocation of resources from more productive to less productive firms in country 1, lowering industry productivity (Φ_{1A}^R decreases from .5564 to .5556).

Next, focus on what happens in industry B when country 1 opens up to trade in industry A. Because wages rise in country 2 (w_1/w_2 decreases from 1.000 to .9801), it becomes less profitable for country 2 firms to export and there is a reallocation of resources from more productive to less productive firms, lowering productivity (Φ_{2B}^R decreases from .5564 to .5505). Because wages fall in country 1 (w_1/w_2 decreases from 1.000 to .9801), the export productivity cutoff falls (φ_{12B}^* decreases from

.3257 to .3144) and there is a reallocation of resources from less productive to more productive firms, raising productivity (Φ_{1B}^R increases from .5564 to .5623).

Finally, turn to the effects of trade liberalization when industry A is larger ($\alpha_A = .5$). We obtain the same qualitative effects in industry B: because wages rise in country 2 (w_1/w_2 decreases from 1.000 to .9707), productivity falls (Φ_{2B}^R decreases from .5564 to .5476) and because wages fall in country 1 (w_1/w_2 decreases from 1.000 to .9707), productivity rises (Φ_{1B}^R increases from .5564 to .5651). But the qualitative effects are different for the industry A that opens up to trade because there is a larger fall in country 1 wages. Even though trade liberalization raises productivity in country 2 (Φ_{2A}^R increases from .5564 to .5694), which by itself makes exporting less attractive for country 1 firms, the larger fall in country 1 wages now dominates and country 1 productivity in industry A actually rises (Φ_{1A}^R increases from .5564 to .5590).

Although the impact of trade liberalization on industrial productivity is the main focus of this paper, we also report the impact of trade liberalization on consumer welfare in the last two rows of Table 3. U_1 and U_2 denote the steady-state utility levels of the representative consumer in countries 1 and 2, respectively. In the $\alpha_A = .5$ case, trade liberalization by country 1 raises consumer welfare in country 2 (U_2 increases from .1230 to .1238) and raises even more consumer welfare in country 1 (U_1 increases from .1230 to .1242). Thus country 2 benefits when country 1 opens up to trade and country 1 benefits even more by unilaterally opening up to trade. Looking at the $\alpha_A = .3$ case, we obtain qualitatively similar welfare effects.

3.4   Symmetric Multilateral Trade Liberalization

In this subsection, we replicate the analysis of symmetric multilateral trade liberalization in Melitz (2003) using our diagrams. The two countries are assumed to be identical as in Melitz (2003) but each industry may have different parameters. We analyze multilateral but non-uniform liberalization by decreasing τ_{12A} and τ_{21A} by the same amount while holding τ_{12B} and τ_{21B} fixed.

Assuming symmetric countries simplifies the model. First, wages are equalized between the two countries: $w_1 = w_2 = 1$. Second, the notation for describing the model takes a simpler form: $X_{is} = X_s$, $\phi_{ijs} = \phi_s$, $T_{ijs} = T_s$, $F_{is} = F_s$, $f_{iis} = f_s$ and $f_{ijs} = f_{xs}$ for $i \neq j$.

Figure 6 describes the impact of liberalization. The employment in sector s becomes $L_{1s} =$

$\alpha_s L_1$ from (13) and (17), so multilateral trade liberalization in sector A ($\tau_{12A} = \tau_{21A} \downarrow$) leads to no equilibrium change in the wage w_1 and the labor allocation.[10] The top-right and bottom-right diagrams show that multilateral liberalization does not affect productivity Φ^k_{1B} in the non-liberalized industry.
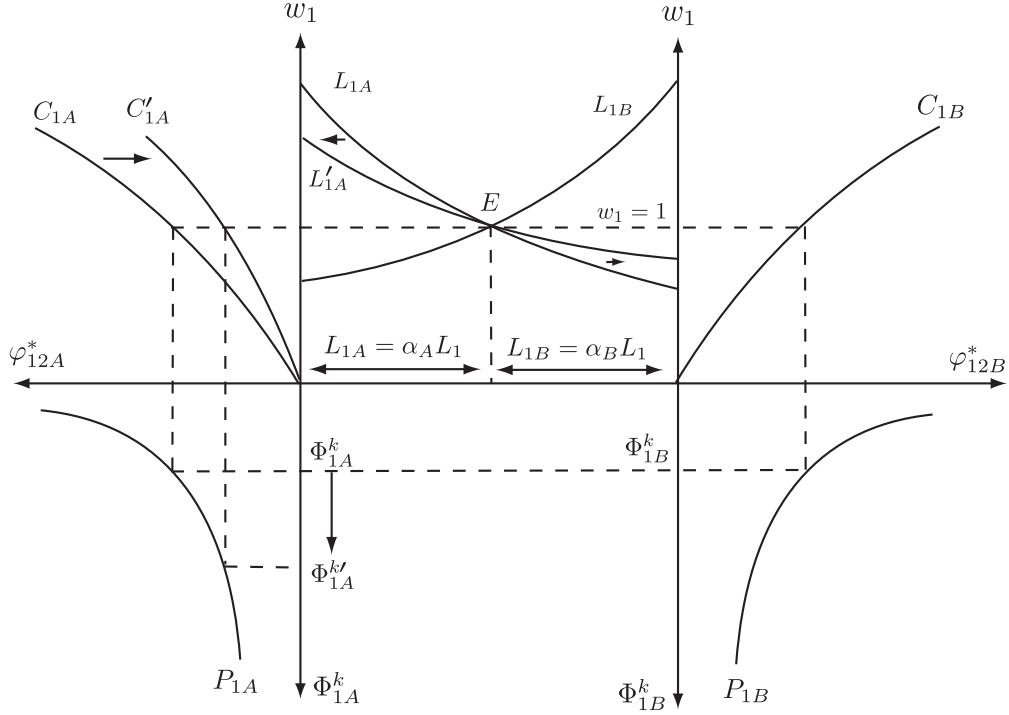

Figure 6: Multilateral trade liberalization in industry A

The impact on the liberalized industry is different from the case of unilateral trade liberalization. Given symmetric countries, the export productivity cut-off in sector A [given by (19)] simplifies to

$$\varphi^*_{12A} = \left[\frac{\gamma_{1A} f_{xA}}{F_A} \left(1 + \frac{1}{\phi_A} \right) \right]^{1/\theta_A} \tag{29}$$

and (14) implies that $\phi_A = (f_{xA}/f_A) T_A^{-\theta_A}$. Thus multilateral trade liberalization leads to a decrease in the export productivity cut-off φ^*_{12A} and an increase in productivity Φ^k_{1A} in the liberalized industry. We have established

Theorem 5 *In the multi-industry Melitz model with symmetric countries, symmetric multilateral trade*

---

[10]The labor demand curve of the liberalized industry becomes flatter as illustrated in Figure 6. This is shown in the Appendix.

liberalization ($\tau_{12A} = \tau_{21A} \downarrow$) increases productivity in the liberalized industry ($\Phi_{1A}^k \uparrow$) but not in the non-liberalized industry (Φ_{1B}^k unchanged).

A comparison of Theorems 2 and 5 confirms that the source of the rise in industrial productivity in the Melitz model is the expansion of export opportunity, not the increased import competition from trade liberalization.

Notice that by setting $\alpha_A = 1$, the model becomes identical to the original Melitz (2003) model with one industry. Therefore, our analysis nests the analysis of multilateral and uniform liberalization in Melitz (2003). We obtain Melitz's original result using new diagrams:

Corollary 1 *(Melitz, 2003) If there is only one industry ($\alpha_A = 1$) and symmetric countries, then symmetric multilateral trade liberalization ($\tau_{12A} = \tau_{21A} \downarrow$) increases industrial productivity ($\Phi_{1A}^k \uparrow$).*

4 Conclusion

In order to establish that trade liberalization improves industrial productivity through within-industry resource reallocation, the empirical literature and the theoretical literature have taken different approaches. On the one hand, empirical studies have typically investigated episodes of unilateral and non-uniform liberalization in order to utilize the cross-industry variations in trade liberalization. On the other hand, theoretical studies have typically analyzed multilateral and uniform liberalization in models with a single representative industry.

We have demonstrated that this apparently small difference in approach between theoretical and empirical studies is not innocuous. Once the Melitz (2003) model is extended to analyze unilateral and non-uniform trade liberalization, it does not predict what researchers find empirically, that industrial productivity increases more strongly in liberalized industries than in non-liberalized industries.

Instead, we find that industrial productivity increases more strongly in non-liberalized industries than in liberalized industries. When one country opens up to trade in one industry, we find that productivity falls if the industry is sufficiently small. If the industry is larger, so productivity rises, we find that it rises even more in the industries that did not open up to trade. So, if we take the Melitz model seriously when looking at the real world, other things being equal, we should observe that productivity

is high in industries with high tariffs and productivity is low in industries with low tariffs. This is the opposite of what reseachers find empirically.

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Appendix: Solving The Model (Not for Publication)

In this Appendix, calculations done to solve the model are spelled out in more detail.

Consumers

First, we solve the within-sector consumer optimization problem

$$\max_{q_s(\cdot)} C_s \equiv \left[\int_{\omega \in \Omega_s} q_s(\omega)^{\rho_s} \, d\omega \right]^{1/\rho_s} \qquad \text{s.t.} \qquad \int_{\omega \in \Omega_s} p_s(\omega) q_s(\omega) \, d\omega = E_s$$

where $q_s(\omega)$ is quantity demanded for variety ω in sector s, $p_s(\omega)$ is the price of variety ω in sector s and E_s is individual consumer expenditure on sector s products. This problem of maximizing a CES utility function subject to a budget constraint can be rewritten as the optimal control problem

$$\max_{q_s(\cdot)} \int_{\omega \in \Omega_s} q_s(\omega)^{\rho_s} \, d\omega \qquad \text{s.t.} \qquad \dot{y}_s(\omega) = p_s(\omega) q_s(\omega), \; y_s(0) = 0, \; y_s(+\infty) = E_s$$

where $y_s(\omega)$ is a new state variable and $\dot{y}_s(\omega)$ is the derivative of y_s with respect to ω. The Hamiltonian function for this optimal control problem is

$$H = q_s(\omega)^{\rho_s} + \gamma(\omega) p_s(\omega) q_s(\omega)$$

where $\gamma(\omega)$ is the costate variable. The costate equation $\frac{\partial H}{\partial y_s} = 0 = -\dot{\gamma}(\omega)$ implies that $\gamma(\omega)$ is constant across ω. $\frac{\partial H}{\partial q_s} = \rho_s q_s(\omega)^{\rho_s - 1} + \gamma \cdot p_s(\omega) = 0$ implies that

$$q_s(\omega) = \left(\frac{\rho_s}{-\gamma \cdot p_s(\omega)} \right)^{1/(1-\rho_s)}.$$

Substituting this back into the budget constraint yields

$$
\begin{aligned}
E_s &= \int_{\omega \in \Omega_s} p_s(\omega) q_s(\omega) \, d\omega = \int_{\omega \in \Omega_s} p_s(\omega) \left(\frac{\rho_s}{-\gamma \cdot p_s(\omega)} \right)^{1/(1-\rho_s)} d\omega \\
&= \left(\frac{\rho_s}{-\gamma} \right)^{1/(1-\rho_s)} \int_{\omega \in \Omega_s} p_s(\omega)^{\frac{1-\rho_s-1}{1-\rho_s}} \, d\omega.
\end{aligned}
$$

Now $\sigma_s \equiv \frac{1}{1-\rho_s}$ implies that $1 - \sigma_s = \frac{1-\rho_s-1}{1-\rho_s} = \frac{-\rho_s}{1-\rho_s}$, so

$$\frac{E_s}{\int_{\omega \in \Omega_s} p_s(\omega)^{1-\sigma_s} d\omega} = \left(\frac{\rho_s}{-\gamma} \right)^{1/(1-\rho_s)}.$$

