Trade and the Internal Labor Markets of Multiproduct Firms

Carsten Eckel and Stephen Yeaple*
University of Munich and Penn State University

Abstract

A large empirical literature has shown that multiproduct firms tend to be larger, more productive, more likely to export, and to pay higher wages than single product firms. The exact source of the advantages of these firms has not been identified, however. In this paper, we attribute the cost advantage of multiproduct firms to their ability to use internal labor markets in a setting of information frictions to better compensate more able workers. Our model can account for many of the stylized facts and can provide an interpretation of the growing wage inequality that has developed with increased international trade.

*PRELIMINARY: Cite with Care

1 Introduction

Economic activity is highly concentrated in large conglomerates. Larger firms tend to produce a wide range of products, tend to display high levels of total factor productivity, pay their workers higher wages, and are more likely to be engaged in international trade and investment (Bernard, Redding, and Schott, 2011). These well documented empirical regularities raise several critical questions. First, what is the source of the competitive advantages of large multiproduct firms? Second, how do these advantages interact to determine the size and scope of these firms? Third, how do changes in product markets caused by lower trade barriers between countries affect the value of these competitive advantages? Finally, why are resources frequently allocated to competing ends within the internal factor markets of large multi-product firms rather than on external factor markets between firms?

In this paper, we develop a model in which multiproduct firms' competitive advantages vis-a-vis single product firms arises exclusively from their ability to overcome labor market frictions. We consider an environment in which workers are heterogeneous in their ability. Prior to employment by a firm, a worker's ability is private information. Subsequent to employment by a firm, a worker's ability is observed by the employer but cannot be verified in a court of law so that an initial employment contract cannot specify a wage as a function of a worker's skill. The advantage of multiproduct firms relative to single product firms in our model lies in the fact that the multiproduct firm has an internal labor market in which different product divisions compete for workers.[1] The existence of this internal labor market allows multiproduct firms to attract the most able workers because it has in place a mechanism for compensating highly skilled workers, while single product firms can only commit to pay a worker a wage that reflects the average skill of its employees.

In our setting, multiproduct firms face an interesting tradeoff in the choice of the

[1]Here we assume that the firm can commit to allow the divisions to compete internally for the workers' services.

number of divisions. On the one hand, adding a division has a positive impact on the Multiproduct firm's aggregate profit as each division generates a positive level of profit. On the other hand, adding a division makes the multiproduct firm's internal labor market more competitive, driving up the firm's marginal cost and lowering profits across all divisions. The firm's optimal scope obtains when the direct effect of adding another division is exactly offset by the inframarginal impact of higher marginal costs.

Our model is able produce many of the stylized facts concerning the features of multiproduct firms relative to those of single product firms. First, multiproduct firms' ability to identify and keep talented workers means that they pay higher wages and exhibit higher labor productivity than single product firms. Second, although multiproduct firms pay higher wages than single product firms, this wage does not entirely reflect worker's ability, allowing these firms to enjoy a rent in the form of lower marginal costs of production relative to single product firms.[2] As such, the sales of the divisions of multiproduct firms will be larger than those of single division firms. Third, although multiproduct firms do not enjoy any inherent productivity advantage over single product firms, their marginal cost advantage allows them to profitability serve foreign markets while single product firms confine their sales to the local market.

We then consider the impact of a multi-lateral trade liberalization. Lower variable costs to international trade lead to a reallocation of resources within multi-product firms away from domestic sales to foreign sales and across firms from single product firms to multiproduct firms. This reallocation has the implication that the real wages of the most skilled workers rise while the real wages of the least talented workers fall. Interestingly, aggregate welfare may rise or fall depending on parameter values. This ambiguous effect of trade on aggregate welfare is due to the inherent labor market imperfections analyzed in the model.

This paper contributes to several strands of the literature. First, the paper contributes to the growing literature that analyzes within firm reallocations in a multiprod-

[2] Silva (2013) wages and labor productivity increasing in the number of divisions managed by multi-product firm.

uct firm setting. As in Eckel and Neary (2010) we analyze a model in which multiproduct firms are those that have technologies that allow them to engage in "flexible manufacturing," firms may introduce additional varieties at progressively higher marginal costs.[3] As in the case of a number of papers in this literature, multiproduct firms in our model internalize the impact of adding product lines. In Feenstra and Ma (2008), and Dhingra (2013) the key internalization issues revolve around cannibalization effects: new products steal demand from existing products, while in Nocke and Yeaple (2014) internalization involves the allocation of a fixed endowment of organizational capital within the internal organizational capital market within a firm. In this paper, firms internalize the endogeneous effect of added product lines on the tightness of an internal labor market. Multiproduct firms are less aggressive adding product lines because doing so bids up the wage that must be paid to their workers.

Second, our paper contributes to the literature on internal versus external labor markets. As in Greenwald (1986), we explore the role of adverse selection in labor markets. As in Greenwald, workers that sort into a "common pool" where their talent cannot be verified are paid a wage that reflects the average talent within the labor force. Unlike Greenwald, we explicitly model the internal labor market as a mechanism for overcoming the problems associated with adverse selection. In contrasting external versus internal labor markets our work is related to a recent literature on the benefit of firm size when labor is allocated within the firm (e.g. Papageorgiou, 2012). In Papageorgiou (2012), larger firms are more diverse in the nature of tasks that they need done. Because workers have idiosyncratic (and initially unknown) task abilities, a large firm offers has a productivity advantage over a small firm and this advantage is partially passed onto its workers in the form of higher wages. In our model, the relevant dimension of firm size is the (endogeneous) number of divisions that it manages as this number determines the "tightness" of its internal labor market and thus its reward to talent.

Our paper also contributes to the literature on the effect of trade on wages in the

[3] Other papers in this literature include Bernard, Redding and Schott (2011); and Mayer, Melitz, and Ottaviano (2012).

context of heterogeneous labor. As in Yeaple (2005) ex ante identical firms choose among different technologies and this choice affects the nature of their work force. We deviate from this setting in two major ways. First, we assume that there are inherent frictions in the labor market. Second, high fixed cost firms have low marginal costs not because they are using technologies for which skilled workers have a comparative advantage but because these firms can commit to compensate workers for at least part of their skill. In fact, from a purely efficiency point of view, firms with endogenously low marginal costs are in fact low productivity producers. While trade leads to a widening income gap in both settings, the aggregate welfare effects are startingly different.[4]

The remainder of this paper is organized into four sections. We begin by specifying the closed economy version of our model in section two and analyze the equilibrium in section three. We focus on how multiproduct firms differ along a number of performance measures in this discussion. In section four, we allow for trade between two identical countries. We extend the model's implications for cross-sectional performance measures, and conclude the section with a comparative static analysis of trade liberalization. In section five, we conclude.

2 Closed Economy Model

The representative consumer's preferences are defined over a composite differentiated good X where the constant elasticity of substitution over varieties of the differentiated good is $\sigma > 1$. There are two factors, capital and labor. Each factor is inelastically supplied by households. The economy's endowment of capital is given by K. We normalize the price of a unit of capital to unity. Labor is supplied by workers who are heterogeneous in terms of their skill z. The distribution of skill in the population of workers, G, is defined on support $[\underline{z}, \infty)$ where $\underline{z} \geq 0$. To develop clean comparative statics, we impose a regularity condition on the distribution of G. Specifically, letting

[4]Another related paper that focuses on labor market frictions (screening technology and search) in a trade setting is Helpman, Itskhoki, and Redding (2010). Unlike that paper, our paper abstracts from complementarity between firm's inherent types and workers skills to focus on the role of firm scope on internal labor markets.

