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Inter-city specialization and trade in functions versus sectors

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Abstract

Our model combines elements of several literatures including the new economic geography, multinational firms, urban economics, and trade theory. A two-city country trades with the larger world, and firms and workers within the country are mobile between the two cities. Firms have two functions or occupations, such as headquarters and plant, which may be located together (integrated firm) or in separate cities (fragmented firm), with fragmentation incurring a cost. This element of the model is similar to Duranton and Puga (2005), but from here we move in a direction more linked to international trade theory. Industries differ in function intensities, and cities differ in Ricardian comparative advantage in functions, or the functions (not industries) have location-specific agglomeration economies. Our approach creates a distribution of fragmented and integrated firms across industries and across cities. We generate a number of economic insights, several of which can be examined empirically. First, as fragmentation costs fall, a city's functional/occupational specialization rises and its sectoral specialization falls. Second, as fragmentation costs fall, there is a fall in the correlation across industries between the share of workers employed in industry z who are doing function i and industry z's overall (integrated firm) i-function intensity. Put differently, a city's industrial mix becomes a weak predictor of its occupational mix, consistent with A. Markusen and Barbour (2003).

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1. Introduction.

The production of final products and services typically requires a number of functions to be performed. Manufactured goods require engineering, finance and marketing; construction requires architects and lawyers, and so on. There may be spatial (e.g. inter-urban) differences in the efficiency with which such functions can be supplied so, if the functions are not perfectly tradable, efficiency differences in functions will translate into a pattern of comparative advantage in the final goods that use these functions. This paper investigates the impact of such differences for firm organisation, city specialisation, trade in goods, and for the associated gains from trade.

The concept of 'function' is fuzzy, depending on how narrowly it is defined. A rather aggregate level is the distinction between headquarters and production, as developed in some of the literature on foreign direct investment (Markusen 2002) and more recent work in the urban context (Duranton and Puga 2005, Rossi-Hansburg et al. 2009). A much finer level is that of a 'task', often thought of as a narrow stage of production and modelled as a continuum (Grossman and Rossi-Hansburg 2008, 2012, Autor 2013). Alternatively, functions could be synonymous with occupations. Indeed, a common statistical breakdown is to divide a firm's workforce into production (or blue-collar) and non-production (or white-collar) workers.

The sort of function we seek to model corresponds to quite broad aggregates, such as engineering, finance, or law. Such functions have several properties. First, most functions are required in most sectors, though in different proportions, which could be referred to as the function intensity of a sector. Second, many large cities appear to have developed quite broad functional specialisms. London and New York in business services: finance, but also legal and advertising; the San Francisco area in both hardware and software; Los Angeles in a range of media and creative sectors. Some cities specialise in quite narrow 'tasks' but, at least for large cities, the broader functional concepts seem more relevant. Third, we think of functions as being associated with labour skills and firm capabilities, and suggest that these may be the fundamental level at which city comparative advantage is based. Cities develop the skill set – through learning or the composition of its labour force – that comes to define what the city is good at. Finally, the different functions of a firm could be located together or geographically separated. Many workers in London and New York may be in the same occupation such as finance, accounting, law, or advertising, but they work for (under contract to) firms in different sectors and places.

The fundamental trade-off that we study arises from the facts that the efficiency with which cities supply functions may vary, and that firms face additional costs if they source functions from different cities: we call these fragmentation costs, arising if e.g. engineering has to acquired in one city, legal services another, and so on. We develop a simple model to show

how the interaction between fragmentation costs, the function intensity of different sectors, and efficiency differences between cities cause firms in some sectors to integrate production in one place, and in others to fragment it between cities. Firms' choices have implications for cities' production structures; to what extent are cities able to specialise in the functions in which they are most efficient, and how does this map into the final product (sectoral) specialisation of cities?

Following from this, we investigate the effects of changes in fragmentation costs (arising perhaps from communication or transport improvements) on the production structure of cities and on their size and the real incomes. Real income gains are particularly large if there are increasing returns to functional concentration in a city, and fragmentation of firms allows cities to develop their functional specialisms. Changes in the production structure of each city may also change the production structure of the economy as a whole. How do changes in the costs of trading functions within an economy shape the external trade of the country?

In order to investigate these questions we develop a model that has elements of economic geography, the literature on vertical multinationals, urban economics, and external economies of scale with some novel twists. There are two regions or cities, with identical workers who are mobile between jobs within and between cities. There are many final products (sectors) and just two functions, each final product requiring the functions in different proportions. There is free trade in final products, capturing the idea that the cities under study are embedded in an integrated market. Firms in each sector may perform both functions in one location, referred to as integrated firms, or one function in each location, referred to as fragmented firms. However, splitting the production of a good between two locations incurs a 'fragmentation cost'. This may be the cost of transporting 'functions' between cities, but is better thought of as coordination costs and the communication costs of maintaining links with suppliers in different cities.