1

It immediately follows that the individual consumer demand function is

$$q_s(\omega) = \frac{p_s(\omega)^{-\sigma_s} E_s}{P_s^{1-\sigma_s}}$$

where $P_s \equiv \left[\int_{\omega \in \Omega_s} p_s(\omega)^{1-\sigma_s} d\omega \right]^{1/(1-\sigma_s)}$ is the price index for sector s. Substituting this consumer demand function back into the CES utility function yields

$$C_s = \left[\int_{\omega \in \Omega_s} q_s(\omega)^{\rho_s} d\omega \right]^{1/\rho_s} = \left[\int_{\omega \in \Omega_s} \frac{p_s(\omega)^{-\sigma_s \rho_s} E_s^{\rho_s}}{P_s^{(1-\sigma_s)\rho_s}} d\omega \right]^{1/\rho_s} = \frac{E_s}{P_s^{1-\sigma_s}} \left[\int_{\omega \in \Omega_s} p_s(\omega)^{-\sigma_s \rho_s} d\omega \right]^{1/\rho_s}.$$

Taking into account that $-\sigma_s \rho_s = \frac{-\rho_s}{1-\rho_s} = 1 - \sigma_s$, the CES utility can be simplified further to

$$C_s = \frac{E_s}{P_s^{1-\sigma_s}} \left[\int_{\omega \in \Omega_s} p_s(\omega)^{1-\sigma_s} d\omega \right]^{1/\rho_s} = \frac{E_s}{P_s^{1-\sigma_s}} \left[P_s^{1-\sigma_s} \right]^{1/\rho_s} = \frac{E_s}{P_s^{1-\sigma_s}} P_s^{-\sigma_s} = \frac{E_s}{P_s}.$$

Thus, we can write the across-sector consumer optimization problem as

$$\max_{E_A, E_B} \; U \equiv C_A^{\alpha_A} C_B^{\alpha_B} = \left(\frac{E_A}{P_A} \right)^{\alpha_A} \left(\frac{E_B}{P_B} \right)^{\alpha_B} \qquad \text{s.t.} \qquad E_A + E_B = E$$

where E is consumer expenditure on products in both sectors combined. The solution to this problem is $E_A = \alpha_A E$ and $E_B = \alpha_B E$.

In country i, workers earn the wage rate w_i and total labor supply is L_i, so total wage income that can be spent on products produced in both sectors is $w_i L_i$. Given free entry, there are no profits earned from entering markets, so consumers spend exactly what they earn in wage income. Let E_{is} denote the expenditure by all consumers in country i on sector s products. It follows that

$$E_{is} = \alpha_s w_i L_i.$$

Firms

Let $\pi_{ijs}(\varphi)$ denote the gross profits (or variable profits) earned by a firm with productivity φ from country i to country j in sector s. It follows that

$$
\begin{aligned}
\pi_{ijs}(\varphi) &= r_{ijs}(\varphi) - \frac{w_i \tau_{ijs}}{\varphi} q_{ijs}(\varphi) \\
&= \frac{p_{ijs}(\varphi)^{1-\sigma_s} \alpha_s w_j L_j}{P_{js}^{1-\sigma_s}} - \frac{w_i \tau_{ijs}}{\varphi} \frac{p_{ijs}(\varphi)^{-\sigma_s} \alpha_s w_j L_j}{P_{js}^{1-\sigma_s}}.
\end{aligned}
$$

2

We obtain the price that maximizes gross profits by solving the first order condition

$$
\begin{aligned}
\frac{\partial \pi_{ijs}(\varphi)}{\partial p_{ijs}(\varphi)} &= \frac{(1-\sigma_s)p_{ijs}(\varphi)^{-\sigma_s}\alpha_s w_j L_j}{P_{js}^{1-\sigma_s}} + \frac{w_i \tau_{ijs}\sigma_s p_{ijs}(\varphi)^{-\sigma_s-1}\alpha_s w_j L_j}{\varphi P_{js}^{1-\sigma_s}} \\
&= \frac{p_{ijs}(\varphi)^{-\sigma_s}\alpha_s w_j L_j}{P_{js}^{1-\sigma_s}}\left[1 - \sigma_s + \frac{w_i \tau_{ijs}\sigma_s}{\varphi p_{ijs}(\varphi)}\right] = 0
\end{aligned}
$$

which yields $\sigma_s - 1 = \frac{w_i \tau_{ijs}\sigma_s}{\varphi p_{ijs}(\varphi)}$. Taking into account that $\frac{\sigma_s}{\sigma_s-1} = \frac{1}{1-\rho_s}/\frac{1-(1-\rho_s)}{1-\rho_s} = \frac{1}{\rho_s}$, we obtain the profit-maximizing price

$$
p_{ijs}(\varphi) = \frac{w_i \tau_{ijs}}{\rho_s \varphi}. \tag{5}
$$

Substituting this expression for price back into gross profits, we obtain

$$
\begin{aligned}
\pi_{ijs}(\varphi) &= \frac{p_{ijs}(\varphi)^{1-\sigma_s}\alpha_s w_j L_j}{P_{js}^{1-\sigma_s}} - \frac{w_i \tau_{ijs}}{\varphi}\frac{p_{ijs}(\varphi)^{-\sigma_s}\alpha_s w_j L_j}{P_{js}^{1-\sigma_s}} \\
&= \frac{p_{ijs}(\varphi)^{1-\sigma_s}\alpha_s w_j L_j}{P_{js}^{1-\sigma_s}}\left[1 - \frac{w_i \tau_{ijs}}{\varphi p_{ijs}(\varphi)}\right] \\
&= r_{ijs}(\varphi)\left[1 - \frac{w_i \tau_{ijs}}{\varphi}\frac{\rho_s \varphi}{w_i \tau_{ijs}}\right] \\
&= r_{ijs}(\varphi)\left[1 - \rho_s\right] \\
&= \frac{r_{ijs}(\varphi)}{\sigma_s}
\end{aligned}
$$

since $\sigma_s = \frac{1}{1-\rho_s}$ implies that $1 - \rho_s = \frac{1}{\sigma_s}$. A firm from country i and sector s needs to have a productivity $\varphi \geq \varphi_{ijs}^*$ to justify paying the fixed "marketing" cost $w_i f_{ijs}$ of serving the country j market. Thus φ_{ijs}^* is determined by the cut-off productivity condition

$$
\frac{r_{ijs}(\varphi_{ijs}^*)}{\sigma_s} = w_i f_{ijs}. \tag{6}
$$

Comparing the cut-off productivity levels of domestic firms and foreign firms in country j, we find

3

that

$$\begin{aligned}
\frac{w_i f_{ijs}}{w_j f_{jjs}} &= \frac{r_{ijs}(\varphi^*_{ijs})/\sigma_s}{r_{jjs}(\varphi^*_{jjs})/\sigma_s} \\
&= \frac{\left(p_{ijs}(\varphi^*_{ijs})/P_{js}\right)^{1-\sigma_s} \alpha_s w_j L_j}{\left(p_{jjs}(\varphi^*_{jjs})/P_{js}\right)^{1-\sigma_s} \alpha_s w_j L_j} \text{ from (4)} \\
&= \frac{\left(w_i \tau_{ijs}/\rho_s \varphi^*_{ijs}\right)^{1-\sigma_s}}{\left(w_j \tau_{jjs}/\rho_s \varphi^*_{jjs}\right)^{1-\sigma_s}} \text{ from (5)} \\
&= \left(\frac{w_i \tau_{ijs}\varphi^*_{jjs}}{w_j \varphi^*_{ijs}}\right)^{1-\sigma_s}.
\end{aligned}$$

Rearranging terms yields

$$\begin{aligned}
\left(\frac{\varphi^*_{jjs}}{\varphi^*_{ijs}}\right)^{1-\sigma_s} &= \tau_{ijs}^{\sigma_s-1}\frac{f_{ijs}}{f_{jjs}}\left(\frac{w_i}{w_j}\right)^{\sigma_s} \\
\frac{\varphi^*_{ijs}}{\varphi^*_{jjs}} &= \left[\tau_{ijs}^{\sigma_s-1}\frac{f_{ijs}}{f_{jjs}}\left(\frac{w_i}{w_j}\right)^{\sigma_s}\right]^{1/(\sigma_s-1)}
\end{aligned}$$

and letting $T_{ijs} \equiv \tau_{ijs}\left(f_{ijs}/f_{jjs}\right)^{1/(\sigma_s-1)}$, it follows that

$$\varphi^*_{ijs} = T_{ijs}\left(\frac{w_i}{w_j}\right)^{1/\rho_s}\varphi^*_{jjs}. \tag{7}$$

The Price Index

Next we solve for the value of the price index P_{js} for country j and sector s. Given the Pareto distribution function $G_{is}(\varphi) \equiv 1 - (b_{is}/\varphi)^{\theta_s}$, let $g_{is}(\varphi) \equiv G'_{is}(\varphi) = b_{is}^{\theta_s}\theta_s\varphi^{-\theta_s-1}$ denote the corresponding productivity density function. Let $\mu_{is}(\varphi)$ denote the equilibrium productivity density function for country i and sector s. Since only firms with productivity $\varphi \geq \varphi^*_{iis}$ produce in equilibrium, firm exit is uncorrelated with productivity and $\varphi^*_{iis} < \varphi^*_{ijs}$, the equilibrium productivity density function is given by

$$\mu_{is}(\varphi) \equiv \begin{cases} \frac{g_{is}(\varphi)}{1-G_{is}(\varphi^*_{iis})} & \text{if } \varphi \geq \varphi^*_{iis} \\ 0 & \text{otherwise.} \end{cases} \tag{8}$$

4

Using $P_s \equiv \left[\int_{\omega \in \Omega_s} p_s(\omega)^{1-\sigma_s} d\omega \right]^{1/(1-\sigma_s)}$ and

$$M_{is}\mu_{is}(\varphi) = \frac{[1 - G_{is}(\varphi_{iis}^*)] M_{ise} g_{is}(\varphi)}{\delta_{is} [1 - G_{is}(\varphi_{iis}^*)]} = \frac{M_{ise}}{\delta_{is}} g_{is}(\varphi), \tag{A.1}$$

the price index P_{is} for country i and sector s satisfies

$$
\begin{aligned}
P_{is}^{1-\sigma_s} &= \int_{\varphi_{iis}^*}^{\infty} p_{iis}(\varphi)^{1-\sigma_s} M_{is}\mu_{is}(\varphi) \, d\varphi + \int_{\varphi_{jis}^*}^{\infty} p_{jis}(\varphi)^{1-\sigma_s} M_{js}\mu_{js}(\varphi) \, d\varphi \\
&= \frac{M_{ise}}{\delta_{is}} \int_{\varphi_{iis}^*}^{\infty} p_{iis}(\varphi)^{1-\sigma_s} \, dG_{is}(\varphi) + \frac{M_{jse}}{\delta_{js}} \int_{\varphi_{jis}^*}^{\infty} p_{jis}(\varphi)^{1-\sigma_s} \, dG_{js}(\varphi).
\end{aligned}
$$

This expression can be written more conveniently by switching indexes i and j

$$P_{js}^{1-\sigma_s} = \frac{M_{jse}}{\delta_{js}} \int_{\varphi_{jjs}^*}^{\infty} p_{jjs}(\varphi)^{1-\sigma_s} \, dG_{js}(\varphi) + \frac{M_{ise}}{\delta_{is}} \int_{\varphi_{ijs}^*}^{\infty} p_{ijs}(\varphi)^{1-\sigma_s} \, dG_{is}(\varphi)$$

and it follows that the price index P_{js} satisfies

$$P_{js}^{1-\sigma_s} = \sum_{k=1,2} \frac{M_{kse}}{\delta_{ks}} \int_{\varphi_{kjs}^*}^{\infty} p_{kjs}(\varphi)^{1-\sigma_s} \, dG_{ks}(\varphi). \tag{9}$$

Free Entry

Free entry implies that the probability of successful entry times the expected profits earned from successful entry must equal the cost of entry, that is, $\text{Prob.}(\varphi \geq \varphi_{iis}^*) \bar{v}_{is} = w_i F_{is}$ or

$$[1 - G_{is}(\varphi_{iis}^*)] \frac{\bar{\pi}_{is}}{\delta_{is}} = w_i F_{is}.$$