$R(a) = \int_0^a G(z)dz$, we require

$$2\frac{R''(a)}{R'(a)} - \frac{R'''(a)}{R''(a)} - \frac{R'(a)}{R(a)} > 0, \text{ for all } a. \tag{1}$$

This assumption requires the density function associated with G, and given by $g(z)$, to be decreasing sufficiently fast relative to $G(z)$. As shown in the appendix, these restrictions are satisfied in the case of the Pareto distribution. We assume that a worker's ability z is initially observable only to the worker.

Varieties of the differentiated good can be produced by two different types of firms. First, there are single product firms (SPF). These firms pay a fixed capital cost f to introduce a single variety of the differentiated good. SPFs require one unit of effective labor to produce one unit of output. Importantly, single product firms cannot direct observe the ability of individuals in its workforce. Second, there are multiproduct firms (MPF) which are free to produce as many products as they want. Like SPF, MPF must pay a fixed cost f for every variety introduced. Unlike SPF, MPF must pay an additional fixed capital cost f^m, which we think of as confering upon the firm greater organizational capability. This greater organizational capability has two features: (1) it allows the firm to manage multiple products and (2) it allows the firm to screen its workers so as to perfectly observe each worker's skill.[5] As in Eckel and Neary (2010) we assume that each firm has a 'core' variety and that each new variety introduced is "further" away from the core variety than the previous variety. We use ω to index the distance of a variety from the firm's 'core' variety (so the core variety has $\omega = 0$). We capture the relative inefficiency of production of non-core varieties as an effective unit labor requirement given by $\alpha(\omega)$, where $\alpha(0) = 1$, $\alpha'(\omega) > 0$.

Markets in our model display two imperfections. First, as is usual in the trade literature, the differentiated good is produced by monopolistically competitive firms that

[5] The first part of this assumption, that MPF can manage multiple products, is made for expositional convenience and can be relaxed. The second part, that paying f^m allows the firm to observe worker skill, is critical in what follows.

charge a mark-up over marginal costs. We assume that all firms, including MPFs, are too small to have an impact on the aggregate price index. Second and unique to our setting, SPF cannot condition wages paid to individual workers on the basis of their skill. MPFs also cannot write contracts based on a worker's skill ex ante but are able to reward high talent workers because they have competitive internal labor markets. We assume that once a worker has been employed by an MPF, the worker's skill is observed and that the individual divisions (associated with a given variety) can commit to compete with one another for labor within a perfectly competitive internal labor market.[6]

The timing of actions is as follows. First, we assume that firms choose to enter either as SPF or MPF, buying capital in the process. Second, independently of their skill types, workers search in undirected fashion for MPF. Each worker meets a single MPF. Upon meeting their MPF they decide whether to accept employment at the MPF. At that time, no wage is promised. Instead, accepting a job at an MPF is accepting a place in the internal labor market of the MPF. Those that do not accept a job at an MPF will then enter a common labor market for SPF firms. Third, MPF observe the effective units of labor available in their internal labor market and decide how many divisions(varieties) to establish.[7] As the marginal cost of production is rising in the distance from the core variety, this choice is akin to choosing the maximum distance variety ω^d from the core variety to produce. At this time additional fixed costs are also incurred by MPF and the capital market must clear. Fourth, internal labor markets clear at MPF and the common SPF labor market clear. Finally, firms sell their output.

[6]In future versions of this paper, we will buttress this feature of the model by introducing a time element to the model. This will allow us to justify the ability to commit to allow competition between divisions by a desire to maintain the firm's reputation.

[7]This timing assumption makes the model much easier to analyze. It can be justified by appealing to the possibility in the real world that a firm may close a division after individual workers have made firm-specific investments.

3 Equilibrium in the Closed Economy

We solve the model backwards beginning with product market competition. We then conclude the section with an analysis of the model's predictions over the differences between MPF and SPF and a discussion of the features of technology that govern the size of the role of MPF in equilibrium.

Product Market Competition Letting $w(z)$ be the equilibrium wage received by a worker of skill z, aggregate expenditure in the economy is given by

$$E = L \int_0^\infty w(z) dG(z) + K. \tag{2}$$

Expenditure on an individual variety of the differentiated good is then given by

$$x(i) = EP^{\sigma-1} p(i)^{-\sigma},$$

where

$$P = \left(\int_{i \in \Omega} p(i)^{1-\sigma} di \right)^{1-\sigma}$$

is the price index and Ω is the set of varieties available.

As each firm is by assumption too small to effect the aggregate price index and each faces iso-elastic demand for each variety, each firm charges a constant mark-up over marginal cost. If $c(i)$ is the marginal cost of producing variety i, then the optimal price charged by the producer of variety i is $p(i) = \sigma c(i)/(\sigma - 1)$ and the reduced form demand for variety i will be given by

$$x(\omega, c) = (\sigma - 1) A c(i)^{-\sigma} \tag{3}$$

where

$$A = \frac{EP^{\sigma-1}}{\sigma} \left(\frac{\sigma}{\sigma - 1} \right)^{1-\sigma}$$

is the (endogenous) mark-up adjusted level of demand.

MPF's internal labor market At this stage, workers have committed to either work for the multiproduct firm (MPF) that they have met or have entered the common labor market pool and will work for a single product firm (SPF). Suppose that $\widetilde{L}$ units of effective labor has chosen to work for a representative MPF firm and that this labor force is composed of workers that vary in terms of their skill. Once they have become part of the internal labor market of a MPF, a worker's skill will be observed by all of the firm's divisions and there will be a common valuation of the worker's skill given by $w(z)$. As workers productivity is linear in their type, no-arbitrage has the implication that the equilibrium wage by the firm to a worker of skill z can be written: $w(z) = cz$, where c is an endogeneous constant that is determined by internal labor market clearing. In fact, c is the price of a unit of effective labor that clears the internal labor market.

Internal labor market clearing for a firm that has chosen to produce the products on the interval $(0, \omega^d)$ and which has $\widetilde{L}$ of effective labor embedded in its work force requires

$$\int_0^{\omega^d} x(\omega, c)\alpha(\omega)d\omega = \widetilde{L}.$$

where $x(\omega, c)$ is the total output of a variety that is distance ω from its core competency, ω^d is the produced variety that is furthest from the firm's core competence, and $\alpha(\omega)$ is the additional units of effective labor required to produce an unit of a variety that is distance ω from the firm's core competence. Hence, the marginal cost of producing a variety that is distance ω from the firm's core product is $c\alpha(\omega)$. Substituting for $x(\omega, c)$ using (3) allows us to rewrite the internal labor market clearing condition as

$$(\sigma - 1)Ac^{-\sigma} \int_0^{\omega^d} \alpha(\omega)^{1-\sigma} d\omega = \widetilde{L}. \tag{4}$$

Everything else equal, the price of labor in an MPF is decreasing in the supply of labor $\widetilde{L}$, increasing in product market demand A, and increasing in the range of goods produced by the firm ω^d.