The efficiency with which functions are produced is city specific, and we start with the simplest case in which there are Ricardian differences in the productivity of functions between cities. This provides a very clean example of how reducing fragmentation costs causes firms (in some sectors) to fragment, and causes cities to move from sectoral towards functional specialisation. Sectors with extreme function intensities are more likely to contain integrated firms, concentrating production in the city with the advantage in the function in which they are intensive. Sectors which draw more equally on both functions will contain firms that are fragmented, performing each in the city with respective efficiency advantage.

The Ricardian model provides a simple introduction, but functional comparative advantage is, we think, more likely to arise endogenously from cities' acquired skills and consequent increasing returns to scale. Economies of scale are, we assume, external to the firm and sector, occurring at the city-function level. In this case city specialisation turns out to be a discontinuous function of fragmentation costs and there is a range of fragmentation costs at which there are multiple equilibria. This arises because of the interaction between firm's location decisions and scale economies: economies of scale large enough to overcome fragmentation costs are achieved only if a wide range of sectors fragment. If firms in all sectors are fully integrated neither city has a large enough comparative advantage to induce fragmentation; but if firms are fragmented then cities are functionally specialised, creating the scale and productivity differences that support fragmentation. Welfare gains from reductions in fragmentation costs can be particularly large if they induce spatial reorganisation and the move from sectoral to functional specialisation.

The questions we pose and the model we develop touches on many strands of international and urban economics. The division of firms' activities (at least, HQ and production) has been studied in the literature on foreign direct investment (see Markusen 2002). Perhaps closest in spirit to this paper is the urban model of Duranton and Puga (2005), the focus of which is precisely the move from sectoral to functional specialisation, although again in the context of the division of HQ and production.[1] The international trade literature has analysed trade in tasks, both in constant returns models (Grossman and Rossi-Hansburg 2008) and under increasing returns (Grossman and Rossi-Hansburg 2012). As well as its international focus, this literature works in a framework of many tasks and few final sectors. This does not capture the ubiquity which, we argued above, distinguishes functions from tasks; our approach therefore works with few functions (tasks) and many final sectors. The present paper also draws on economic geography modelling (Fujita et al. 1999), particularly in its analysis of the multiplicity of equilibria occurring at intermediate levels of spatial frictions. Finally, there are literatures on the impact of internal geography on external trade. Uneven distribution of factors of production within a country is studied by Courant and Deardorff (1992) and following literature (Brakman and van Marrewijk 2013), and the physical geography of proximity to ports is studied by Limao and Venables (2002). In the present paper the internal geography arises from city variation in efficiency in the production of functions.

We generate a number of economic insights, several of which we can examine empirically. First, as fragmentation costs fall, a city's functional specialization rises and its sectoral specialization falls. A corollary is that a city's industrial mix becomes a weak predictor of its functional mix. Interpreting function mix as occupational mix, this is consistent with the findings of A. Markusen and Barbour (2003). So, for example, a city may be relatively

[1] See Rossi-Hansburg et al. (2009) for intra-urban separation of HQ and production.

specialized in blue-collar-intensive industries, but the workers in these industries are doing primarily white-collar functions.

2. The model

The ingredients of the model are locations, focussing on two cities; a primary factor, labour, which is mobile between cities; sectors, which we model as a continuum; and two functions, each using labour and being used as input by sectors. We build the model in stages, initially focusing on sectors and functions. In section 3 we use this to draw out results on fragmentation and specialisation, whilst keeping the general equilibrium side of the model in the background; we are able to do this by making sufficient assumptions to ensure that the two cities are symmetric, with the same wages. Section 4 then adds the general equilibrium side of the model enabling analysis of a richer set of possibilities.

The wage rate in city j is w_j, $j = 1, 2$. The functions are labelled A and B, and we assume that producing one unit of function i in city j requires λ_{ij} ($i = A, B,\ j = 1, 2$) units of labour. We look at cases where these productivity differences are Ricardian and where they are endogenous due to increasing returns. Functions are used in the production of freely traded final goods. There is a continuum of such goods, indexed $z \in [0, 1]$, with price $p(z)$ the same in both cities. Each final goods sector contains a number of firms each of which produces one unit of output using as inputs $a(z)$ units of function A and $b(z)$ of B. These input coefficients are fixed, the same in each city and, for simplicity we assume that firms use no labour. Internal returns to scale are constant, so setting firm scale at unity is without loss of generality. The input of each function varies across sectors, and we rank sectors such that low z sectors are A-intensive, $a'(z) < 0$, $b'(z) > 0$.