The average profits across all domestic firms (exporters and non-exporters) is given by

$$
\begin{aligned}
\bar{\pi}_{is} &= \frac{1}{M_{is}} \left\{ \int_{\varphi_{iis}^*}^{\infty} [\pi_{iis}(\varphi) - w_i f_{iis}] M_{is}\mu_{is}(\varphi) \, d\varphi + \int_{\varphi_{ijs}^*}^{\infty} [\pi_{ijs}(\varphi) - w_i f_{ijs}] M_{is}\mu_{is}(\varphi) \, d\varphi \right\} \\
&= \int_{\varphi_{iis}^*}^{\infty} \left[\frac{r_{iis}(\varphi)}{\sigma_s} - w_i f_{iis} \right] \frac{g_{is}(\varphi)}{1 - G_{is}(\varphi_{iis}^*)} \, d\varphi + \int_{\varphi_{ijs}^*}^{\infty} \left[\frac{r_{ijs}(\varphi)}{\sigma_s} - w_i f_{ijs} \right] \frac{g_{is}(\varphi)}{1 - G_{is}(\varphi_{iis}^*)} \, d\varphi.
\end{aligned}
$$

Substituting yields

$$[1 - G_{is}(\varphi_{iis}^*)] \bar{\pi}_{is} = \int_{\varphi_{iis}^*}^{\infty} \left[\frac{r_{iis}(\varphi)}{\sigma_s} - w_i f_{iis} \right] g_{is}(\varphi) \, d\varphi + \int_{\varphi_{ijs}^*}^{\infty} \left[\frac{r_{ijs}(\varphi)}{\sigma_s} - w_i f_{ijs} \right] g_{is}(\varphi) \, d\varphi = \delta_{is} w_i F_{is}.$$

Thus we obtain

$$\frac{1}{\delta_{is}} \sum_{j=1,2} \int_{\varphi_{ijs}^*}^{\infty} \left[\frac{r_{ijs}(\varphi)}{\sigma_s} - w_i f_{ijs} \right] dG_{is}(\varphi) = w_i F_{is}. \tag{10}$$

To evaluate the integrals, next note that from (4) and (5),

$$\frac{r_{ijs}(\varphi)}{r_{ijs}(\varphi_{ijs}^*)} = \frac{p_{ijs}(\varphi)^{1-\sigma_s} (\alpha_s w_j L_j)/P_{js}^{1-\sigma_s}}{p_{ijs}(\varphi_{ijs}^*)^{1-\sigma_s} (\alpha_s w_j L_j)/P_{js}^{1-\sigma_s}} = \left(\frac{p_{ijs}(\varphi)}{p_{ijs}(\varphi_{ijs}^*)} \right)^{1-\sigma_s} = \left(\frac{w_i \tau_{ijs}}{\rho_s \varphi} \frac{\rho_s \varphi_{ijs}^*}{w_i \tau_{ijs}} \right)^{1-\sigma_s} = \left(\frac{\varphi}{\varphi_{ijs}^*} \right)^{\sigma_s-1}.$$

Using the cut-off productivity condition, it follows that

$$\frac{r_{ijs}(\varphi)}{\sigma_s} = \frac{r_{ijs}(\varphi_{ijs}^*)}{\sigma_s} \left(\frac{\varphi}{\varphi_{ijs}^*} \right)^{\sigma_s-1} = \frac{\sigma_s w_i f_{ijs}}{\sigma_s} \left(\frac{\varphi}{\varphi_{ijs}^*} \right)^{\sigma_s-1} = w_i f_{ijs} \left(\frac{\varphi}{\varphi_{ijs}^*} \right)^{\sigma_s-1} \tag{A.2}$$

and

$$\int_{\varphi_{ijs}^*}^{\infty} \left[\frac{r_{ijs}(\varphi)}{\sigma_s} - w_i f_{ijs} \right] dG_{is}(\varphi) = \int_{\varphi_{ijs}^*}^{\infty} \left[w_i f_{ijs} \left(\frac{\varphi}{\varphi_{ijs}^*} \right)^{\sigma_s-1} - w_i f_{ijs} \right] dG_{is}(\varphi)$$

$$= w_i f_{ijs} \int_{\varphi_{ijs}^*}^{\infty} \left[\left(\frac{\varphi}{\varphi_{ijs}^*} \right)^{\sigma_s-1} - 1 \right] dG_{is}(\varphi)$$

$$= w_i f_{ijs} J_{is}(\varphi_{ijs}^*), \tag{A.3}$$

where the function $J_{is}(\cdot)$ is given by

$$J_{is}(x) \equiv \int_x^{\infty} \left[\left(\frac{\varphi}{x} \right)^{\sigma_s-1} - 1 \right] dG_{is}(\varphi)$$

$$= \int_x^{\infty} \left(\frac{\varphi}{x} \right)^{\sigma_s-1} b_{is}^{\theta_s} \theta_s \varphi^{-\theta_s-1} d\varphi - [1 - G_{is}(x)]$$

$$= b_{is}^{\theta_s} \theta_s x^{1-\sigma_s} \int_x^{\infty} \varphi^{\sigma_s-1-\theta_s-1} d\varphi - \left(\frac{b_{is}}{x} \right)^{\theta_s}$$

$$= b_{is}^{\theta_s} \theta_s x^{1-\sigma_s} \frac{x^{\sigma_s-1-\theta_s}}{\theta_s - \sigma_s + 1} - \left(\frac{b_{is}}{x} \right)^{\theta_s}$$

$$= \frac{\theta_s - (\theta_s - \sigma_s + 1)}{\theta_s - \sigma_s + 1} \left(\frac{b_{is}}{x} \right)^{\theta_s}$$

$$= \frac{\sigma_s - 1}{\theta_s - \sigma_s + 1} \left(\frac{b_{is}}{x} \right)^{\theta_s}. \tag{A.4}$$

We assume that $\theta_s > \sigma_s - 1$ to guarantee that expected profits are finite. Making substitutions and

6

rearranging terms, it follows that

$$\sum_{j=1,2} \int_{\varphi_{ijs}^*}^{\infty} \left[\frac{r_{ijs}(\varphi)}{\sigma_s} - w_i f_{ijs} \right] dG_{is}(\varphi) = \delta_{is} w_i F_{is}$$

$$\sum_{j=1,2} w_i f_{ijs} J_{is}(\varphi_{ijs}^*) = \delta_{is} w_i F_{is} \quad \text{from (A.3)}$$

$$\sum_{j=1,2} f_{ijs} J_{is}(\varphi_{ijs}^*) = \delta_{is} F_{is}$$

$$\sum_{j=1,2} f_{ijs} \frac{\sigma_s - 1}{\theta_s - \sigma_s + 1} \left(\frac{b_{is}}{\varphi_{ijs}^*} \right)^{\theta_s} = \delta_{is} F_{is} \quad \text{from (A.4)} \tag{A.5}$$

and using $\gamma_{is} \equiv b_{is}^{\theta_s} (\sigma_s - 1) / [\delta_{is} (\theta_s - \sigma_s + 1)]$, yields the free entry condition

$$\sum_{j=1,2} \gamma_{is} f_{ijs} \varphi_{ijs}^{*-\theta_s} = F_{is}. \tag{11}$$

Labor Demand

Let L_{is} denote labor demand by all firms in country i and sector s. We use a three step argument to solve for labor demand.

First, we show that the fixed costs (the entry costs plus the marketing costs) are proportional to the mass of entrants in each country i and sector s.

$$
\begin{aligned}
w_i \left(M_{ise} F_{is} + \sum_{j=1,2} \int_{\varphi_{ijs}^*}^{\infty} f_{ijs} M_{is} \mu_{is}(\varphi) \, d\varphi \right) &= w_i \left(M_{ise} F_{is} + \sum_{j=1,2} \int_{\varphi_{ijs}^*}^{\infty} f_{ijs} \frac{M_{ise}}{\delta_{is}} g_{is}(\varphi) \, d\varphi \right) \text{ from (A.1)} \\
&= w_i \left(M_{ise} F_{is} + \frac{M_{ise}}{\delta_{is}} \sum_{j=1,2} f_{ijs} [1 - G_{is}(\varphi_{ijs}^*)] \right) \\
&= w_i \left(M_{ise} F_{is} + \frac{M_{ise}}{\delta_{is}} \sum_{j=1,2} f_{ijs} \left(\frac{b_{is}}{\varphi_{ijs}^*} \right)^{\theta_s} \right) \\
&= w_i \left(M_{ise} F_{is} + \frac{M_{ise}}{\delta_{is}} \delta_{is} F_{is} \frac{\theta_s - \sigma_s + 1}{\sigma_s - 1} \right) \text{ from (A.5)} \\
&= w_i M_{ise} F_{is} \left(\frac{\sigma_s - 1 + \theta_s - \sigma_s + 1}{\sigma_s - 1} \right)
\end{aligned}
$$

7

from which it follows that

$$
w_i \left(M_{ise} F_{is} + \sum_{j=1,2} \int_{\varphi_{ijs}^*}^\infty f_{ijs} M_{is} \mu_{is}(\varphi)\, d\varphi \right) = w_i M_{ise} \left(\frac{\theta_s F_{is}}{\sigma_s - 1} \right). \tag{12}
$$

Second, we show that the fixed costs are equal to the gross profits in each country i and sector s. From the free entry condition (10), we obtain

$$
\begin{aligned}
\delta_{is} w_i F_{is} &= \sum_{j=1,2} \int_{\varphi_{ijs}^*}^\infty \left[\frac{r_{ijs}(\varphi)}{\sigma_s} - w_i f_{ijs} \right] dG_{is}(\varphi) \\
w_i \left(\delta_{is} F_{is} + \sum_{j=1,2} f_{ijs}[1 - G_{is}(\varphi_{ijs}^*)] \right) &= \sum_{j=1,2} \int_{\varphi_{ijs}^*}^\infty \frac{r_{ijs}(\varphi)}{\sigma_s} dG_{is}(\varphi) \\
w_i \left(M_{ise} F_{is} + \frac{M_{ise}}{\delta_{is}} \sum_{j=1,2} f_{ijs}[1 - G_{is}(\varphi_{ijs}^*)] \right) &= \frac{M_{ise}}{\delta_{is}} \sum_{j=1,2} \int_{\varphi_{ijs}^*}^\infty \frac{r_{ijs}(\varphi)}{\sigma_s} dG_{is}(\varphi) \\
w_i M_{ise} \left(\frac{\theta_s F_{is}}{\sigma_s - 1} \right) &= \frac{M_{is}}{1 - G_{is}(\varphi_{iis}^*)} \sum_{j=1,2} \int_{\varphi_{ijs}^*}^\infty \frac{r_{ijs}(\varphi)}{\sigma_s} dG_{is}(\varphi) \text{ from (12)} \\
&= \frac{1}{\sigma_s} \sum_{j=1,2} \int_{\varphi_{ijs}^*}^\infty r_{ijs}(\varphi) M_{is} \mu_{is}(\varphi) d\varphi \text{ from (A.1)} \\
&= \frac{1}{\sigma_s} \sum_{j=1,2} R_{ijs}
\end{aligned}
$$

where $R_{ijs} \equiv \int_{\varphi_{ijs}^*}^\infty r_{ijs}(\varphi) M_{is} \mu_{is}(\varphi) d\varphi$ is the total revenue associated with shipments from country i to country j in sector s.

Third, we show that the wage payments to labor equals the total revenue in each country i and sector s. Firms use labor for market entry, for the production of goods sold to domestic consumers and for the production of goods sold to foreign consumers. Taking into account both the marginal and

8

fixed costs of production, we obtain

$$
\begin{aligned}
w_i L_{is} &= w_i M_{ise} F_{is} + w_i \sum_{j=1,2} \int_{\varphi_{ijs}^*}^{\infty} \left[f_{ijs} + q_{ijs}(\varphi) \frac{\tau_{ijs}}{\varphi} \right] M_{is} \mu_{is}(\varphi) \, d\varphi \\
&= w_i \left(M_{ise} F_{is} + \sum_{j=1,2} \int_{\varphi_{ijs}^*}^{\infty} f_{ijs} M_{is} \mu_{is}(\varphi) \, d\varphi \right) + \sum_{j=1,2} \int_{\varphi_{ijs}^*}^{\infty} q_{ijs}(\varphi) \frac{w_i \tau_{ijs}}{\rho_s \varphi} \rho_s M_{is} \mu_{is}(\varphi) \, d\varphi \\
&= w_i M_{ise} \left(\frac{\theta_s F_{is}}{\sigma_s - 1} \right) + \rho_s \sum_{j=1,2} \int_{\varphi_{ijs}^*}^{\infty} r_{ijs}(\varphi) M_{is} \mu_{is}(\varphi) \, d\varphi \text{ from (4), (5) and (12)} \\
&= \frac{1}{\sigma_s} \sum_{j=1,2} R_{ijs} + \rho_s \sum_{j=1,2} R_{ijs} \\
&= (1 - \rho_s + \rho_s) \sum_{j=1,2} R_{ijs} \\
&= \sum_{j=1,2} R_{ijs}.
\end{aligned}
$$

Thus

$$
L_{is} = \frac{1}{w_i} \sum_{j=1,2} R_{ijs} = \frac{1}{w_i} w_i M_{ise} \left(\frac{\theta_s F_{is}}{\sigma_s - 1} \right) \sigma_s
$$

and it immediately follows that

$$
L_{is} = \frac{1}{w_i} \sum_{j=1,2} R_{ijs} = M_{ise} X_{is} \tag{13}
$$

where $X_{is} \equiv \theta_s F_{is} / \rho_s$ is the labor demand per entrant in country i and sector s.