MPF's choice of product scope We now consider the firm's choice of the number of varieties to produce. Given firms' optimal pricing strategies, the reduced form profits that a MPF could earn from variety that is distance ω from the core product is given by

$$\pi(\omega) = Ac^{1-\sigma}\alpha(\omega)^{1-\sigma} - f$$

Aggregating over all varieties produced by the MPF and accounting for the initial entry cost, the total profits earned by a MPF are given by

$$\Pi = \max_{\omega^d}\left\{Ac^{1-\sigma}\int_0^{\omega^d}\alpha(\omega)^{1-\sigma}d\omega - \omega^d f - f^m\right\}, \tag{5}$$

where c is endogenous and determined by the firm's internal labor market clearing condition, given by (4). The first-order condition for the optimal choice of product range, ω^d, is

$$\left[Ac(\omega^d)^{1-\sigma}\alpha(\omega^d)^{1-\sigma} - f\right] - (\sigma-1)Ac(\omega^d)^{-\sigma}\int_0^{\omega^d}\alpha(\omega)^{1-\sigma}d\omega\frac{\partial c}{\partial\omega^d} = 0$$

where the term in brackets is the direct effect of having an additional product line and the second term is the inframarginal of the higher labor costs on incumbent product lines. Expanding along one dimension necessarily results in a "cannibalization-like" effect on incumbent products due to the tighter internal labor market conditions. Critically, these costs are internalized by the firm. By totally differentiating (4) with respect to c and ω^d, substituting the resulting expression into the first-order condition, and simplifying, we obtain

$$Ac(\omega^d)^{1-\sigma}\alpha(\omega^d)^{1-\sigma} = \sigma f. \tag{6}$$

The inframarginal effect can be seen at work in this expression as the profits earned by the marginal product line ω^d are strictly positive (i.e. $Ac(\omega^d)^{1-\sigma}\alpha(\omega^d)^{1-\sigma} - f > 0$). Given their captive labor force, firms exploit their monospony power by restricting the number of divisions that they manage. The size of this effect is mediated by σ, which governs the extent to which higher labor costs can be passed onto consumers.

External Labor Markets Having characterized the equilibrium in the MPF and SPF labor markets given an arbitrary assignment of workers into those markets, we now step back to the first part of the period and derive the sorting behavior of individual workers on the basis of their type. To begin, recall that workers are randomly matched with MPFs at the start of the labor market. The workers anticipate the number of products to be managed by the MPF and use this information to infer the wage that they would receive in the internal labor market of the firm, which as noted earlier must be $w_m(z) = cz$ for a worker of ability z. The alternative to becoming employed by the MPF is to reject the job offer and instead to enter the common labor pool for SPF. As SPFs cannot condition their wage on a worker's skill, perfect competition means that they must be paid a wage that reflects their average productivity. Hence, workers in the common labor pool for SPF receiving a wage of $w_s = c_s \overline{z}$, where $\overline{z}$ is the average ability of workers that have chosen to enter the SPF labor market pool and c_s is the endogenous cost of a unit of effective labor to a SPF. In equilibrium a worker's wage as function of her skill is given by

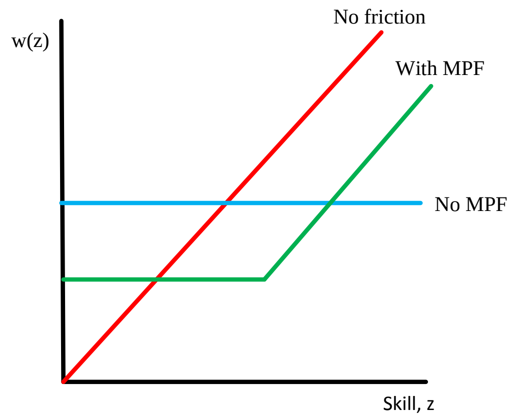
$$w(z) = \max\{cz, c_s\overline{z}\}. \tag{7}$$

As Figure 0 makes clear, the wage of a worker is strictly increasing in her skill in MPF and is constant in SPF. Hence, there must exist $\tilde{z}$ that satisfies $w_m(\tilde{z}) = w_s$ such that all workers with $z > \widetilde{z}$ choose to accept a job at the MPF that they met while all others choose to enter the common labor pool. Substituting for the wage schedules, we obtain a key equilibrium condition of the model:

$$c_s\overline{z}(\widetilde{z}) = \widetilde{z}c. \tag{8}$$

where

$$\bar{z}(\tilde{z}) \equiv \frac{1}{G(\tilde{z})} \int_{\underline{z}}^{\tilde{z}} z dG(z).$$



Note that the assumption given in (1) implies that $\bar{z}(\tilde{z})$ is strictly concave in $\tilde{z}$ so that there is must be a unique relationship between $\tilde{z}$ and c/c_s.

Given the sorting behavior implied by equation (8), we can fully characterize labor market clearing. If there are n SPFs that have entered, then labor market clearing in the common labor pool can be written

$$n(\sigma-1)A\left(c_s\right)^{-\sigma}=L\int_{\underline{z}}^{\widetilde{z}}zdG(z). \tag{9}$$

Turning to labor market clearing among MPF, undirected seach has the implication that L/m workers are matched with each firm.[8] These workers will have the same distribution of skill across all firms as they are randomly matched with MPF. As only workers with $z>\widetilde{z}$ choose to accept employment anticipating that they will earn a wage in the internal labor market of the firm that exceeds the wage they can get in the common pool labor market for SPF. Hence, the total supply of effective labor available to a MPF will be

$$\widetilde{L}=\frac{L}{m}\int_{\widetilde{z}}^{\infty}zdG(z).$$

[8] In fact, as workers cannot observe each others' skill they cannot coordinate on which firm to apply for a job. Hence, the composition of the work force must be the same for each MPF even if there is no search process.

and aggregating over the individual labor markets for MPF, we have

$$m(\sigma-1)Ac^{-\sigma}\int_0^{\omega^d}\alpha(\omega)^{1-\sigma}d\omega = L\int_{\widetilde{z}}^{\infty} zdG(z). \tag{10}$$

Free Entry MPF and SPF can coexist in equilibrium because they have distinct advantages over employing different types of workers. For MPF, the high fixed cost of entry is compensated by earning a rent on the high skill workers. While SPF have higher marginal costs, this disadvantage is offset by their low fixed costs of being in operation. Firms enter until MPF and SPF firms each earn zero profits. For MPF, this implies that the profits given in equation (5) must equal zero so that

$$Ac^{1-\sigma}\int_0^{\omega^d}\alpha(\omega)^{1-\sigma}d\omega = \omega^d f + f^m. \tag{11}$$

Similarly, zero profits for SPF require

$$A(c_s)^{1-\sigma} = f. \tag{12}$$

As is the case in the standard Krugman model with CES preferences, the size of a SPF is pinned down by the free entry condition. This is not the case, however, for MPF as they optimally choose the number of varieties to produce. Instead, their size is pinned down by the first-order condition for the choice of the number of products. The collective mass of varieties depends on the choices of MPF firms and entry of both type of firms and capital market clearing:

$$nf + m\left(\omega^d f + f^m\right) = K. \tag{13}$$

Definition 1 *An equilibrium in the closed economy are (1) a number of entrants, m and n, and a firm scope ω_d, (2) marginal costs c, c_s and a cutoff skill level $\widetilde{z}$, (3) a mark-up adjusted demand level A that satisfy the free entry conditions (12) and (22), the first order conditions for firm scope (6), labor market clearing conditions (9), (10), and (8),* *and the capital market clearing condition (13).*

MPF versus SPF This section serves two purposes. First, we compare the differences in observable characteristics between MPF and SPF that arise in equilibrium. Second, we the features of technology that determine the allocation of workers between these types of firms.