Since the technology with which functions are combined into final goods, $a(z)$, $b(z)$, is the same in both cities, urban comparative advantage is determined entirely by the efficiency with which cities use labour to produce functions, λ_{ij}. Cities are labelled such that productivity differences (if any) give city 1 a comparative advantage in function A, i.e. $\lambda_{A1}/\lambda_{B1} < \lambda_{A2}/\lambda_{B2}$. Since low z sectors are A-intensive, city 1 will be attractive (other things equal) for firms in low z sectors, and city 2 attractive for high z sectors.

Firms in each sector can source functions from either city, but if the two functions come from different cities then a fragmentation cost is incurred.[2] Each firm can therefore operate in one of three modes, choosing to operate entirely in city 1, entirely in 2, or to locate one function in city 1 and the other in city 2.[3] Firms that produce in a single city are 'integrated' and will

[2] We think of functions as being produced within the organisational boundaries of each firm, although they could just as well be outsourced and purchased through an arms-length relationship.

[3] The assignment of which function to which city will become clear, and does not merit additional

be labelled by subscript 1, 2 according to city of operation; those operating in both are 'fragmented' (subscript F). Fragmented firms incur additional cost, T, of operating in two locations. The profits of a firm in sector z for each of the three production modes are therefore

$$
\begin{aligned}
\pi_1(z) &= p(z) - [a(z)\lambda_{A1} + b(z)\lambda_{B1}]w_1, \\
\pi_F(z) &= p(z) - [a(z)\lambda_{A1}w_1 + b(z)\lambda_{B2}w_2] - T, \qquad (1) \\
\pi_2(z) &= p(z) - [a(z)\lambda_{A2} + b(z)\lambda_{B2}]w_2 .
\end{aligned}
$$

The term in square brackets is unit production cost. Thus, a firm in sector z uses $a(z)$ units of function A and $b(z)$ units of B. The functions use labour, with input per unit output given by the λ_{ij}, depending on the city (j = 1, 2) in which the sector performs the function ($i = A, B$). Wage costs depend on where the functions are performed, and hence on the sector's function intensity and chosen mode.

Firms' choice of mode partitions sectors into three groups. First is a range of z in which firms are integrated and produce both functions in city 1; as we show below, these will be low z sectors, intensive in function A. Second is a range of sectors in which firms are fragmented producing function A in city 1 and function B in city 2; if such sectors exist they will be those with intermediate values of z (i.e. using both functions in similar proportions). Third are high z (B-intensive) sectors in which firms are integrated and operate only in city 2.

The boundaries between these ranges are denoted z_1, z_2, and are the sectors for which different modes of operation are equi-profitable, i.e. $\pi_1(z_1) = \pi_F(z_1),\ \ \pi_F(z_2) = \pi_2(z_2)$. Using (1), these mode-boundaries are implicitly defined by

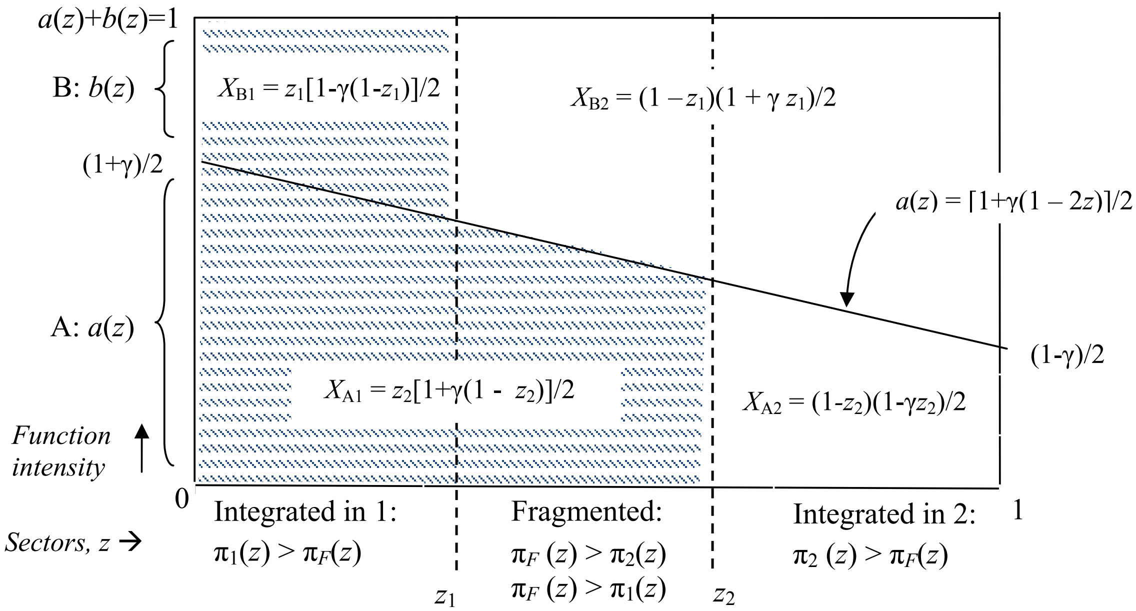
$$
\begin{aligned}
\pi_F(z_1) - \pi_1(z_1) &= b(z_1)[w_1\lambda_{B1} - w_2\lambda_{B2}] - T = 0, \qquad (2) \\
\pi_F(z_2) - \pi_2(z_2) &= a(z_2)[w_2\lambda_{A2} - w_1\lambda_{A1}] - T = 0 .
\end{aligned}
$$