Relative Expected Profit

The expected profit of an entrant in country i from selling to country j in sector s (after the entrant has paid the entry cost $w_i F_{is}$) is

$$
\frac{[1 - G_{is}(\varphi_{iis}^*)]}{\delta_{is}} \int_{\varphi_{ijs}^*}^{\infty} \left[\frac{r_{ijs}(\varphi)}{\sigma_s} - w_i f_{ijs} \right] \frac{g_{is}(\varphi)}{1 - G_{is}(\varphi_{iis}^*)} \, d\varphi = \delta_{is}^{-1} \int_{\varphi_{ijs}^*}^{\infty} \left[\frac{r_{ijs}(\varphi)}{\sigma_s} - w_i f_{ijs} \right] dG_{is}(\varphi).
$$

The expected profit of an entrant in country j from selling to country j in sector s (after the entrant has paid the entry cost $w_i F_{is}$) is

$$
\frac{\left[1 - G_{js}(\varphi_{jjs}^*) \right]}{\delta_{js}} \int_{\varphi_{jjs}^*}^{\infty} \left[\frac{r_{jjs}(\varphi)}{\sigma_s} - w_j f_{jjs} \right] \frac{g_{js}(\varphi)}{1 - G_{js}(\varphi_{jjs}^*)} \, d\varphi = \delta_{js}^{-1} \int_{\varphi_{jjs}^*}^{\infty} \left[\frac{r_{jjs}(\varphi)}{\sigma_s} - w_j f_{jjs} \right] dG_{js}(\varphi).
$$

Thus the expected profit of an entrant in country i from selling to country j in sector s relative to that captured by an entrant in country j from selling to country j (or the relative expected profit) is given by

$$
\begin{aligned}
\phi_{ijs} &\equiv \frac{\delta_{is}^{-1}\int_{\varphi_{ijs}^*}^{\infty}\left[\frac{r_{ijs}(\varphi)}{\sigma_s} - w_i f_{ijs}\right]dG_{is}(\varphi)}{\delta_{js}^{-1}\int_{\varphi_{jjs}^*}^{\infty}\left[\frac{r_{jjs}(\varphi)}{\sigma_s} - w_j f_{jjs}\right]dG_{js}(\varphi)} \\
&= \frac{\delta_{is}^{-1}w_i f_{ijs}J_{is}(\varphi_{ijs}^*)}{\delta_{js}^{-1}w_j f_{jjs}J_{js}(\varphi_{jjs}^*)} \text{ from (A.3)} \\
&= \frac{\delta_{js}w_i f_{ijs}\frac{\sigma_s-1}{\theta_s-\sigma_s+1}\left(\frac{b_{is}}{\varphi_{ijs}^*}\right)^{\theta_s}}{\delta_{is}w_j f_{jjs}\frac{\sigma_s-1}{\theta_s-\sigma_s+1}\left(\frac{b_{js}}{\varphi_{jjs}^*}\right)^{\theta_s}} \text{ from (A.4)} \\
&= \frac{\delta_{js}w_i f_{ijs}}{\delta_{is}w_j f_{jjs}}\left(\frac{b_{is}}{b_{js}}\right)^{\theta_s}\left[T_{ijs}^{-1}\left(\frac{w_i}{w_j}\right)^{-1/\rho_s}\right]^{\theta_s} \text{ from (7)}
\end{aligned}
\tag{A.6}
$$

or

$$
\phi_{ijs} = \frac{\delta_{js}f_{ijs}}{\delta_{is}f_{jjs}}\left(\frac{b_{is}}{b_{js}}\right)^{\theta_s}T_{ijs}^{-\theta_s}\left(\frac{w_i}{w_j}\right)^{1-\theta_s/\rho_s}.
\tag{14}
$$

It follows that

$$
\begin{aligned}
\phi_{12s}\phi_{21s} &= \frac{\delta_{2s}f_{12s}}{\delta_{1s}f_{22s}}\left(\frac{b_{1s}}{b_{2s}}\right)^{\theta_s}T_{12s}^{-\theta_s}w_1^{1-\theta_s/\rho_s}\frac{\delta_{1s}f_{21s}}{\delta_{2s}f_{11s}}\left(\frac{b_{2s}}{b_{1s}}\right)^{\theta_s}T_{21s}^{-\theta_s}w_1^{-1+\theta_s/\rho_s} \\
&= \frac{f_{12s}f_{21s}}{f_{11s}f_{22s}}\left[T_{12s}T_{21s}\right]^{-\theta_s} \\
&= \frac{f_{12s}f_{21s}}{f_{11s}f_{22s}}\left[\tau_{12s}\left(\frac{f_{12s}}{f_{22s}}\right)^{1/(\sigma_s-1)}\tau_{21s}\left(\frac{f_{21s}}{f_{11s}}\right)^{1/(\sigma_s-1)}\right]^{-\theta_s} \\
&= \frac{1}{(\tau_{12s}\tau_{21s})^{\theta_s}}\left(\frac{f_{11s}f_{22s}}{f_{12s}f_{21s}}\right)^{(\theta_s-\sigma_s+1)/(\sigma_s-1)} < 1
\end{aligned}
$$

since $\tau_{12s} > 1$, $\tau_{21s} > 1$, $f_{12s} > f_{11s}$ and $f_{21s} > f_{22s}$.

Total Revenue

To solve for total revenue R_{ijs} associated with shipments from country i to country j in sector s, we first establish three properties:

$$p_{ijs}(\varphi_{ijs}^*) = \frac{w_i \tau_{ijs}}{\rho_s \varphi_{ijs}^*}$$

$$= \frac{w_i T_{ijs}}{\rho_s T_{ijs} \left(\frac{w_i}{w_j}\right)^{1/\rho_s} \varphi_{jjs}^*} \left(\frac{f_{ijs}}{f_{jjs}}\right)^{1/(1-\sigma_s)} \quad \text{from (7)}$$

$$= \frac{w_i^{-1/(\sigma_s-1)} w_j^{\sigma_s/(\sigma_s-1)}}{\rho_s \varphi_{jjs}^*} \left(\frac{f_{ijs}}{f_{jjs}}\right)^{1/(1-\sigma_s)} \tag{A.7}$$

since $1 - \frac{1}{\rho_s} = \frac{(\sigma_s-1)-\sigma_s}{\sigma_s-1} = \frac{-1}{\sigma_s-1}$,

$$J_{is}(x) + 1 - G_{is}(x) = \int_x^\infty \left[\left(\frac{\varphi}{x}\right)^{\sigma_s-1} - 1\right] dG_{is}(\varphi) + 1 - G_{is}(x)$$

$$= \int_x^\infty \left(\frac{\varphi}{x}\right)^{\sigma_s-1} dG_{is}(\varphi) - [1 - G_{is}(x)] + 1 - G_{is}(x)$$

$$= \int_x^\infty \left(\frac{\varphi}{x}\right)^{\sigma_s-1} dG_{is}(\varphi)$$

$$= \frac{\sigma_s - 1}{\theta_s - \sigma_s + 1} \left(\frac{b_{is}}{x}\right)^{\theta_s} + 1 - G_{is}(x)$$

$$= \frac{\sigma_s - 1 + \theta_s - \sigma_s + 1}{\theta_s - \sigma_s + 1} [1 - G_{is}(x)]$$

$$= \frac{\theta_s}{\theta_s - \sigma_s + 1} \frac{\theta_s - \sigma_s + 1}{\sigma_s - 1} J_{is}(x)$$

$$= \frac{\theta_s}{\sigma_s - 1} J_{is}(x), \tag{A.8}$$

11

and

$$\int_{\varphi^*_{ijs}}^{\infty} p_{ijs}(\varphi)^{1-\sigma_s}\, dG_{is}(\varphi) = \int_{\varphi^*_{ijs}}^{\infty} p_{ijs}(\varphi^*_{ijs})^{1-\sigma_s} \left(\frac{\varphi}{\varphi^*_{ijs}}\right)^{\sigma_s-1} dG_{is}(\varphi)$$

$$= p_{ijs}(\varphi^*_{ijs})^{1-\sigma_s} \int_{\varphi^*_{ijs}}^{\infty} \left(\frac{\varphi}{\varphi^*_{ijs}}\right)^{\sigma_s-1} dG_{is}(\varphi)$$

$$= p_{ijs}(\varphi^*_{ijs})^{1-\sigma_s}[J_{is}(\varphi^*_{ijs}) + 1 - G_{is}(\varphi^*_{ijs})] \quad \text{from (A.4)}$$

$$= \left[\frac{w_i^{-1/(\sigma_s-1)} w_j^{\sigma_s/(\sigma_s-1)}}{\rho_s \varphi^*_{jjs}} \left(\frac{f_{ijs}}{f_{jjs}}\right)^{1/(1-\sigma_s)}\right]^{1-\sigma_s} [J_{is}(\varphi^*_{ijs}) + 1 - G_{is}(\varphi^*_{ijs})] \text{ from (A.7)}$$

$$= \frac{w_i w_j^{-\sigma_s} f_{ijs}}{\rho_s^{1-\sigma_s} \varphi^{*1-\sigma_s}_{jjs} f_{jjs}} \left(\frac{\theta_s}{\sigma_s - 1}\right) J_{is}(\varphi^*_{ijs}) \text{ from (A.8)}$$

$$= \frac{w_i w_j^{-\sigma_s} f_{ijs}}{\rho_s^{1-\sigma_s} \varphi^{*1-\sigma_s}_{jjs} f_{jjs}} \left(\frac{\theta_s}{\sigma_s - 1}\right) \frac{\delta_{js}^{-1} w_j f_{jjs} J_{js}(\varphi^*_{jjs})}{\delta_{is}^{-1} w_i f_{ijs}} \phi_{ijs} \text{ from (A.6)}$$

$$= \frac{\theta_s}{\sigma_s - 1} \left(\frac{w_j}{\rho_s \varphi^*_{jjs}}\right)^{1-\sigma_s} \frac{J_{js}(\varphi^*_{jjs})}{\delta_{js}} \delta_{is} \phi_{ijs}. \tag{A.9}$$