As noted in the introduction, MPF tend to pay higher wages and exhibit higher productivity than SPF. As we now show, our model is qualitatively consistent with these observations. We begin by characterizing the difference between firms in terms of the wages paid. To pin down the wage gradiant given in (7) we first must solve for c and c_s. Combining equations (12) through (13) we obtain the following relationship between c_s, c, and $\widetilde{z}$:

$$c_s \int_0^{\widetilde{z}} z dG(z) + c \int_{\widetilde{z}}^{\infty} z dG(z) = \frac{\sigma - 1}{\sigma} \frac{K}{L}. \tag{14}$$

This expression, which we refer to as the combined factor market clearing condition or FMC, reflects the fact that in this model, labor income accounts for a constant fraction of aggregate income due to the Cobb-Douglas expenditure shares and the monopolistic competition result that fixed costs account for a constant fraction of firm costs.

Expression (14) combined with the Sorting condition, given by (8), determine c_s and c as functions of $\tilde{z}$. Given a cutoff $\tilde{z}$, the FMC is a linear downward sloping curve in a $c - c_s$ space. Changes in $\tilde{z}$ lead to a rotation of the curve around the point of intersection with the 45° line. The Sorting condition $c\tilde{z} = c_s \bar{z}(\tilde{z})$ is an upward sloping ray from the origin. Since $\tilde{z} > \bar{z}(\tilde{z})$, the slope of this curve is less than one, and the intersection with the FMC must be below the 45° line.

We can now state the following proposition:

Proposition 1 *Multiproduct firms pay higher wages and yet face a lower effective cost of labor relative to single product firms.*

By being able to commit to reward skill, multiproduct firms earn a rent on their work force. That is, the wage premium offered by MPF over SPF for any given level of worker skill does not fully reflect the worker's true ability. This rent confers a marginal cost advantage to MPF that compensates these firms for their higher overhead costs f^m.

MPF have lower effective labor costs relative to SPF so that they must have lower marginal costs in their core product relative to SPF. Unlike SPF, MPF also produce a range of goods in which they are technically less proficient than SPF. The following proposition shows that despite this technical disadvantage, MPF ultimately have a lower marginal cost of production in all of their products relative to a SPF.

Proposition 2 *Multiproduct firms sell more units per product than single product firms.*

Proof. The proposition follows immediately from the optimal scope equation (6) and SPF free entry condition (12):

$$c\alpha(\omega^d) = \left(\frac{A}{\sigma f}\right)^{\frac{1}{\sigma-1}} < \left(\frac{A}{f}\right)^{\frac{1}{\sigma-1}} = c_s.$$

Hence, the marginal cost of all products sold by a MPF are lower than the product sold by a SPF an MPF sells more of every good it produces than a SPF. ■

The fact that MPF tend to appear more productive in all of the products that they produce relative to a SPF reflects the fact that MPF exploit monopsony power within their internal labor market. Internalizing the "supply side cannibalization", they restrict their product offering sufficiently that even their lowest productivity product has a lower marginal cost (but in fact lower inherent TFP) than SPF. The proposition means that MPF are larger than SPF not only because they can expand along the extensive margin by adding products, but they also sell more per product than a SPF. This is consistent with existing empirical evidence (see Bernard, Redding, and Schott, 2010).

We now turn to solving for the allocation of workers by skill to MPF and to SPF. We begin working the free entry conditions for MPF (11) and SPF (12). Dividing the free entry condition for MPF by the free entry condition for SPF, we obtain

$$\left(\frac{c}{c_s}\right)^{1-\sigma} \int_0^{\omega^d} \alpha(\omega)^{1-\sigma} d\omega = \omega^d + \frac{f^m}{f}$$

using (6) and simplifying, this condition becomes

$$\left(\frac{c}{c_s}\right)^{1-\sigma} = \sigma\alpha(\omega^d)^{\sigma-1}. \tag{15}$$

Now, substituting using (8) we obtain

$$\widetilde{z} = \sigma^{\frac{1}{\sigma-1}}\alpha(\omega^d)\overline{z}(\widetilde{z}). \tag{16}$$

Note assumption (1) combined with the $\sigma^{\frac{1}{\sigma-1}}\alpha(\omega^d) > 1$ and $\lim_{\widetilde{z}\longrightarrow\underline{z}}\overline{z}(\widetilde{z}) = \underline{z}$ imply an unique $\widetilde{z}$ that satisfies this expression as a function of ω^d. Knowing what determines the optimal scope of a multiproduct firm is sufficient for knowing what determines the share of the labor force employed at MPF. This relationship is summarized in the following proposition.

Proposition 3 *The share of the labor force employed in MPF, summarized by $1-G(\widetilde{z})$, is independent of a country's size and its capital stock. It is strictly decreasing in the fixed cost of managing a multiproduct firm, f^m, and strictly increasing in the fixed cost of adding an additional product, f.*

Proof. From equation (16), it follows that $\widetilde{z}$ is strictly increasing in the optimal scope of a MPF, ω^d. The optimal scope of a MPF can be solved for directly by dividing the the free entry condition (11) by first order condition (6) to obtain

$$\Lambda \equiv \sigma\alpha(\omega^d)^{\sigma-1}\int_0^{\omega^d}\alpha(\omega)^{1-\sigma}d\omega - \omega^d - \frac{f^m}{f} = 0$$

Differentiating Λ with respect to ω^d, we obtain

$$\frac{\partial\Lambda(\omega^d)}{\partial\omega^d} = (\sigma-1)\left(1+\sigma\frac{\int_0^{\omega^d}\alpha(\omega)^{1-\sigma}d\omega}{\alpha(\omega^d)^{1-\sigma}}\frac{\alpha'(\omega^d)}{\alpha(\omega^d)}\right) > 0$$

Note that $\lim_{\omega^d\to 0}\Lambda(\omega^d) < 0$ so to establish that there is an optimal equilibrium ω^d we need to establish that $\lim_{\omega^d \to \infty} \Lambda(\omega^d) > 0$. We have

$$\lim_{\omega^d \to \infty} \Lambda(\omega^d) = \lim_{\omega^d \to \infty} \left[\omega^d \left(\frac{\sigma \alpha(\omega^d)^{\sigma-1} \int_0^{\omega^d} \alpha(\omega)^{1-\sigma} d\omega}{\omega^d} - 1 \right) \right] - \frac{f^m}{f}$$
$$= \lim_{\omega^d \to \infty} \omega^d \left(\lim_{\omega^d \to \infty} \frac{\sigma \alpha(\omega^d)^{\sigma-1} \int_0^{\omega^d} \alpha(\omega)^{1-\sigma} d\omega}{\omega^d} - 1 \right) - \frac{f^m}{f}$$

Applying l'Hopital's rule to the terms in brackets on the last line, we obtain

$$\lim_{\omega^d \to \infty} \frac{\sigma \alpha(\omega^d)^{\sigma-1} \int_0^{\omega^d} \alpha(\omega)^{1-\sigma} d\omega}{\omega^d} = \sigma + \sigma(\sigma - 1) \lim_{\omega^d \to \infty} \frac{\int_0^{\omega^d} \alpha(\omega)^{1-\sigma} d\omega}{\alpha(\omega^d)^{1-\sigma}} \frac{\alpha'(\omega^d)}{\alpha(\omega^d)} > \sigma$$

Hence, $\lim_{\omega^d \to \infty} \Lambda(\omega^d) > 0$. As ω^d is independent of the size of the labor force, so too is the share of labor employed by MPF. ■

In the closed economy, the share of MPF relative to SPF depends only on technological parameters such as the share of $\alpha(\cdot)$ and the relative size of fixed costs f^m/f, the elasticity of substitution σ, and the distribution of skill in the population as summarized by G. This last observation has the implication that countries with a greater share of unobserved skill in the population will have a larger share of production done by MPF than countries with a smaller share of unobserved skill.