The relationship between sectors, functions, and chosen modes of production is illustrated on figure 1, where the horizontal axis is the range of sectors, z ε [0, 1], and the vertical is input of each function per firm. This is illustrated for an example in which $a(z) = [1 + \gamma(1 - 2z)]/2$ and $b(z) = [1 - \gamma(1 - 2z)]/2$, so that $a(z) + b(z) = 1$. The inequalities at the bottom indicate the relative profitability of operating each mode, with mode-boundaries z_1, z_2, indicated by the vertical dashed lines. The shaded area gives total city 1 use (and hence production) of function B (with output denoted X_{B1}) and function A (output X_{A1}) under the assumption that

notation.

there is one firm active in each sector. This framework of sectors, functions, and firms provides the basis for investigating patterns of firm organisation and urban specialisation.

Figure 1: Sectors, functions, firm types and employment



3. Sectoral and functional specialisation in symmetric equilibria.

We start by looking at the way in which firms' mode of operation and the consequent location of sectors and functions depend on technology and fragmentation costs. Throughout this section we make a number of strong assumptions which make cities and sectors symmetrical. Function intensity of sectors is linear in z so, as in figure 1, $a(z) = [1+\gamma(1-2z)]/2$ and $b(z) = [1-\gamma(1-2z)]/2$. This form is symmetric, with middle sector, z = ½, equally intensive in A and B; parameter γ measures the heterogeneity of function intensities across sectors. The labour input requirements of functions, λ_{ij}, are described below, and will be constructed to be symmetrical (so city 1's productivity advantage in A will be equal to city 2's advantage in B). Together with the assumption of symmetry of cities (developed explicitly in section 4), these conditions imply equality of wages in each city, $w_1 = w_2$, with common value denoted w. These assumptions enable us to derive a number of key results in this section. The full general equilibrium is set out in section 4 and asymmetric cases analysed in section 5.

3.1: Ricardian functional advantage.

Ricardian productivity differentials are captured by assuming that the labour input coefficients λ_{ij}, ($i = A, B, j = 1, 2$) are exogenous. City 1 has a productivity advantage in function A and city 2 has an equal advantage in function B, so we define $\Delta\lambda \equiv \lambda_{A2} - \lambda_{A1} = \lambda_{B1} - \lambda_{B2} > 0$. This supports full symmetry of cities and functions, so equilibrium will have $z_1 = 1 - z_2$, i.e. the mode-boundaries are equi-distant above and below the mid-sector, $z_2 - 1/2 = 1/2 - z_1$ (see figure 1). Explicit values for the mode-boundaries, z_1, z_2, then come from eqns. (2),

$$b(z_1) = T / w\Delta\lambda\,, \qquad\qquad a(z_2) = T / w\Delta\lambda\,.$$

We assume that fragmentation costs are incurred in labour, so $T = tw$.[4] Using our specific functional forms for $a(z)$, $b(z)$ this gives

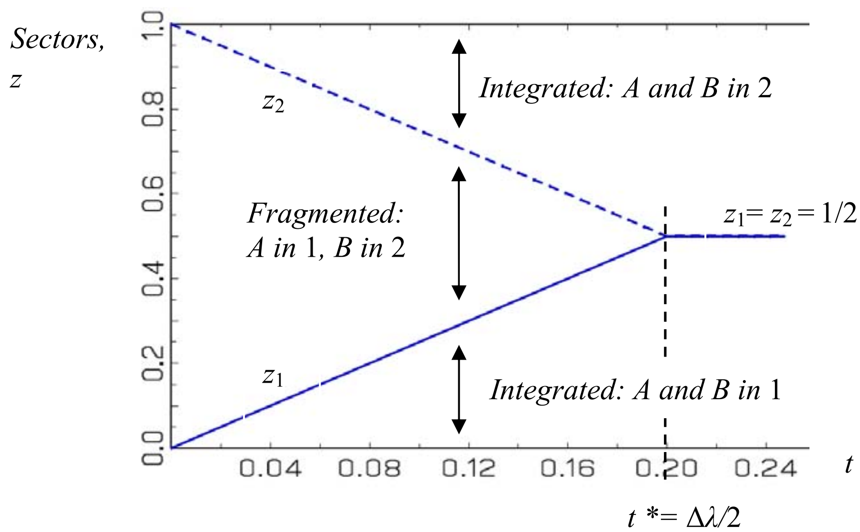
$$z_1 = \frac{1}{2}\left[1 - \frac{1}{\gamma}\left\{1 - \frac{2t}{\Delta\lambda}\right\}\right], \qquad\qquad z_2 = \frac{1}{2}\left[1 + \frac{1}{\gamma}\left\{1 - \frac{2t}{\Delta\lambda}\right\}\right]. \qquad (3)$$