Using these properties, we can solve for total revenue

$$R_{ijs} \equiv \int_{\varphi^*_{ijs}}^{\infty} r_{ijs}(\varphi) M_{is} \mu_{is}(\varphi)\, d\varphi$$

$$= \frac{M_{is}}{1 - G_{is}(\varphi^*_{iis})} \int_{\varphi^*_{ijs}}^{\infty} r_{ijs}(\varphi)\, dG_{is}(\varphi) \text{ from (A.1)}$$

$$= \frac{[1 - G_{is}(\varphi^*_{iis})] M_{ise}}{\delta_{is}[1 - G_{is}(\varphi^*_{iis})]} \int_{\varphi^*_{ijs}}^{\infty} p_{ijs}(\varphi) q_{ijs}(\varphi)\, dG_{is}(\varphi)$$

$$= \frac{M_{ise}}{\delta_{is}} \int_{\varphi^*_{ijs}}^{\infty} p_{ijs}(\varphi) \frac{p_{ijs}(\varphi)^{-\sigma_s} \alpha_s w_j L_j}{P_{js}^{1-\sigma_s}}\, dG_{is}(\varphi) \text{ from (4)}$$

$$= \frac{\alpha_s w_j L_j}{P_{js}^{1-\sigma_s}} \frac{M_{ise}}{\delta_{is}} \int_{\varphi^*_{ijs}}^{\infty} p_{ijs}(\varphi)^{1-\sigma_s}\, dG_{is}(\varphi)$$

$$= \alpha_s w_j L_j \frac{\frac{M_{ise}}{\delta_{is}} \int_{\varphi^*_{ijs}}^{\infty} p_{ijs}(\varphi)^{1-\sigma_s}\, dG_{is}(\varphi)}{\sum_{k=1,2} \frac{M_{kse}}{\delta_{ks}} \int_{\varphi^*_{kjs}}^{\infty} p_{kjs}(\varphi)^{1-\sigma_s}\, dG_{ks}(\varphi)} \text{ from (9)}$$

$$= \alpha_s w_j L_j \frac{\frac{M_{ise}}{\delta_{is}} \frac{\theta_s}{\sigma_s-1} \left(\frac{w_j}{\rho_s \varphi^*_{jjs}}\right)^{1-\sigma_s} \frac{J_{js}(\varphi^*_{jjs})}{\delta_{js}} \delta_{is} \phi_{ijs}}{\sum_{k=1,2} \frac{M_{kse}}{\delta_{ks}} \frac{\theta_s}{\sigma_s-1} \left(\frac{w_j}{\rho_s \varphi^*_{jjs}}\right)^{1-\sigma_s} \frac{J_{js}(\varphi^*_{jjs})}{\delta_{js}} \delta_{ks} \phi_{kjs}} \text{ from (A.9)}$$

12

and it follows that total revenue can be written simply as

$$R_{ijs} = \alpha_s w_j L_j \frac{M_{ise}\phi_{ijs}}{\sum_{k=1,2} M_{kse}\phi_{kjs}}. \tag{15}$$

The Mass of Entrants

We are now in a position to solve for the mass of entrants using the property that labor demand is proportional to the mass of entrants. From $L_{is} = \frac{1}{w_i} \sum_{j=1,2} R_{ijs} = M_{ise} X_{is}$, we obtain

$$\sum_{j=1,2} R_{ijs} = w_i M_{ise} X_{is}$$

$$\sum_{j=1,2} \alpha_s w_j L_j \frac{M_{ise}\phi_{ijs}}{\sum_{k=1,2} M_{kse}\phi_{kjs}} = w_i M_{ise} X_{is} \text{ from (15)}$$

from which it follows that

$$\sum_{j=1,2} \alpha_s w_j L_j \frac{\phi_{ijs}}{\sum_{k=1,2} M_{kse}\phi_{kjs}} = w_i X_{is}. \tag{16}$$

Now $\phi_{ijs} = \frac{\delta_{js} f_{ijs}}{\delta_{is} f_{jjs}} \left(\frac{b_{is}}{b_{js}}\right)^{\theta_s} T_{ijs}^{-\theta_s} \left(\frac{w_i}{w_j}\right)^{1-\theta_s/\rho_s}$ and $T_{ijs} \equiv \tau_{ijs} \left(\frac{f_{ijs}}{f_{jjs}}\right)^{1/(\sigma_s-1)}$ imply that $T_{iis} = 1$ and $\phi_{iis} = 1$. Thus equation (16) can be written out as

$$\frac{\alpha_s w_1 L_1}{G_{1s}} + \frac{\alpha_s L_2}{G_{2s}}\phi_{12s} = w_1 X_{1s}$$

$$\frac{\alpha_s w_1 L_1}{G_{1s}}\phi_{21s} + \frac{\alpha_s L_2}{G_{2s}} = X_{2s}$$

where

$$G_{1s} \equiv M_{1se} + M_{2se}\phi_{21s}$$

$$G_{2s} \equiv M_{1se}\phi_{12s} + M_{2se}.$$

Written in matrix form, these systems of linear equations become

$$\begin{pmatrix} 1 & \phi_{12s} \\ \phi_{21s} & 1 \end{pmatrix} \begin{pmatrix} \frac{\alpha_s w_1 L_1}{G_{1s}} \\ \frac{\alpha_s L_2}{G_{2s}} \end{pmatrix} = \begin{pmatrix} w_1 X_{1s} \\ X_{2s} \end{pmatrix}$$

13

$$
\begin{pmatrix} 1 & \phi_{21s} \\ \\ \phi_{12s} & 1 \end{pmatrix} \begin{pmatrix} M_{1se} \\ \\ M_{2se} \end{pmatrix} = \begin{pmatrix} G_{1s} \\ \\ G_{2s} \end{pmatrix}.
$$

Solving using Cramer's Rule yields

$$
\begin{aligned}
\frac{\alpha_s w_1 L_1}{G_{1s}} &= \frac{1}{\Delta_s}(w_1 X_{1s} - \phi_{12s} X_{2s}) \\
\frac{\alpha_s L_2}{G_{2s}} &= \frac{1}{\Delta_s}(X_{2s} - \phi_{21s} w_1 X_{1s})
\end{aligned}
\tag{A.9}
$$

where $\Delta_s \equiv 1 - \phi_{12s}\phi_{21s} > 0$ is the common determinant and

$$
\begin{aligned}
M_{1se} &= \frac{1}{\Delta_s}(G_{1s} - \phi_{21s}G_{2s}) \\
&= \frac{1}{\Delta_s}\left(\frac{\alpha_s w_1 L_1 \Delta_s}{w_1 X_{1s} - \phi_{12s} X_{2s}} - \phi_{21s}\frac{\alpha_s L_2 \Delta_s}{X_{2s} - \phi_{21s} w_1 X_{1s}} \right).
\end{aligned}
$$

Thus the mass of entrants is given by

$$
M_{1se} = \alpha_s \left(\frac{w_1 L_1}{w_1 X_{1s} - \phi_{12s} X_{2s}} - \frac{\phi_{21s} L_2}{X_{2s} - \phi_{21s} w_1 X_{1s}} \right)
\tag{17}
$$

where

$$
\phi_{12s} = \frac{\delta_{2s} f_{12s}}{\delta_{1s} f_{22s}} \left(\frac{b_{1s}}{b_{2s}} \right)^{\theta_s} T_{12s}^{-\theta_s} w_1^{1-\theta_s/\rho_s}
$$

and

$$
\phi_{21s} = \frac{\delta_{1s} f_{21s}}{\delta_{2s} f_{11s}} \left(\frac{b_{2s}}{b_{1s}} \right)^{\theta_s} T_{21s}^{-\theta_s} w_1^{-1+\theta_s/\rho_s}.
$$

Equation (17) defines M_{1se} as a function of w_1, T_{12s} and T_{21s}, and can be written in function form as $M_{1se}(w_1, T_{12s}, T_{21s})$. To determine the properties of this function, we calculate the partial derivatives. Given $\theta_s > \sigma_s - 1 = \rho_s \sigma_s > \rho_s$, we obtain $\frac{\partial \phi_{12s}}{\partial w_1} < 0$, $\frac{\partial \phi_{12s}}{\partial T_{12s}} < 0$, $\frac{\partial \phi_{21s}}{\partial w_1} > 0$, $\frac{\partial \phi_{21s}}{\partial T_{21s}} < 0$,

and it follows that

$$
\begin{aligned}
\frac{\partial M_{1se}}{\partial w_1} &= \alpha_s \left[\frac{(w_1 X_{1s} - \phi_{12s} X_{2s})L_1 - w_1 L_1 (X_{1s} - \frac{\partial \phi_{12s}}{\partial w_1} X_{2s})}{(w_1 X_{1s} - \phi_{12s} X_{2s})^2} \right. \\
&\qquad \left. - \frac{(X_{2s} - \phi_{21s} w_1 X_{1s})\frac{\partial \phi_{21s}}{\partial w_1} L_2 + \phi_{21s} L_2 (\phi_{21s} + \frac{\partial \phi_{21s}}{\partial w_1} w_1)X_{1s}}{(X_{2s} - \phi_{21s} w_1 X_{1s})^2} \right] \\
&= \alpha_s \left[\frac{-\phi_{12s} X_{2s} L_1 + w_1 L_1 \frac{\partial \phi_{12s}}{\partial w_1} X_{2s}}{(w_1 X_{1s} - \phi_{12s} X_{2s})^2} - \frac{X_{2s} \frac{\partial \phi_{21s}}{\partial w_1} L_2 + (\phi_{21s})^2 L_2 X_{1s}}{(X_{2s} - \phi_{21s} w_1 X_{1s})^2} \right] < 0 \\
\frac{\partial M_{1se}}{\partial T_{12s}} &= \alpha_s \left[\frac{(w_1 X_{1s} - \phi_{12s} X_{2s})0 + w_1 L_1 \frac{\partial \phi_{12s}}{\partial T_{12s}} X_{2s}}{(w_1 X_{1s} - \phi_{12s} X_{2s})^2} - 0 \right] \\
&= \alpha_s \left[\frac{w_1 L_1 \frac{\partial \phi_{12s}}{\partial T_{12s}} X_{2s}}{(w_1 X_{1s} - \phi_{12s} X_{2s})^2} \right] < 0 \\
\frac{\partial M_{1se}}{\partial T_{21s}} &= \alpha_s \left[0 - \frac{(X_{2s} - \phi_{21s} w_1 X_{1s})\frac{\partial \phi_{21s}}{\partial T_{21s}} L_2 + \phi_{21s} L_2 \frac{\partial \phi_{21s}}{\partial T_{21s}} w_1 X_{1s}}{(X_{2s} - \phi_{21s} w_1 X_{1s})^2} \right] \\
&= -\alpha_s \left[\frac{X_{2s} \frac{\partial \phi_{21s}}{\partial T_{21s}} L_2}{(X_{2s} - \phi_{21s} w_1 X_{1s})^2} \right] > 0.
\end{aligned}
$$

Thus, the function $M_{1se}(w_1, T_{12s}, T_{21s})$ has the properties $\frac{\partial M_{1se}}{\partial w_1} < 0$, $\frac{\partial M_{1se}}{\partial T_{12s}} < 0$ and $\frac{\partial M_{1se}}{\partial T_{21s}} > 0$.

Equilibrium Cut-off Productivities

Having found the equilibrium wage rate w_1, we can now solve for the equilibrium cut-off productivities. Writing out the free entry conditions $\sum_{j=1,2} w_i f_{ijs} J_{is}(\varphi_{ijs}^*) = \delta_{is} w_i F_{is}$, we obtain

$$
\begin{aligned}
\frac{w_1 f_{11s}}{\delta_{1s}} J_{1s}(\varphi_{11s}^*) + \frac{w_1 f_{12s}}{\delta_{1s}} J_{1s}(\varphi_{12s}^*) &= w_1 F_{1s} \\
\frac{f_{21s}}{\delta_{2s}} J_{2s}(\varphi_{21s}^*) + \frac{f_{22s}}{\delta_{2s}} J_{2s}(\varphi_{22s}^*) &= F_{2s}.
\end{aligned}
$$

Writing out the relative expected profit conditions $\phi_{ijs} = \frac{\delta_{js} w_i f_{ijs} J_{is}(\varphi_{ijs}^*)}{\delta_{is} w_j f_{jjs} J_{js}(\varphi_{jjs}^*)}$, we obtain

$$
\begin{aligned}
\phi_{12s} &= \frac{\delta_{2s} w_1 f_{12s} J_{1s}(\varphi_{12s}^*)}{\delta_{1s} f_{22s} J_{2s}(\varphi_{22s}^*)} \\
\phi_{21s} &= \frac{\delta_{1s} f_{21s} J_{2s}(\varphi_{21s}^*)}{\delta_{2s} w_1 f_{11s} J_{1s}(\varphi_{11s}^*)}.
\end{aligned}
$$