It is worth noting at this point that the equilibrium to our model is suboptimal from a planner's perspective because MPFs are not efficient producers relative to SPF. They use more capital per variety produced than an SPF and their unit labor requirements are higher as $\alpha(\omega) > 1$ for $\omega > 0$. A social planner tasked with the job of maximizing aggregate welfare would not allow the formation of multi-product firms. In equilibrium, however, as high skill workers would not be compensated for their skill by an SPF, they choose to work for MPF. The MPF is willing to incur high fixed costs because its will earn some of the rent on the talent in its internal labor market. The following proposition demonstrates that aggregate welfare would be improved were MPF banned.

Proposition 4 *Aggregate output is strictly higher in equilibrium in an economy where*

MPF are banned relative to the same economy in which MPF are allowed.

Proof. We first solve for aggregate output in an equilibrium in which MPF are banned and then show that this output is higher than in the equilibrium solved for above. Aggregate output in a no MPF equilibrium is given by

$$X_{\text{noMPF}} = n^{\frac{\sigma}{\sigma-1}} X_s.$$

In an equilibrium with no MPF, we have only n homogeneous single product firms producing X_s units of output. From factor market clearing we have

$$\begin{aligned} n &= K/f, \text{ and} \\ X_s &= \frac{L}{n} \int_0^\infty z dG(z). \end{aligned}$$

Hence, aggregate output must be

$$X_{\text{noMPF}} = \left(\frac{K}{f}\right)^{\frac{\sigma}{\sigma-1}} \frac{Lf}{K} \int_0^\infty z dG(z).$$

In an MPF equilibrium, aggregate output must be

$$X_{\text{MPF}} = \left(m \int_0^{\omega_d} X(\omega)^{\frac{\sigma-1}{\sigma}} d\omega + n X_s^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}},$$

where $X(\omega) = (\sigma-1)Ac^{-\sigma}\alpha(\omega)^{-\sigma}d\omega$ is the output of a variety that is distance ω from the MPF core competency. Rearranging the expression for aggregate output and substituting, we have

$$\begin{aligned} X_{\text{MPF}} &= X_s \left(m \frac{\int_0^{\omega_d} X(\omega)^{\frac{\sigma-1}{\sigma}} d\omega}{X_s^{\frac{\sigma-1}{\sigma}}} + n \right)^{\frac{\sigma}{\sigma-1}} \\ &= X_s \left(m \frac{c^{1-\sigma} \int_0^{\omega_d} \alpha(\omega)^{1-\sigma} d\omega}{c_s^{1-\sigma}} + n \right)^{\frac{\sigma}{\sigma-1}} \end{aligned}$$

Free entry for MPF and SPF implies

$$\frac{c^{1-\sigma}\int_0^{\omega_d}\alpha(\omega)^{1-\sigma}d\omega}{c_s^{1-\sigma}} = \frac{\omega_d f + f^m}{f}$$

and so we can write

$$X_{\text{MPF}} = X_s \left(\frac{m(\omega_d f + f^m) + nf}{f}\right)^{\frac{\sigma}{\sigma-1}}.$$

Factor market clearing in an equilibrium with MPF requires

$$\begin{aligned} m(\omega_d f + f^m) + nf &= K \\ X_s &= \frac{Lf}{nf}\int_0^{\tilde{z}} zdG(z) \end{aligned}$$

and so we have

$$X_{\text{MPF}} = \left(\frac{K}{f}\right)^{\frac{\sigma}{\sigma-1}} \frac{Lf}{nf}\int_0^{\tilde{z}} zdG(z).$$

Suppose that the claim in the proposition is wrong and an MPF equilibrium generates at least weakly higher aggregate output, or $X_{\text{MPF}} \geq X_{\text{noMPF}}$. This requires

$$\begin{aligned} \left(\frac{K}{f}\right)^{\frac{\sigma}{\sigma-1}} \frac{Lf}{nf}\int_0^{\tilde{z}} zdG(z) &\geq \left(\frac{K}{f}\right)^{\frac{\sigma}{\sigma-1}} \frac{Lf}{K}\int_0^{\infty} zdG(z) \\ \frac{1}{nf}\int_0^{\tilde{z}} zdG(z) &\geq \frac{1}{K}\int_0^{\infty} zdG(z). \end{aligned}$$

Rearranging this inequality yields

$$\frac{K}{nf} = \frac{m(\omega_d f + f^m) + nf}{nf} \geq \frac{\int_0^{\infty} zdG(z)}{\int_0^{\tilde{z}} zdG(z)} = 1 + \frac{\int_{\tilde{z}}^{\infty} zdG(z)}{\int_0^{\tilde{z}} zdG(z)}.$$

$$\frac{m(\omega_d f + f^m)}{nf} \geq \frac{\int_{\tilde{z}}^{\infty} zdG(z)}{\int_0^{\tilde{z}} zdG(z)}$$

where we have used factor market clearing on the L.H.S. of this inequality. Finally, using free entry and labor market clearing for both SPF and MPF, we have

$$\frac{m(\omega_d f + f^m)}{nf} = \frac{c}{c_s}\frac{\int_{\tilde{z}}^{\infty} zdG(z)}{\int_0^{\tilde{z}} zdG(z)}.$$

Plugging this into the inequality we obtain

$$\frac{c}{c_s}\frac{\int_{\tilde{z}}^{\infty} zdG(z)}{\int_0^{\tilde{z}} zdG(z)} \geq \frac{\int_{\tilde{z}}^{\infty} zdG(z)}{\int_0^{\tilde{z}} zdG(z)}$$
$$c \geq c_s.$$

But, in an MPF equilibrium, we have shown that it has to be the case that $c < c_s$. Hence, we contradicted the claim that an MPF equilibrium generates at least as much aggregate output as a no-MPF equilibrium. ■

4 The Open Economy

We now consider a simple open economy version of our model in which two identical countries trade varieties of the differentiated good. Given the symmetry of the problem, the extension is relatively straightforward. The remainder of this section is organized as follows. We first introduce the additional assumptions governing international trade. We then explore the optimal exporting decisions of SPF and MPF in an open economy equilibrium and update the critical equilibrium conditions accordingly. Finally, we consider the effect of a reduction in variable trade costs between countries on resource allocation both within and across firms and on the distribution of income.

4.1 Assumptions

We assume that at the time that a firm chooses how many products to manage it also chooses how many of its goods to export. Further, we assume that exporting requires the firm to incur fixed cost f_x (again in terms of the numeraire, capital) and iceberg-type shipping costs per unit exported $\tau > 1$. Following Melitz, we make the following restriction on parameter values:

$$f^x \tau^{\sigma-1} > f \tag{17}$$

As in Melitz, this assumption is necessary to generate varieties that are produced but not traded. We begin our adjustments of the model to the trading environment by first considering the optimal export choices of SPF and MPF.

4.2 Open Economy Equilibrium

We begin this section by investigating the exporting decisions of SPF and MPF. Then, after having characterized the optimal behavior of firms as a function of industry level variables, we update the equilibrium conditions that determine these variables.