Integration to fragmentation: Equations (3) capture the way in which the sourcing of functions by firms in each sector depends on fragmentation costs, technological differences, and function intensities. If $t = \Delta\lambda / 2$ then $z_1 = z_2 = ½$; this is the highest value of t at which any sector fragments; we refer to it as the critical value and denote it t^*. For $t \geq t^*$ all firms are integrated and sectors are partitioned between cities; city 1 has sectors $z < ½$, i.e. sectors intensive in function A, and city 2 has sectors $z > ½$.

If $t < t^*$, fragmented firms emerge, first in sectors that have similar use of both functions, i.e. z in an interval around ½ and of width $z_2 - z_1 = \frac{1}{\gamma}\left[1 - \frac{2t}{\Delta\lambda}\right]$, wider the smaller is t and the larger are productivity differences, $\Delta\lambda$. This and eqns. (3) are illustrated on figure 2, which has sectors on the vertical axis and fragmentation costs, t, on the horizontal. Thus, at $t < t^*$ the most A-intensive sectors operate with integrated firms in city 1, the most B-intensive are integrated in city 2, and those with intermediate function intensities are fragmented.

[4] In general this could be a combination of labour from both cities. With symmetric cities, the sourcing of this labour is irrelevant for profits.

Figure 2: Sectoral mode of operation.



The figure is constructed with $\gamma = 1$ and $\Delta\lambda = 0.4$. The critical value t^* is proportional to $\Delta\lambda$ and, for a given value of $t/\Delta\lambda$ the range of fragmented firms is larger the smaller is γ, the parameter that measures the range of function intensities.[5]

Sectoral to functional specialisation: Preceding paragraphs established where firms in each sector locate their activities. The dual question is: what activities take place in which cities? There are two aspects of this; what sectors are present in each city and, more importantly, what is the output of each function in each city, X_{Aj}, X_{Bj}?

The number of firms of each mode in sector z is denoted $n_k(z) \geq 0$, $k = 1, 2, F$. Only one mode is active in each range of z, so sectors are active (the number of firms non-zero) in city 1, in both, or in city 2 according as:

$$n_1(z) > 0 \text{ for } z < z_1; \qquad n_F(z) > 0 \text{ for } z_1 < z < z_2; \qquad n_2(z) > 0 \text{ for } z > z_2. \tag{4}$$

Output levels of each function in each city, X_{ij}, depend on demand from firms in each sector and city, their function intensity and their mode. They are given by

$$X_{A1} = \int_0^1 a(z)[n_1(z) + n_F(z)]dz\,, \qquad X_{B1} = \int_0^1 b(z)n_1(z)dz\,, \tag{5}$$

$$X_{A2} = \int_0^1 a(z)n_2(z)dx\,, \qquad X_{B2} = b(z)\int_0^1 [n_2(z) + n_F(z)]dz$$

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[5] The figure has $\gamma = 1$, this being the special case in which all sectors become fragmented ($z_1 = 0$ and $z_2 = 1$) at $t = 0$. If sectors are more similar in function intensity, $\gamma < 1$, then all sectors become fragmented at some positive value of t; if $\gamma > 1$ then extreme sectors use only one function (see fig 1).

For the remainder of this section we assume that the number of firms is constant and the same for all sectors, denoted N, so for each z, $n_1(z) + n_F(z) + n_2(z) = N$. The partition of sectors between cities is given by (3) so this, with (4) and (5) gives the following output levels. (See fig. 2 above for easy read-off; derivatives come from routine calculation and hold for $t \le t^*$).