15

Thus the free entry conditions can be rewritten as

$$\frac{f_{21s}}{\phi_{21s}\delta_{2s}}J_{2s}(\varphi_{21s}^*) + \frac{w_1 f_{12s}}{\delta_{1s}}J_{1s}(\varphi_{12s}^*) = w_1 F_{1s}$$

$$\frac{f_{21s}}{\delta_{2s}}J_{2s}(\varphi_{21s}^*) + \frac{w_1 f_{12s}}{\phi_{12s}\delta_{1s}}J_{1s}(\varphi_{12s}^*) = F_{2s}$$

and in matrix form become

$$\begin{pmatrix} \frac{1}{\phi_{21s}} & 1 \\ 1 & \frac{1}{\phi_{12s}} \end{pmatrix} \begin{pmatrix} \frac{f_{21s}J_{2s}(\varphi_{21s}^*)}{\delta_{2s}} \\ \frac{w_1 f_{12s}J_{1s}(\varphi_{12s}^*)}{\delta_{1s}} \end{pmatrix} = \begin{pmatrix} w_1 F_{1s} \\ F_{2s} \end{pmatrix}.$$

Solving using Cramer's Rule yields

$$\frac{w_1 f_{12s}J_{1s}(\varphi_{12s}^*)}{\delta_{1s}} = \frac{\frac{F_{2s}}{\phi_{21s}} - w_1 F_{1s}}{\frac{1}{\phi_{12s}\phi_{21s}} - 1}$$

$$\frac{w_1 f_{12s}}{\delta_{1s}}\frac{\sigma_s - 1}{\theta_s - \sigma_s + 1}\left(\frac{b_{1s}}{\varphi_{12s}^*}\right)^{\theta_s} = \frac{F_{2s}\phi_{12s} - \phi_{12s}\phi_{21s}w_1 F_{1s}}{1 - \phi_{12s}\phi_{21s}}$$

$$\frac{w_1 f_{12s}}{\delta_{1s}}\frac{\sigma_s - 1}{\theta_s - \sigma_s + 1}b_{1s}^{\theta_s}\frac{1 - \phi_{12s}\phi_{21s}}{F_{2s}\phi_{12s} - \phi_{12s}\phi_{21s}w_1 F_{1s}} = \varphi_{12s}^{*\theta_s}.$$

Letting $\gamma_{1s} \equiv b_{1s}^{\theta_s}(\sigma_s - 1)/[\delta_{1s}(\theta_s - \sigma_s + 1)]$, we can write the last expression more simply as

$$\varphi_{12s}^* = \left[\frac{\gamma_{1s}f_{12s}(1 - \phi_{12s}\phi_{21s})}{F_{2s}(\phi_{12s}/w_1) - \phi_{12s}\phi_{21s}F_{1s}}\right]^{1/\theta_s}. \tag{19}$$

Equation (19) shows the export productivity cut-off φ_{12s}^* for country 1 in sector s as a function of the country 1 wage rate w_1. To determine the slope of this function, note that

$$\phi_{12s}\phi_{21s} = \frac{1}{(\tau_{12s}\tau_{21s})^{\theta_s}}\left(\frac{f_{11s}f_{22s}}{f_{12s}f_{21s}}\right)^{(\theta_s - \sigma_s + 1)/(\sigma_s - 1)}$$

does not depend on w_1 and

$$\frac{\phi_{12s}}{w_1} = \left[\frac{\delta_{2s}f_{12s}}{\delta_{1s}f_{22s}}\left(\frac{b_{1s}}{b_{2s}}\right)^{\theta_s}T_{12s}^{-\theta_s}w_1^{1-\theta_s/\rho_s}\right]w_1^{-1} = \left[\frac{\delta_{2s}f_{12s}}{\delta_{1s}f_{22s}}\left(\frac{b_{1s}}{b_{2s}}\right)^{\theta_s}T_{12s}^{-\theta_s}\right]w_1^{-\theta_s/\rho_s}$$

is decreasing in w_1. Thus the export productivity cut-off φ_{12s}^* is an unambiguously increasing function of w_1.

16

Proof of Lemma 1

(Part 1) One measure of industrial productivity Φ_{1s}^R is the industrial average of firm productivity φ weighted by each firm's revenue share in the industry and is given by

$$\Phi_{1s}^R \equiv \int_0^\infty \varphi v_{1s}(\varphi)\, d\varphi \quad \text{where} \quad v_{1s}(\varphi) \equiv \frac{\sum_{j=1,2} I(\varphi \geq \varphi_{1js}^*) r_{1js}(\varphi) M_{1s}\mu_{1s}(\varphi)}{\sum_{k=1,2} R_{1ks}}.$$

The function $v_{1s}(\varphi)$ is a revenue-weighted density function for φ since

$$
\begin{aligned}
\int_0^\infty v_{1s}(\varphi)\, d\varphi &= \int_0^\infty \frac{\sum_{j=1,2} I(\varphi \geq \varphi_{1js}^*) r_{1js}(\varphi) M_{1s}\mu_{1s}(\varphi)}{\sum_{k=1,2} R_{1ks}}\, d\varphi \\
&= \frac{1}{\sum_{k=1,2} R_{1ks}} \sum_{j=1,2} \int_0^\infty I(\varphi \geq \varphi_{1js}^*) r_{1js}(\varphi) M_{1s}\mu_{1s}(\varphi)\, d\varphi \\
&= \frac{1}{\sum_{k=1,2} R_{1ks}} \sum_{j=1,2} \int_{\varphi_{1js}^*}^\infty r_{1js}(\varphi) M_{1s}\mu_{1s}(\varphi)\, d\varphi \\
&= \frac{1}{\sum_{k=1,2} R_{1ks}} \sum_{j=1,2} R_{1js} \\
&= 1.
\end{aligned}
$$

From (3), (4), (5), (6) and (8), R_{1js} can be written as

$$
\begin{aligned}
R_{1js} &= \int_{\varphi_{1js}^*}^\infty r_{1js}(\varphi) M_{1s}\mu_{1s}(\varphi)\, d\varphi \\
&= \frac{[1 - G_{1s}(\varphi_{11s}^*)]M_{1se}}{\delta_{1s}} \int_{\varphi_{1js}^*}^\infty r_{1js}(\varphi_{1js}^*) \left(\frac{\varphi}{\varphi_{1js}^*}\right)^{\sigma_s - 1} \frac{g_{1s}(\varphi)}{1 - G_{1s}(\varphi_{11s}^*)}\, d\varphi \text{ from (A.1, A.2)} \\
&= \frac{M_{1se}\sigma_s w_1 f_{1js}}{\delta_{1s}} \int_{\varphi_{1js}^*}^\infty \left(\frac{\varphi}{\varphi_{1js}^*}\right)^{\sigma_s - 1} g_{1s}(\varphi)\, d\varphi \text{ from (6)} \\
&= \frac{M_{1se}\sigma_s w_1 f_{1js}}{\delta_{1s}} \frac{\theta_s b_{1s}^{\theta_s}}{\varphi_{1js}^{*\sigma_s - 1}} \int_{\varphi_{1js}^*}^\infty \varphi^{\sigma_s - 1 - \theta_s - 1}\, d\varphi \\
&= \frac{M_{1se}\sigma_s w_1 f_{1js}}{\delta_{1s}} \frac{\theta_s b_{1s}^{\theta_s}}{\varphi_{1js}^{*\sigma_s - 1}} \left(\frac{-\varphi_{1js}^{*\sigma_s - 1 - \theta_s}}{\sigma_s - 1 - \theta_s}\right) \\
&= \frac{M_{1se}\sigma_s w_1 b_{1s}^{\theta_s}\theta_s}{\delta_{1s}(\theta_s - \sigma_s + 1)} \left(\frac{f_{1js}}{\varphi_{1js}^{*\theta_s}}\right).
\end{aligned}
$$

By following similar steps, we can rewrite Φ_{1s}^R as

$$
\begin{aligned}
\Phi_{1s}^R &= \int_0^\infty \varphi \frac{\sum_{j=1,2} I(\varphi \geq \varphi_{1js}^*) r_{1js}(\varphi) M_{1s}\mu_{1s}(\varphi)}{\sum_{k=1,2} R_{1ks}} d\varphi \\
&= \frac{1}{\sum_{k=1,2} R_{1ks}} \sum_{j=1,2} \int_{\varphi_{1js}^*}^\infty \varphi \, r_{1js}(\varphi) \, M_{1s}\mu_{1s}(\varphi) \, d\varphi \\
&= \frac{1}{\sum_{k=1,2} R_{1ks}} \left[\sum_{j=1,2} \frac{M_{1se}\sigma_s w_1 f_{1js}}{\delta_{1s}} \int_{\varphi_{1js}^*}^\infty \varphi \left(\frac{\varphi}{\varphi_{1js}^*} \right)^{\sigma_s - 1} g_{1s}(\varphi) \, d\varphi \right] \\
&= \frac{1}{\sum_{k=1,2} R_{1ks}} \left[\sum_{j=1,2} \frac{M_{1se}\sigma_s w_1 f_{1js}}{\delta_{1s}} \frac{\theta_s b_{1s}^{\theta_s}}{\varphi_{1js}^{*\sigma_s - 1}} \int_{\varphi_{1js}^*}^\infty \varphi^{\sigma_s - \theta_s - 1} d\varphi \right] \\
&= \frac{1}{\sum_{k=1,2} R_{1ks}} \left[\sum_{j=1,2} \frac{M_{1se}\sigma_s w_1 f_{1js}}{\delta_{1s}} \frac{\theta_s b_{1s}^{\theta_s}}{\varphi_{1js}^{*\sigma_s - 1}} \left(\frac{-\varphi_{1js}^{*\sigma_s - \theta_s - 1 + 1}}{\sigma_s - \theta_s - 1 + 1} \right) \right] \\
&= \left[\sum_{k=1,2} \frac{M_{1se}\sigma_s w_1 b_{1s}^{\theta_s} \theta_s}{\delta_{1s}(\theta_s - \sigma_s + 1)} \left(\frac{f_{1ks}}{\varphi_{1ks}^{*\theta_s}} \right) \right]^{-1} \left[\frac{M_{1se}\sigma_s w_1 b_{1s}^{\theta_s} \theta_s}{\delta_{1s}(\theta_s - \sigma_s)} \sum_{j=1,2} \left(\frac{f_{1js}}{\varphi_{1js}^{*\theta_s}} \right) \varphi_{1js}^* \right] \\
&= \frac{\theta_s - \sigma_s + 1}{\theta_s - \sigma_s} \left[\sum_{j=1,2} \frac{\left(f_{1js}/\varphi_{1js}^{*\theta_s} \right)}{\sum_{k=1,2} \left(f_{1ks}/\varphi_{1ks}^{*\theta_s} \right)} \varphi_{1js}^* \right] \\
&= \xi_s \left[h_{11s}\varphi_{11s}^* + h_{12s}\varphi_{12s}^* \right],
\end{aligned}
$$

where $\xi_s \equiv (\theta_s - \sigma_s + 1)/(\theta_s - \sigma_s) > 0$ holds given our assumption that $\theta_s > \sigma_s$,

$$
h_{11s} \equiv \frac{1}{1 + z_s}, \quad h_{12s} \equiv \frac{z_s}{1 + z_s} \quad \text{and} \quad z_s \equiv \frac{f_{12s}}{f_{11s}} \left(\frac{\varphi_{11s}^*}{\varphi_{12s}^*} \right)^{\theta_s}.
$$

Notice that

$$
\frac{\partial h_{11s}}{\partial \varphi_{11s}^*} = -\frac{\partial h_{12s}}{\partial \varphi_{11s}^*} = -\left(\frac{1}{1 + z_s} \right) \left(\frac{z_s}{1 + z_s} \right) \left(\frac{\theta_s}{\varphi_{11s}^*} \right) = -h_{11s}h_{12s} \left(\frac{\theta_s}{\varphi_{11s}^*} \right) < 0
$$

and

$$
\frac{\partial h_{11s}}{\partial \varphi_{12s}^*} = -\frac{\partial h_{12s}}{\partial \varphi_{12s}^*} = \left(\frac{1}{1 + z_s} \right) \left(\frac{z_s}{1 + z_s} \right) \left(\frac{\theta_s}{\varphi_{12s}^*} \right) = h_{11s}h_{12s} \left(\frac{\theta_s}{\varphi_{12s}^*} \right) > 0.
$$