Lemma 1 *Single product firms do not export.*

Proof. Given the fixed (f_x) and variable trade costs (τ), the potential additional profits that a SPF could earn are given by

$$\pi^* (c_s) = A (\tau c_s)^{1-\sigma} - f_x.$$

Suppose that SPF firms exported. Then, they must break even in equilibrium so that

$$A(1 + \tau^{1-\sigma})c_s^{1-\sigma} = f + f_x.$$

But, assumption (17) implies that firms that do not export make positive profits which cannot happen with free entry. To see this, suppose they did not make positive profits. Then, $Ac_s^{1-\sigma} \leq f$. From the zero profit condition, we have $f_x \leq f\tau^{1-\sigma}$, which violates the assumption in (17). ■

We now consider the profits that MPF could earn by exporting a variety that is distance ω from its core product. These profits are

$$\pi^* (\omega, c) = A [\tau c \alpha (\omega)]^{1-\sigma} - f_x.$$

The profits of a MPF that operates varieties on the interval $(0,\omega^d)$ and exports on the varieties on the interval $(0,\omega^f)$ are then

$$\Pi = Ac^{1-\sigma}\left(\int_0^{\omega^d}\alpha(\omega)^{1-\sigma}\,d\omega + \tau^{1-\sigma}\int_0^{\omega^f}\alpha(\omega)^{1-\sigma}\,d\omega\right) - \omega^d f - \omega^f f_x - f^m. \qquad (18)$$

The internal labor market clearing condition of firm that operates varieties on the interval $(0,\omega^d)$ and exports on the varieties on the interval $(0,\omega^f)$ now becomes

$$\int_0^{\omega^d} x(\omega,c)\,\alpha(\omega)\,d\omega + \tau\int_0^{\omega^f} x^*(\omega,c)\,\alpha(\omega)\,d\omega = \widetilde{L}.$$

Substituting for $x(\omega,c)$ and for $x^*(\omega,c)$ using (3) and simplifying, the internal labor market clearing in the open economy becomes

$$(\sigma-1)\,Ac^{-\sigma}\left(\int_0^{\omega^d}\alpha(\omega)^{1-\sigma}\,d\omega + \tau^{1-\sigma}\int_0^{\omega^f}\alpha(\omega)^{1-\sigma}\,d\omega\right) = \widetilde{L}. \qquad (19)$$

As in the case of the closed economy, the choice of the optimal product range to produce and to export depends on how the marginal cost of production is affected through the internal labor market. The first order condition for the range of domestic production continues to be given by equation (6), while the first order condition for the optimal range of exports (assuming provisionally for an internal solution) is implicitly given by

$$Ac^{1-\sigma}\tau^{1-\sigma}\alpha\left(\omega^f\right)^{1-\sigma} = \sigma f_x. \qquad (20)$$

We are now in a position to characterize the conditions under which MPF export a set of goods.

Lemma 2 *There exists a level of the fixed entry cost for MPF $\widehat{f^m}$ such that all MPF export if $f^m > \widehat{f^m}$.*

Proof. We establish that for sufficiently large entry cost that an MPF would optimally deviate from an equilibrium in which MPF did not export. Suppose that MPF did not export and were breaking even. In a no-export equilibrium, the free entry condition would be the same as in the closed economy, and given by equation (11). Further, the optimal product range condition would also continue to be given by equation (6). For a MPF to not want to export it would have to be true that an increase in the profit due to exporting the core variety ($\omega = 0$) would be less than the additional strain on the firm's internal labor market, or

$$A\left(\tau c\right)^{1-\sigma} < \sigma f_x.$$

Combining the optimal product range condition with this expression, we find that for exporting to be unprofitable, we must have

$$\alpha(\omega^d)^{\sigma-1} < \frac{f^x}{f}\tau^{\sigma-1}.$$

From the proof of proposition 3 we know that for non-exporting MPF ω^d is strictly increasing and unbounded in f^m. As the function $\alpha(\omega)$ is strictly increasing, it follows that $\alpha(\omega^d)^{\sigma-1}$ is strictly increasing and unbounded in f^m. Hence, there must exist a critical level $\widehat{f^m}$ such that for $f^m > \widehat{f^m}$ MPF would want to deviate from a no-export equilibrium. ■

Henceforth, we restrict attention to equilibria in which f^m is sufficiently large to guarantee that MPF export. We now characterize the range of products that are produced for the local market and that are exported. Using equations (6) and (20), we have

$$\alpha\left(\omega^f\right)^{\sigma-1} = \frac{f}{f_x}\tau^{1-\sigma}\alpha\left(\omega^d\right)^{\sigma-1}. \tag{21}$$

The parameter restriction given by (17) guarantees that the range of goods exported is strictly smaller than the range of goods sold domestically: $\omega^f < \omega^d$.

Finally, as was the case in the closed economy, the first order conditions for the cutoff product combined with the free entry condition for MPF pins down the range of products produced and exported. In the open economy case, the free entry condition for MPF becomes

$$Ac^{1-\sigma}\left(\int_0^{\omega^d}\alpha(\omega)^{1-\sigma}d\omega+\tau^{1-\sigma}\int_0^{\omega^f}\alpha(\omega)^{1-\sigma}d\omega\right)=\omega^d f+\omega^f f_x+f^m. \tag{22}$$

Using equations (6) and (20) we can rewrite this expression as

$$\sigma f\int_0^{\omega^d}\left(\frac{\alpha(\omega)}{\alpha(\omega^d)}\right)^{1-\sigma}d\omega+\sigma f_x\int_0^{\omega^f}\left(\frac{\alpha(\omega)}{\alpha(\omega^f)}\right)^{1-\sigma}d\omega=\omega^d f+\omega^f f_x+f^m \tag{23}$$

The optimal range of products produced and exported can be derived from equations (21) and (23). Note that equation (21) implies a positive relationship between ω^d and ω^f while equation (23) implies a negative relationship.

As the sorting condition continues to be given by equation (8), the labor market clearing condition for MPF must only be amended to include exported goods, or

$$m(\sigma-1)Ac^{-\sigma}\left(\int_0^{\omega^d}\alpha(\omega)^{1-\sigma}d\omega+\tau^{1-\sigma}\int_0^{\omega^f}\alpha(\omega)^{1-\sigma}d\omega\right)=L\int_{\tilde{z}}^{\infty}zdG(z). \tag{24}$$

Finally, as MPF exporters require capital to export their products, we must adjust the capital market clearing condition to account for this additional source of demand. The open economy capital market clearing condition is now given by

$$K=m\left(f^m+\omega^d f+\omega^f f_x\right)+nf \tag{25}$$

4.3 The Impact of Falling Trade Costs

We now consider the impact of globalization on labor markets and on the structure of production across firms. We begin our analysis with the effect of lower trade costs on the portfolio of products produced by MPF. We then consider the reallocation of resources across firms, showing that trade liberalization tends to suppress SPF, lowering the share of labor that works for SPF and lowering the number of varieties produced by SPF. We conclude the section with a discussion of the impact of trade liberalization on the real income of workers of differing levels of skill.

As in many of the models of multiproduct firms in the literature, trade induces firms to expand the range of goods that they export and to consolidate the number of varieties produced by the firm. This result is shown in the following proposition.