$$X_{A1} = N\int_0^{z_2} a(z)dz = z_2[1+\gamma(1-z_2)]N/2, \qquad \frac{dX_{A1}}{dt} = -[1+\gamma(1-z_2)]\frac{N}{2\gamma\Delta\lambda} < 0. \qquad (6)$$

$$X_{B1} = N\int_0^{z_1} b(z)dz = z_1[1-\gamma(1-z_1)]N/2, \qquad \frac{dX_{B1}}{dt} = [1-\gamma(1-z_1)]\frac{N}{2\gamma\Delta\lambda} > 0.$$

$$X_{A2} = N\int_{z_2}^1 a(z)dz = (1-z_2)(1-\gamma z_2)N/2, \qquad \frac{dX_{A2}}{dt} = [1+\gamma(1-z_2)]\frac{N}{2\gamma\Delta\lambda} > 0.$$

$$X_{B2} = N\int_{z_1}^1 b(z)dz = (1-z_1)(1+\gamma z_1)N/2, \qquad \frac{dX_{B2}}{dt} = -[1-\gamma(1-z_1)]\frac{N}{2\gamma\Delta\lambda} < 0.$$

Given the dependence of mode-boundaries $\{z_1, z_2\}$ on t (eqn. 3), equations (6) indicate how varying fragmentation costs changes the pattern of activity in the economy. We summarise results in proposition 1.

Proposition 1:

i) If $t \ge t^* = \Delta\lambda/2$ then $z_2 = z_1 = 1/2$:

a) Mode: All firms in all sectors are integrated.

b) Sectors: Each sector operates in a single city.

c) Functions: Fraction $\frac{1}{2}\left[1+\frac{\gamma}{2}\right] \in [0.5, 0.75]$ of each function is produced in the city where it has comparative advantage.

ii) If $t < t^* = \Delta\lambda/2$ then $z_2 - z_1 = \frac{1}{\gamma}\left[1 - \frac{2t}{\Delta\lambda}\right] > 0$: reductions in t bring,

a) Mode: An increase in the range of sectors $z_2 - z_1$ in which firms are fragmented.

b) Sectors: A decrease in sectoral specialisation, i.e. an increase in the range of sectors, to which each city contributes at least one function.

c) Functions: An increase in functional specialisation as outputs of functions move further in line with cities' comparative advantage (see derivatives in (6)).

3.2. External economies of scale.

Ricardian efficiency differences might be due to differences in cities' history or physical geography but are exogenous. We now suppose that productivity is endogenous, determined by the scale of activity of each function in each city. Given the substantial evidence base on the presence of urban agglomeration economies this case is empirically relevant. It is also more complex although, since economies of scale are assumed to be external to the firm, we can keep the description of firms simple, as above.

Labour input coefficients are function and city specific, and are now assumed to be based on an endogenous part deriving from productivity spillovers in the same function and city, as well as a possible Ricardian component, which we now denote, Λ_{A1}, Λ_{B1}, Λ_{A2}, Λ_{B2}, and $\Delta\Lambda \equiv \Lambda_{A2} - \Lambda_{A1} = \Lambda_{B1} - \Lambda_{B2} \geq 0$. These productivity spillovers are denoted, s_{A1}, s_{B1}, s_{A2}, s_{B2}, with parameter σ_A, σ_B measuring the impact of spillovers in each function. The endogenous and Ricardian components of labour input coefficients are additive, giving

$$\lambda_{A1} = \Lambda_{A1} - \sigma_A s_{A1}, \quad \lambda_{A2} = \Lambda_{A2} - \sigma_A s_{A2}, \tag{7}$$

$$\lambda_{B1} = \Lambda_{B1} - \sigma_B s_{B1}, \quad \lambda_{B2} = \Lambda_{B2} - \sigma_B s_{B2}.$$

We assume that the spillovers generated by each function in each city are equal to output in the function/ city pair, so $s_{ij} = X_{ij}$, i = A, B, j = 1, 2. Hence, productivity differentials are, using eqns. (6) in (7),

$$\lambda_{B1} - \lambda_{B2} = \Delta\Lambda - \sigma_B N[-1/2 + z_1\{1 - \gamma(1 - z_1)\}], \tag{8a}$$

$$\lambda_{A2} - \lambda_{A1} = \Delta\Lambda - \sigma_A N[1/2 - z_2\{1 + \gamma(1 - z_2)\}], \tag{8b}$$

Thus, if z_2 is large a relatively small range of sectors undertake function A in city 2, this reducing city 2 productivity in A, i.e. raising $\lambda_{A2} - \lambda_{A1}$. If these spillovers are equally powerful in both functions, $\sigma \equiv \sigma_A = \sigma_B > 0$, and there is symmetry so $w = w_1 = w_2$, then the mode-boundaries (2) become,