18

Using these results, we can calculate the partial derivatives of Φ_{1s}^R with respect to the domestic productivity cut-off and the export productivity cut-off:

$$
\begin{aligned}
\frac{\partial \Phi_{1s}^R}{\partial \varphi_{11s}^*} &= \xi_s \left[h_{11s} + \frac{\partial h_{11s}}{\partial \varphi_{11s}^*} (\varphi_{11s}^* - \varphi_{12s}^*) \right] \\
&= \xi_s \left[h_{11s} - h_{11s} h_{12s} \left(\frac{\theta_s}{\varphi_{11s}^*} \right) (\varphi_{11s}^* - \varphi_{12s}^*) \right] \\
&= \xi_s h_{11s} \left[1 + h_{12s} \theta_s \left(\frac{\varphi_{12s}^*}{\varphi_{11s}^*} - 1 \right) \right] > 0
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\partial \Phi_{1s}^R}{\partial \varphi_{12s}^*} &= \xi_s \left[\frac{\partial h_{11s}}{\partial \varphi_{12s}^*} (\varphi_{11s}^* - \varphi_{12s}^*) + h_{12s} \right] \\
&= \xi_s \left[h_{11s} h_{12s} \left(\frac{\theta_s}{\varphi_{12s}^*} \right) (\varphi_{11s}^* - \varphi_{12s}^*) + h_{12s} \right] \\
&= \xi_s h_{12s} \left[h_{11s} \theta_s \left(\frac{\varphi_{11s}^*}{\varphi_{12s}^*} - 1 \right) + 1 \right].
\end{aligned}
$$

The free entry condition

$$
\frac{f_{11s}}{(\varphi_{11s}^*)^{\theta_s}} + \frac{f_{12s}}{(\varphi_{12s}^*)^{\theta_s}} = \frac{F_{1s}}{\gamma_{1s}}
$$

determines φ_{11s}^* as an implicit function of φ_{12s}^*. We can solve for its derivative by totally differentiating the free entry condition. This yields $-f_{11s} \theta_s \varphi_{11s}^{*-\theta_s-1} d\varphi_{11s}^* - f_{12s} \theta_s \varphi_{12s}^{*-\theta_s-1} d\varphi_{12s}^* = 0$ and rearranging terms, we obtain the derivative

$$
\frac{d\varphi_{11s}^*}{d\varphi_{12s}^*} = -\frac{f_{12s}}{f_{11s}} \left(\frac{\varphi_{11s}^*}{\varphi_{12s}^*} \right)^{\theta_s+1} = -z_s \left(\frac{\varphi_{11s}^*}{\varphi_{12s}^*} \right).
$$

It follows that

$$
\begin{aligned}
\frac{\partial \Phi_{1s}^R}{\partial \varphi_{11s}^*} \frac{d\varphi_{11s}^*}{d\varphi_{12s}^*} &= -\xi_s h_{11s} \left[1 + h_{12s} \theta_s \left(\frac{\varphi_{12s}^*}{\varphi_{11s}^*} - 1 \right) \right] z_s \left(\frac{\varphi_{11s}^*}{\varphi_{12s}^*} \right) \\
&= -\xi_s (1 - h_{11s}) \left[\frac{\varphi_{11s}^*}{\varphi_{12s}^*} + h_{12s} \theta_s \left(1 - \frac{\varphi_{11s}^*}{\varphi_{12s}^*} \right) \right] \\
&= -\xi_s h_{12s} \left[\frac{\varphi_{11s}^*}{\varphi_{12s}^*} + h_{12s} \theta_s \left(1 - \frac{\varphi_{11s}^*}{\varphi_{12s}^*} \right) \right].
\end{aligned}
$$

We can now express Φ_{1s}^R as a function of φ_{12s}^* and take its derivative:

$$
\begin{aligned}
\frac{d\Phi_{1s}^R}{d\varphi_{12s}^*} &= \frac{\partial \Phi_{1s}^R}{\partial \varphi_{11s}^*} \frac{d\varphi_{11s}^*}{d\varphi_{12s}^*} + \frac{\partial \Phi_{1s}^R}{\partial \varphi_{12s}^*} \\
&= -\xi_s h_{12s} \left[\frac{\varphi_{11s}^*}{\varphi_{12s}^*} + h_{12s}\theta_s \left(1 - \frac{\varphi_{11s}^*}{\varphi_{12s}^*} \right) \right] + \xi_s h_{12s} \left[h_{11s}\theta_s \left(\frac{\varphi_{11s}^*}{\varphi_{12s}^*} - 1 \right) + 1 \right] \\
&= -\xi_s h_{12s} \left[\left(\frac{\varphi_{11s}^*}{\varphi_{12s}^*} - 1 \right) + (h_{12s}\theta_s + h_{11s}\theta_s) \left(1 - \frac{\varphi_{11s}^*}{\varphi_{12s}^*} \right) \right] \\
&= -\xi_s h_{12s} (\theta_s - 1) \left(1 - \frac{\varphi_{11s}^*}{\varphi_{12s}^*} \right) < 0.
\end{aligned}
$$

Therefore, Φ_{1s}^R is a decreasing function in φ_{12s}^*.

Notice that the expression $\Phi_{1s}^R = \xi_s \left[h_{11s}\varphi_{11s}^* + h_{12s}\varphi_{12s}^* \right]$ does not include either variable trade costs τ_{ijs} or w_1. Thus changes in variable trade costs affect Φ_{1s}^R only through the change in φ_{12s}^*.

(Part 2) The second measure of industrial productivity Φ_{1s}^L is industrial labor productivity:

$$
\Phi_{1s}^L \equiv \frac{\sum_{j=1,2} R_{1js}}{\tilde{P}_{1s} L_{1s}} \quad \text{where } \tilde{P}_{1s} = \int_{\varphi_{11s}^*}^{\infty} p_{11s}(\varphi) \mu_{1s}(\varphi) d\varphi.
$$

From $w_1 L_{1s} = \sum_{j=1,2} R_{1js}$ and

$$
\begin{aligned}
\tilde{P}_{1s} &= \int_{\varphi_{11s}^*}^{\infty} \left(\frac{w_1}{\rho_s \varphi} \right) \frac{g_{1s}(\varphi)}{1 - G_{1s}(\varphi_{11s}^*)} d\varphi \\
&= \frac{w_1}{\rho_s (b_{1s}/\varphi_{11s}^*)^{\theta_s}} \int_{\varphi_{11s}^*}^{\infty} \frac{\theta_s b_{1s}^{\theta_s}}{\varphi^{\theta_s + 2}} d\varphi \\
&= \frac{w_1 \theta_s \varphi_{11s}^{*\theta_s}}{\rho_s} \left[\frac{-\varphi_{11s}^{*-(\theta_s+2)+1}}{-(\theta_s + 2) + 1} \right] \\
&= \frac{w_1}{\rho_s \varphi_{11s}^*} \left(\frac{\theta_s}{\theta_s + 1} \right),
\end{aligned}
$$

industrial labor productivity becomes

$$
\Phi_{1s}^L = \left(\frac{\theta_s + 1}{\theta_s} \right) \rho_s \varphi_{11s}^*.
$$

From the free entry condition $\sum_{j=1,2} f_{ijs} \varphi_{ijs}^{*-\theta_s} = F_{is}/\gamma_{is}$, φ_{11s}^* decreases when φ_{12s}^* increases. Therefore, Φ_{1s}^L decreases when φ_{12s}^* increases. Furthermore, a change in variable trade costs only affects industrial productivity Φ_{1s}^L through its influence on φ_{12s}^* since the trade costs τ_{ijs} and the wage w_1 do not appear separately in the above expression for Φ_{1s}^L or the free entry condition.

(Part 3) Another measure of industrial productivity Φ_{1s}^W is industrial labor productivity calculated using a theoretically consistent "exact" price index:

$$\Phi_{1s}^W \equiv \frac{\sum_{j=1,2} R_{1js}}{P_{1s} L_{1s}}.$$

It is easy to calculate how a change in φ_{12s}^* affects this measure of industrial productivity. Starting from the cut-off productivity condition (6)

$$\frac{r_{11s}(\varphi_{11s}^*)}{\sigma_s} = w_1 f_{11s}$$

$$\frac{p_{11s}(\varphi_{11s}^*)^{1-\sigma_s} \alpha_s w_1 L_1}{P_{1s}^{1-\sigma_s}} = \sigma_s w_1 f_{11s} \text{ from (4)}$$

$$\left(\frac{w_1 \tau_{11s}}{\rho_s \varphi_{11s}^* P_{1s}} \right)^{1-\sigma_s} \alpha_s w_1 L_1 = \sigma_s w_1 f_{11s} \text{ from (5)}$$

$$\left(\frac{w_1}{P_{1s}} \right)^{1-\sigma_s} = \frac{\sigma_s f_{11s}}{\alpha_s L_1} (\rho_s \varphi_{11s}^*)^{1-\sigma_s}$$

$$\frac{w_1}{P_{1s}} = \left(\frac{\sigma_s f_{11s}}{\alpha_s L_1} \right)^{1/(1-\sigma_s)} \rho_s \varphi_{11s}^*$$

and then using $w_1 L_{1s} = \sum_{j=1,2} R_{1js}$, we obtain

$$\Phi_{1s}^W \equiv \frac{\sum_{j=1,2} R_{1js}}{P_{1s} L_{1s}} = \frac{w_1}{P_{1s}} = \left(\frac{\alpha_s L_1}{\sigma_s f_{11s}} \right)^{1/(\sigma_s - 1)} \rho_s \varphi_{11s}^*.$$

From the free entry condition $\sum_{j=1,2} f_{ijs} \varphi_{ijs}^{*-\theta_s} = F_{is}/\gamma_{is}$, φ_{11s}^* decreases when φ_{12s}^* increases. Therefore, Φ_{1s}^W decreases when φ_{12s}^* increases. Furthermore, a change in variable trade costs only affects Φ_{1s}^W through its influence on φ_{12s}^* since the trade costs τ_{ijs} and the wage w_1 do not appear separately in the above expression for Φ_{1s}^W or the free entry condition.

Finally, we derive the welfare formula (23) for the representative consumer in country 1 who supplies one unit of labor. Since her income is w_1, her aggregate consumption over varieties in sector s is

$$C_{1s} = \frac{\alpha_s w_1}{P_{1s}}.$$

From the utility function (1) and $\Phi_{1s}^W = w_1/P_{1s}$, her utility is written as:

$$
\begin{aligned}
U &= \left(\frac{\alpha_A w_1}{P_{1A}}\right)^{\alpha_A} \left(\frac{\alpha_B w_1}{P_{1B}}\right)^{\alpha_B} \\
&= \left(\alpha_A \Phi_{1A}^W\right)^{\alpha_A} \left(\alpha_B \Phi_{1B}^W\right)^{\alpha_B}.
\end{aligned}
$$

Footnote 5

Local consumer demand for an individual firm's product is given by

$$
\begin{aligned}
q_{11s}(\varphi) &= \frac{p_{11s}(\varphi)^{-\sigma_s} \alpha_s w_1 L_1}{P_{1s}^{1-\sigma_s}} \\
&= \left(\frac{w_1 \tau_{11s}}{\rho_s \varphi}\right)^{-\sigma_s} \frac{\alpha_s w_1 L_1}{P_{1s}^{1-\sigma_s}} \\
&= (\rho_s \varphi)^{\sigma_s} \left(\frac{w_1}{P_{1s}}\right)^{1-\sigma_s} \alpha_s L_1 \\
&= (\rho_s \varphi)^{\sigma_s} \left(\Phi_{1s}^W\right)^{1-\sigma_s} \alpha_s L_1.
\end{aligned}
$$