Proposition 5 *A reduction in trade costs induces firms to consolidate the range of products produced (ω^d falls) and increases the range of products that the firm exports (ω^f increases).*

Proof. The proposition is proven by somewhat tedious computation. Totally differentiating equations (21) and (23) and rearranging yields

$$\frac{d\ln\omega^d}{d\ln\tau} = \Delta^{-1}\left(\sigma\alpha\left(\omega^f\right)^{\sigma-1}\int_0^{\omega^f}\alpha\left(\omega\right)^{1-\sigma}d\omega\varepsilon_\alpha\left(\omega^f\right)+\omega^f\right)f_x > 0,$$

$$\frac{d\ln\omega^f}{d\ln\tau} = -\Delta^{-1}\left(\sigma\alpha\left(\omega^d\right)^{\sigma-1}\int_0^{\omega^d}\alpha\left(\omega\right)^{1-\sigma}d\omega\varepsilon_\alpha\left(\omega^d\right)+\omega^d\right)f < 0,$$

where $\varepsilon_\alpha\left(\omega\right) \equiv \omega\alpha'(\omega)/\alpha(\omega) > 0$ and

$$\begin{aligned}\Delta = {} & \varepsilon_\alpha\left(\omega^d\right)\left(\sigma\alpha\left(\omega^f\right)^{\sigma-1}\int_0^{\omega^f}\alpha\left(\omega\right)^{1-\sigma}d\omega\varepsilon_\alpha\left(\omega^f\right)+\omega^f\right)f_x \\ & +\varepsilon_\alpha\left(\omega^f\right)\left(\sigma\alpha\left(\omega^d\right)^{\sigma-1}\int_0^{\omega^d}\alpha\left(\omega\right)^{1-\sigma}d\omega\varepsilon_\alpha\left(\omega^d\right)+\omega^d\right)f \\ > {} & 0\end{aligned}$$

■

If trade costs fall, MPFs expand the range of products exported and reduce the product range at home. In our Figure 2, a reduction in τ leads to an upward rotation of the Optimal Scope locus, leading to an increase in ω^f and a reduction in ω^d.

Dividing equation for the optimal domestic production range, given by (6), by the condition for SPF free entry, given by (12), we see that the relative cost difference between MPF and SPF, c/c_s, continues to be given by equation (15). As a decrease in τ decreases ω^d, it follows immediately that c/c_s must fall. Further, using (16), which also continues to hold in the open economy, it follows immediately that a fall in trade costs is associated with a decrease in $\widetilde{z}$ as higher relative wages that will be paid by MPFs attract workers out of SPFs. Mathematically we have

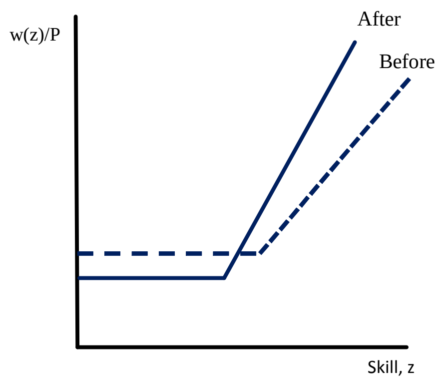
$$\frac{d\ln\tilde{z}}{d\ln\tau} = \frac{\bar{z}(\tilde{z})}{\tilde{z}}\left(1 - \frac{\tilde{z}g(\tilde{z})}{G(\tilde{z})}\frac{[\tilde{z}-\bar{z}(\tilde{z})]}{\bar{z}(\tilde{z})}\right)^{-1}\varepsilon_\alpha\left(\omega^d\right)\frac{d\ln\omega^d}{d\ln\tau} > 0, \tag{26}$$

where $\frac{\tilde{z}g(\tilde{z})}{G(\tilde{z})}\frac{[\tilde{z}-\bar{z}(\tilde{z})]}{\bar{z}(\tilde{z})} < 1$ by assumption. We summarize these results in the following proposition.

Proposition 6 *A reduction in trade costs, τ, induces a shift of labor out of single product firms into multi-product firms.*

The proposition tells us that the labor available to SPF must shrink as a result of falling trade barriers. Not surprisingly, it can be shown that the number of SPF falls as trade costs fall. Without further assumptions it is not possible to know whether the number of MPF rises, however. Whether the number of MPF expands or contracts depend on the shape of the skill distribution G and on the shape of the competency function α. Looking at the capital market clearing condition (25), it is clear that either m, ω^f, or both must rise as both n and ω^d have fallen.

The rising demand for labor by MPF intuitively is associated with an increase in the reward of skilled workers (workers that were previously employed by MPF) relative to the reward of unskilled workers (workers that remain employed by SPF). In fact, it is not only relative wages that move against unskilled workers. To see this, recall that workers for MPF are paid a wage of $w(z) = cz$ so that their wage (measured in terms of capital)



depend only on the level of c. From (8) and (14) we have

$$\frac{d\ln c}{d\ln\tau} = -\frac{\tilde{z}G\left(\tilde{z}\right)}{\tilde{z}G\left(\tilde{z}\right)+\int_{\tilde{z}}^{\infty} z dG\left(z\right)}\frac{d\ln\tilde{z}}{d\ln\tau} < 0. \tag{27}$$

As a reduction in transport costs lowers the cutoff, the wage of workers who previously worked for MPF must rise. Now consider the wage of workers that had previously worked for a SPF. These workers recieve a wage of $w_s = c\tilde{z}$. Totally differentiating this expression, and substituting (27), we obtain

$$\frac{d\ln w_s}{d\ln\tau} = \frac{\int_{\tilde{z}}^{\infty} z dG\left(z\right)}{\tilde{z}G\left(\tilde{z}\right)+\int_{\tilde{z}}^{\infty} z dG\left(z\right)}\frac{d\ln\tilde{z}}{d\ln\tau} > 0.$$

The impact of a reduction in tariffs on the wage distribution is illustrated in the following figure.

The solid lines correspond to the post trade liberalization wage distribution while the broken lines correspond to the pre-liberalization wage distribution. Comparing the wages of SPF and MPF workers before and after the trade liberalization, it is clear that the gap widens. As in the standard Melitz (2003) framework, a reduction in trade costs raises demand for labor for export production but lowers demand for local production. As skilled labor is used by the exporting MPF while unskilled labor is used by the SPF that serve only the domestic market, trade liberalization widens the wage gap.

We conclude with an analysis of the effect of trade liberalization on real income for individual workers and for the economy as a whole. The following proposition shows that trade liberalization induces an increase in the real income of incumbent MPF employees and a decrease in the real income of incumbent SPF employees:

Proposition 7 *A reduction in trade costs lowers the real wage of a SPF worker and increases the real wage of an MPF worker.*

Proof. Starting with the free entry condition

$$A(c_s)^{1-\sigma} = f$$

From the definition of A and the result that E is a constant, we have

$$\frac{c_s}{P} = \Psi$$

where $\Psi \equiv \left((\sigma-1)^{\sigma-1}\sigma^{-\sigma}E/f\right)^{1/(\sigma-1)}$ is a constant. This expression says that the free entry condition pins down the real marginal cost facing a SPF. Note that the wage of SPF worker is just $w_s = c_s\overline{z}(\widetilde{z})$, so we can write the real wage of an SPF worker as

$$\frac{w_s}{P} = \Psi\overline{z}(\widetilde{z}).$$

A reduction in τ lowers $\overline{z}(\widetilde{z})$, hence SPF workers are made worse off. Now consider the implications for a worker at an MPF. Using the maximization condition for domestic product range, we have

$$\frac{c}{P} = \frac{\Psi}{\sigma}\frac{1}{\alpha(\omega^d)}.$$

The wage of an MPF worker before and after trade liberalization is $w(z) = cz$ so the real wage is then

$$\frac{w(z)}{P} = \frac{\Psi}{\sigma}\frac{z}{\alpha(\omega^d)}.$$

A reduction in trade cost lowers ω^d and so lowers $\alpha(\omega^d)$ thereby making the real wage of MPF workers rise. ■

Trade creates winners and losers, but whether the winners gain more than losers lose depends critically on the initial conditions. A reduction in trade costs shift resources to the relatively less efficient MPF, which tends to lower aggregate output, but it induces the incumbent MPF to trim their least efficient products, which tends to raise aggregate output. Which effect dominates depends on how prevalent MPF were in the initial equilibrium as summarized in the following proposition:

Proposition 8 *If trade costs fall, welfare increases when the share of multi-product firms is large and decreases when this share is small.*

Proof. Aggregate welfare can be written $U = X = \frac{\sigma}{\sigma-1}\frac{\bar{w}}{P}L$. As noted earlier aggregate expenditure is fixed in terms of capital (the numeraire) and so we need only sign the effect on the price index P, but $d\ln U = -d\ln P = -d\ln c_s$, so we only need to sign the impact on c_s. Because

$$c_s \int_{\underline{z}}^{\tilde{z}} zdG(z) + c\int_{\tilde{z}}^{\infty} zdG(z) = (\sigma-1)\frac{K}{L},$$

it follows that from differentiation that

$$\frac{d\ln c_s}{d\ln \tilde{z}} \int_{\underline{z}}^{\tilde{z}} zdG(z) = -\frac{d\ln c}{d\ln \tilde{z}} \frac{\bar{z}(\tilde{z})}{\tilde{z}} \int_{\tilde{z}}^{\infty} zdG(z) - [\tilde{z} - \bar{z}(\tilde{z})]\,\tilde{z}g(\tilde{z}),$$

which is positive for small values of $\widetilde{z}$ and negative for large values of $\widetilde{z}$. The proposition then follows directly from the observation that $d\widetilde{z}d\tau > 0$. ■

Intuitively, when MPF dominate the economy initially, most of the action in labor reallocation is due to the internal trimming of products of these firms. When MPF are not a large part of the economy initially then most of the reallocation of labor is from SPF to MPF.

5 Conclusion

Much of the reallocation of resources due to economic shocks occurs within the boundaries of large multiproduct firms rather than on independent factor markets. This paper explicitly models exactly such a phenomenon in the context of industrial conglomerates. Consistent with the stylized facts, multiproduct firms in our paper are larger, appear more productive, are more likely to export, and pay higher wages than single product firms. This occurs in equilibrium despite the fact that multiproduct firms are inherently less efficient than single product firms. Multiproduct firms appear to be successful because their ability to reward skill allows them to earn a rent on their skilled employees. The existence of multiproduct firms is clearly in the interest of skilled workers because it allows them to earn higher wages than their less skilled peers, but it is does not mean that internal labor markets are good for economic efficiency.

6 Appendix: Desirable properties of pareto

The point of this note is to demonstrate that the Pareto distribution with parameters $(1, \kappa)$ will imply that the function $\overline{z}(\widetilde{z})/\widetilde{z}$ is monotonically decreasing. By definition, we have

$$\frac{\overline{z}(\widetilde{z})}{\widetilde{z}} = \frac{1}{\widetilde{z}G(\widetilde{z})}\int_1^{\widetilde{z}} z dG(z).$$

Differentiating with respect to $\widetilde{z}$, the condition that must be satisfied is

$$\frac{g(\widetilde{z})}{G(\widetilde{z})} - \frac{\overline{z}(\widetilde{z})}{\widetilde{z}}\left(\frac{1}{\widetilde{z}} + \frac{g(\widetilde{z})}{G(\widetilde{z})}\right) < 0$$

$$\frac{\widetilde{z}g(\widetilde{z})}{G(\widetilde{z})} < \frac{\overline{z}(\widetilde{z})}{\widetilde{z} - \overline{z}(\widetilde{z})}$$

Using the Pareto assumption with $G(z) = 1 - z^{-\kappa}$ and $g(z) = \kappa z^{-\kappa-1}$, we have

$$\overline{z}(\widetilde{z}) = \frac{\kappa}{\kappa - 1}\frac{1 - \widetilde{z}^{1-\kappa}}{1 - \widetilde{z}^{-\kappa}}.$$

Now, substituting this into the condition, we have

$$\frac{\kappa \widetilde{z}^{-\kappa}}{1-\widetilde{z}^{-\kappa}} < \frac{\frac{\kappa}{\kappa-1}\frac{1-\widetilde{z}^{1-\kappa}}{1-\widetilde{z}^{-\kappa}}}{\widetilde{z}-\frac{\kappa}{\kappa-1}\frac{1-\widetilde{z}^{1-\kappa}}{1-\widetilde{z}^{-\kappa}}}$$

After substantial manipulation, this simplifies to

$$\Theta(\widetilde{z}) \equiv \frac{\widetilde{z}^{\kappa}-1}{\widetilde{z}-1}$$

Using l'Hopital's rule, it can easily be established that

$$\begin{aligned}\lim_{\widetilde{z}\downarrow 1}\Theta(\widetilde{z}) &= \lim_{\widetilde{z}\downarrow 1}\kappa\widetilde{z}^{\kappa-1} = \kappa,\\ \lim_{\widetilde{z}\downarrow \infty}\Theta(\widetilde{z}) &= \lim_{\widetilde{z}\downarrow \infty}\kappa\widetilde{z}^{\kappa-1} = \infty.\end{aligned}$$

Hence, to establish that $\overline{z}(\widetilde{z})/\widetilde{z}$ is strictly increasing, we only need to show that $\Theta(\widetilde{z})$ is monotonic. Differentiating $\Theta(\widetilde{z})$, we obtain

$$\Theta'(\widetilde{z}) = \frac{1+(\kappa-1)\widetilde{z}^{\kappa}-\kappa\widetilde{z}^{\kappa-1}}{(\widetilde{z}-1)^2}.$$

By L'hopital's rule, we have $\lim_{\widetilde{z}\downarrow 1}\Theta'(\widetilde{z}) > 0$. Further, we have $\Theta''(\widetilde{z}) > 0$ for $\widetilde{z} > 1$. So, we have $\frac{\overline{z}(\widetilde{z})}{\widetilde{z}}$ strictly decreasing.

References

[1] Bernard, Andrew, Stephen Redding, and Peter Schott. 2011. "Multiproduct Firms and Trade Liberalization." *Quarterly Journal of Economics* 126(3):1271-1318

[2] Dhingra, Swati. 2013. "Trading Away Wide Brands for Cheap Brands." *American Economic Review* 103(6): 2554-84.

[3] Eckel, Carsten and Peter Neary. 2010. "Multi-product Firms and Flexible Manufacturing in the Global Economy," *Review of Economic Studies* 77(1), 188–217.

[4] Feenstra, Robert, and Ma. 2007."Optimal Choice of Product Scope for Multiproduct Firm Under Monopolistic Competition," NBER Working Paper Series 13703.

[5] Greenwald, Bruce. 1986. "Adverse Selection in the Labour Market." *Review of Economic Studies* 53: 325-347.

[6] Helpman, Elhanan, Oleg Itskhoki, and Stephen Redding. 2010. "Inequality and Unemployment in a Global Economy." *Econometrica* 78(4), 1239-1283.

[7] Mayer, Thierry, Marc Melitz, and G. Ottaviano. 2014. "Market Size, Competition, and the Product Mix of Exporters." *American Economic Review* 104(2): 495-536

[8] Melitz, Marc. 2003. "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity." *Econometrica* 71: 1695-1725

[9] Nocke, Volker, and Stephen Yeaple. 2014. "Globalization and Multiproduct Firms." Forthcoming, *International Economic Review.*

[10] Papageorgiou, Theodore. 2012. "Large Firms and Internal Labor Markets," mimeo Penn State University.

[11] Yeaple, Stephen. 2005. "A Simple Model of Firm Heterogeneity, International Trade, and Wages." *Journal of International Economics* 65(1):1-20.