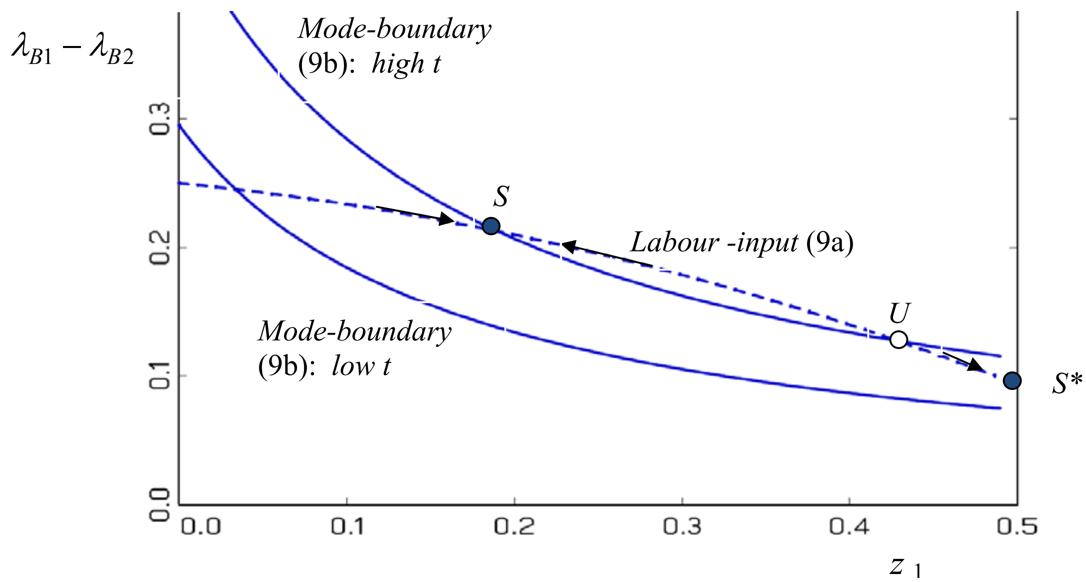
$$\pi_F(z_1) - \pi_1(z_1) = [1 - \gamma(1 - 2z_1)][\lambda_{B1} - \lambda_{B2}]w/2 - tw = 0, \tag{9a}$$

$$\pi_F(z_2) - \pi_2(z_2) = [1 + \gamma(1 - 2z_2)][\lambda_{A2} - \lambda_{A1}]w/2 - tw = 0. \tag{9b}$$

To analyse these relationships, we focus on (9a) and (9b), two equations in z_1 and $\lambda_{B1} - \lambda_{B2}$ (the other pair being symmetric). Figure 3 illustrates these equations. The labour-input relationship (9a, dashed line) is downward sloping as a higher value of z_1 increases city 1 output of function B, thereby reducing λ_{B1}, the labour input requirement in 1 (and increasing λ_{B2}; with constant returns and Ricardian differences, this line would be horizontal). The mode-boundary relationship is also downwards sloping, because higher productivity in function B will enlarge the set of sectors operating in integrated mode in city 1. The mode-boundary depends on fragmentation costs, and is drawn for two values of t. Given labour-input requirements, a higher t means that more sectors are integrated, hence the curve lies to the right.

Looking at the mode boundary for the higher value of t, there are three equilibria. The left-most, labelled S, is 'stable', as sectors to the left of this point have profits in integrated mode higher than in fragmented mode; starting to the left of this point sectors become integrated, increasing z_1. Equilibrium U is unstable, and the right-most equilibrium is S*, where $z_1 = 1/2 = z_2$, the boundary at which all sectors are integrated.

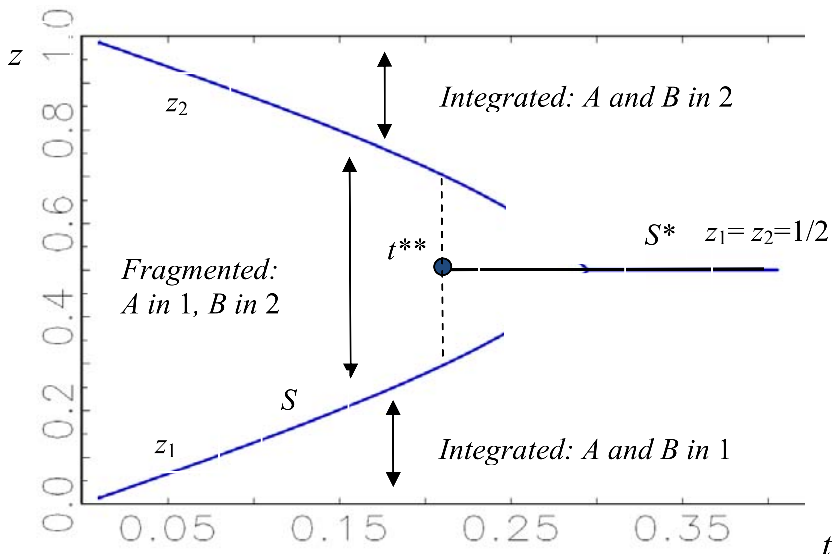
Figure 3: Productivity and mode boundaries