Footnote 6

The weighted average productivity measure in Melitz (2003) satisfies

$$
\begin{aligned}
\tilde{\varphi}_{1s} &\equiv \left[\int_{\varphi_{11s}^*}^{\infty} \varphi^{\sigma_s - 1} \mu_{1s}(\varphi) d\varphi\right]^{1/(\sigma_s - 1)} \\
&= \left[\int_{\varphi_{11s}^*}^{\infty} \varphi^{\sigma_s - 1} \frac{g_{1s}(\varphi)}{1 - G_{1s}(\varphi_{11s}^*)} d\varphi\right]^{1/(\sigma_s - 1)} \\
&= \left[\int_{\varphi_{11s}^*}^{\infty} \varphi^{\sigma_s - 1} \frac{\theta_s b_{1s}^{\theta_s}}{\varphi^{\theta_s + 1} (b_{1s}/\varphi_{11s}^*)^{\theta_s}} d\varphi\right]^{1/(\sigma_s - 1)} \\
&= \left[\varphi_{11s}^{*\theta_s} \theta_s \int_{\varphi_{11s}^*}^{\infty} \varphi^{\sigma_s - 1 - \theta_s - 1} d\varphi\right]^{1/(\sigma_s - 1)} \\
&= \left[\varphi_{11s}^{*\theta_s} \theta_s \left(\frac{-\varphi_{11s}^{*\sigma_s - 1 - \theta_s - 1 + 1}}{\sigma_s - 1 - \theta_s - 1 + 1}\right)\right]^{1/(\sigma_s - 1)} \\
&= \left[\frac{\theta_s}{\theta_s - \sigma_s + 1} \varphi_{11s}^{*\sigma_s - 1}\right]^{1/(\sigma_s - 1)} \\
&= \left[\frac{\theta_s}{\theta_s - \sigma_s + 1}\right]^{1/(\sigma_s - 1)} \varphi_{11s}^*.
\end{aligned}
$$

How the Competitiveness Curve Shifts Due to Trade Liberalization

We can determine how the competitiveness curve shifts due to unilateral trade liberalization by country 1 in sector A. When τ_{21A} decreases holding all other parameter values fixed and holding φ_{12A}^* fixed, the free entry condition

$$f_{11A}\varphi_{11A}^{*-\theta_A} + f_{12A}\varphi_{12A}^{*-\theta_A} = F_{1A}/\gamma_{1A}$$

implies that φ_{11A}^* remains fixed. The other free entry condition

$$f_{21A}\varphi_{21A}^{*-\theta_A} + f_{22A}\varphi_{22A}^{*-\theta_A} = F_{2A}/\gamma_{2A}$$

implies that φ_{21A}^* and φ_{22A}^* move in opposite directions. Because the cut-off productivity levels satisfy

$$\varphi_{12A}^*\varphi_{21A}^* = \left(T_{12A}w_1^{1/\rho_A}\varphi_{22A}^*\right)\left(T_{21A}w_1^{-1/\rho_A}\varphi_{11A}^*\right) = T_{12A}\varphi_{22A}^*T_{21A}\varphi_{11A}^*,$$

φ_{12A}^* is fixed, T_{12A} is fixed, T_{21A} decreases and φ_{11A}^* is fixed, it follows that φ_{21A}^* decreases if φ_{22A}^* decreases. But this is impossible since φ_{21A}^* and φ_{22A}^* move in opposite directions. If φ_{22A}^* remains fixed, then φ_{21A}^* decreases. This is also impossible. The only possibility left is that φ_{22A}^* increases and φ_{21A}^* decreases. Thus, a decrease in τ_{21A} holding φ_{12A}^* fixed leads to φ_{11A}^* remaining fixed, φ_{22A}^* increasing and φ_{21A}^* decreasing. But then the cut-off productivity condition $\varphi_{12A}^* = T_{12A}w_1^{1/\rho_A}\varphi_{22A}^*$ implies that w_1 must decrease. It follows that when τ_{21A} decreases holding φ_{12A}^* fixed, then the wage rate w_1 must decrease and the competitiveness curve shifts down.

Balanced Trade

From $\sum_{j=1,2} R_{1js} = w_1 L_{1s}$ and $\sum_{j=1,2} E_{1js} = E_{11s} + E_{12s} = \alpha_s w_1 L_1$, the excess exports of sector s for country 1 is

$$
\begin{aligned}
\left(\sum_{j=1,2} R_{1js} - E_{11s}\right) - E_{12s} &= w_1 L_{1s}(\cdot) - (\alpha_s w_1 L_1 - E_{12s}) - E_{12s} \\
&= w_1 \alpha_s \left(\frac{L_{1s}(\cdot)}{\alpha_s} - L_1\right).
\end{aligned}
$$

Summing up for both industries, we obtain that the balanced trade condition is equivalent to the labor market clearing condition:

$$
\begin{aligned}
0 = \sum_{s=A,B} \left[\left(\sum_{j=1,2} R_{1js} - E_{11s} \right) - E_{12s} \right] &= \sum_{s=A,B} \left[w_1 \alpha_s \left(\frac{L_{1s}(\cdot)}{\alpha_s} - L_1 \right) \right] \\
&= w_1 \left[\alpha_A \left(\frac{L_{1A}(\cdot)}{\alpha_A} - L_1 \right) + \alpha_B \left(\frac{L_{1B}(\cdot)}{\alpha_B} - L_1 \right) \right] \\
&= w_1 \left[L_{1A}(\cdot) + L_{1B}(\cdot) - L_1 \right].
\end{aligned}
$$

Multilateral Trade Liberalization

With symmetric countries and $w_1 = w_2 = 1$,

$$
\begin{aligned}
L_{1s} &= M_{1se} X_{1s} \\
&= \alpha_s \left(\frac{w_1 L_1}{w_1 X_{1s} - \phi_{12s} X_{2s}} - \frac{\phi_{21s} L_2}{X_{2s} - \phi_{21s} w_1 X_{1s}} \right) X_{1s} \\
&= \alpha_s \left(\frac{L_1}{X_s - \phi_s X_s} - \frac{\phi_s L_1}{X_s - \phi_s X_s} \right) X_s \\
&= \alpha_s L_1,
\end{aligned}
$$

and

$$
\begin{aligned}
\varphi_{12s}^* &= \left[\frac{\gamma_{1s} f_{12s} (1 - \phi_{12s} \phi_{21s})}{F_{2s}(\phi_{12s}/w_1) - \phi_{12s} \phi_{21s} F_{1s}} \right]^{1/\theta_s} \\
\varphi_{12A}^* &= \left[\frac{\gamma_{1A} f_{xA} (1 - \phi_A \phi_A)}{F_A(\phi_A/1) - \phi_A \phi_A F_A} \right]^{1/\theta_A} \\
&= \left[\frac{\gamma_{1A} f_{xA} (1 - \phi_A)(1 + \phi_A)}{F_A \phi_A (1 - \phi_A)} \right]^{1/\theta_A},
\end{aligned}
$$

from which it follows that

$$
\varphi_{12A}^* = \left[\frac{\gamma_{1A} f_{xA}}{F_A} \left(1 + \frac{1}{\phi_A} \right) \right]^{1/\theta_A}. \tag{27}
$$

Since

$$
\phi_{ijs} = \frac{\delta_{js} f_{ijs}}{\delta_{is} f_{jjs}} \left(\frac{b_{is}}{b_{js}} \right)^{\theta_s} T_{ijs}^{-\theta_s} \left(\frac{w_i}{w_j} \right)^{1 - \theta_s/\rho_s}
$$

simplifies to

$$
\phi_A = \frac{f_{xA}}{f_A} T_A^{-\theta_A},
$$

a decrease in T_A leads to an increase in ϕ_A and a decrease in φ_{12A}^* for fixed $w_1 = 1$.

Finally, we show the labor demand curve of industry A, L_{1A}, becomes flatter in response to liberalization of industry A as illustrated in Figure 6. To draw the labor demand curve, we allow w_1 can be different from one; therefore ϕ_{12s} can be different from ϕ_{21s}. The labor demand by sector s in country 1 is

$$
\begin{aligned}
L_{1s} = M_{1se}X_s &= \alpha_s X_s \left[\frac{w_1 L_1}{w_1 X_s - \phi_{12s}X_s} - \frac{\phi_{21s}L_1}{X_s - \phi_{21s}w_1 X_s} \right] \\
&= \alpha_s L_1 \left[\frac{1}{1 - \phi_{12s}/w_1} - \left(\frac{1}{w_1}\right) \frac{\phi_{21s}w_1}{1 - \phi_{21s}w_1} \right].
\end{aligned}
$$

Notice that $w_1 > \phi_{12s}$ and $1 > \phi_{21s}w_1$ are required for an interior solution from (A.9).

Let $\varpi \equiv w_1^{\theta_s/\rho_s}$ and $\kappa_s \equiv \tau_s^{-\theta_s} \left(\frac{f_s}{f_{xs}}\right)^{(\theta_s - \sigma_s + 1)/(\sigma_s - 1)}$. Then ϕ_{12s} and ϕ_{21s} become

$$
\frac{\phi_{12s}}{w_1} = \frac{\kappa_s}{\varpi} < 1 \quad \text{and} \quad \phi_{21s}w_1 = \kappa_s \varpi < 1,
$$

from which it follows that $\kappa_s < 1$. By substituting these into the labor demand, we obtain

$$
L_{1s} = \alpha_s L_1 \left[\frac{\varpi}{\varpi - \kappa_s} - \left(\frac{1}{\varpi^{\sigma_s/\theta_s}}\right) \frac{\kappa_s \varpi}{1 - \kappa_s \varpi} \right].
$$

We take its derivative with respect to κ_s

$$
\begin{aligned}
\frac{\partial L_{1s}}{\partial \kappa_s} &= \varpi \alpha_s L_1 \left[\frac{1}{(\varpi - \kappa_s)^2} - \left(\frac{1}{\varpi^{\sigma_s/\theta_s}}\right) \frac{1}{(1 - \kappa_s \varpi)^2} \right] \\
&= \frac{\varpi^{1 - \sigma_s/\theta_s} \alpha_s L_1}{(\varpi - \kappa_s)^2} \left[\varpi^{\sigma_s/\theta_s} - \left(\frac{\varpi - \kappa_s}{1 - \kappa_s \varpi}\right)^2 \right].
\end{aligned}
$$

Since $\partial \kappa_s / \partial \tau_s < 0$,

$$
\begin{aligned}
\frac{\partial L_{1s}}{\partial \tau_s} &> 0 \text{ if } LHS(\varpi) \equiv \varpi^{\sigma_s/2\theta_s} < \frac{\varpi - \kappa_s}{1 - \kappa_s \varpi} \equiv RHS(\varpi) \\
\frac{\partial L_{1s}}{\partial \tau_s} &= 0 \text{ if } LHS(\varpi) = RHS(\varpi) \\
\frac{\partial L_{1s}}{\partial \tau_s} &< 0 \text{ if } LHS(\varpi) > RHS(\varpi).
\end{aligned}
$$

25

Since

$$\begin{aligned}
\frac{dLHS(\varpi)}{d\varpi} &= \frac{\sigma_s}{2\theta_s}\varpi^{\sigma_s/2\theta_s - 1} > 0, \\
\frac{d^2 LHS(\varpi)}{d\varpi^2} &= -\frac{\sigma_s}{2\theta_s}\left(\frac{2\theta_s - \sigma_s}{2\theta_s}\right)\varpi^{-(2\theta_s - \sigma_s)/2\theta_s - 1} < 0, \\
\frac{dRHS(\varpi)}{d\varpi} &= \frac{1 - \kappa_s^2}{(1 - \kappa_s\varpi)^2} > 0, \\
\frac{d^2 RHS(\varpi)}{d\varpi^2} &= \frac{2\kappa_s\left(1 - \kappa_s^2\right)}{(1 - \kappa_s\varpi)^3} > 0,
\end{aligned}$$

and

$$\begin{aligned}
LHS(\varpi = 1) &= RHS(\varpi = 1) \\
\frac{dLHS(\varpi = 1)}{d\varpi} &= \frac{\sigma_s}{2\theta_s} < 1 < \frac{1 + \kappa_s}{1 - \kappa_s} = \frac{dRHS(\varpi = 1)}{d\varpi},
\end{aligned}$$

we have

$$\begin{aligned}
LHS(\varpi) &< RHS(\varpi) \text{ if } \varpi > 1 \\
LHS(\varpi) &= RHS(\varpi) \text{ if } \varpi = 1 \\
LHS(\varpi) &> RHS(\varpi) \text{ if } \varpi < 1.
\end{aligned}$$

Since $\varpi = w_1^{\theta_s/\rho_s}$, we obtain

$$\begin{aligned}
\frac{\partial L_{1s}}{\partial \tau_s} &> 0 \text{ for } w_1 > 1 \\
\frac{\partial L_{1s}}{\partial \tau_s} &= 0 \text{ for } w_1 = 1 \\
\frac{\partial L_{1s}}{\partial \tau_s} &< 0 \text{ for } w_1 < 1.
\end{aligned}$$

Therefore, a reduction in τ_A makes L_{1A} flatter and tilt counterclockwise around point E in Figure 6.