From this figure we can see how varying t changes equilibrium outcomes. At very high t the curves do not intersect, so the unique equilibrium is at S*, with all sectors integrated. At somewhat lower levels there are three equilibria, as discussed above. There is a critical value t^{**} at which points U and S* merge. This is easily found analytically; it is the value of t at which both (9a) and (9b) hold at $z_1 = 1/2$, giving $t^{**} = \left[\Delta\Lambda + \sigma N\gamma / 4\right]/2$. This reduces to the Ricardian case if $\sigma = 0$, while $\sigma > 0$ implies a strictly higher critical point t^{**}. At values of $t < t^{**}$ full integration ceases to be an equilibrium, and there is a unique equilibrium value of z_1. This tracks to the left – fewer sectors integrated – as t falls, possibly reaching the boundary with all sectors fragmented, $z_1 = 0$.

This is illustrated in figure 4, giving mode-boundaries for the fully integrated equilibrium S^* and for the equilibrium with fragmentation, S. Solid lines on the figure are equilibrium values of $\{z_1, z_2\}$. The appendix gives further details on parameter values at which various outcomes occur, and results are summarised in proposition 2.

Figure 4: Sectoral mode of operation with increasing returns to scale



Proposition 2:

i) If $t > t^{**} = [\Delta\Lambda + \sigma N\gamma / 4]/2$ there is an equilibrium in which all sectors are integrated.

ii) If $t < t^{**}$ there is a unique equilibrium, in which a range of firms are fragmented.

iii) There is a range of values of $t > t^{**}$ at which there are multiple equilibria. In this range integration of all sectors is an equilibrium, and so too is fragmentation of an intermediate range of sectors.

iv) Increasing returns ($\sigma > 0$) means that, should fragmentation occur, the range of sectors that are fragmented is wider, at each t and for each $\Delta\Lambda$, than if $\sigma = 0$.

4. General Equilibrium: wages, prices, and industry scale

To this point we have assumed that product prices are constant, that there is a fixed and equal number of firms in all sectors, and that there is sufficient symmetry for wages to be the same in both cities. We now lay out the general equilibrium framework needed to move beyond these cases.

4.1 Geographical structure and wages.

The single country on which we focus contains three locations. One is a hinterland region, producing an 'outside good' using labour alone under constant returns to scale. This good is numeraire, and labour productivity in the sector gives fixed wage w_0. The other two are cities, home to sectors *z* and functions *A* and *B*. Labour is perfectly mobile between cities and the outside region.

City workers face additional urban costs of commuting and high land prices. This means that the cost of living may vary across locations, in which case labour mobility implies that the equilibrium wages paid by producers in each city, w_1, w_2, may differ from w_0 and from each other. Urban costs depend on city size as described by the simplest form of the standard urban model (the Alonso-Mills-Muth model, Henderson and Thisse 2004). Each household occupies one unit of land and the rent in city *j* at distance *r* from the centre is $h_j(r)$. All urban jobs are in the city centre (CBD), and commuting costs are c_j per unit distance. Workers choose residential location within and between cities so (since final goods prices are the same everywhere), real wages are equalised when $w_j - c_j r - h_j(r) = w_0$ for all *j*, *z*. In a linear city in which there are *K* spokes from the CBD, along which people live and commuting takes place, population is $L_j = K r_j^*$, where r_j^* is the edge of the city (length of each spoke). At the city edge land rent must be zero, so $w_0 = w_j - c_j r_j^* = w_j - c_j L_j / K$ giving the city-size equations

$$L_1 = (w_1 - w_0)K / c_1, \qquad L_2 = (w_2 - w_0)K / c_2 . \tag{11}$$

It should be noted that L_j denotes both the number of residents and the number of workers in the city.

These equations simply say that larger cities have to pay higher wages in order to cover the commuting costs and rents incurred by workers. Finally, we note that rent in each city can be expressed as, $h_j(z) = w_j - w_0 - c_j r = c_j(L_j / K - r)$, so integrating over *r* and adding over all spokes, total rent in a city of size L_j is

$$H_j = c_j L_j^2 / 2K . \tag{12}$$

Thus, while workers' utility is equalised across all locations, the productivity gap associated with $w_1, w_2 > w_0$ is partly dissipated in commuting costs, with the rest going to recipients of land rents.

4.2 Free entry and trade

We now turn to specification of a full general-equilibrium model in which prices, outputs numbers of firms, and trade are all endogenous. We present the equations for the spillover case, since that is the more complicated of the two. Most of the notation needed for this is already in place. We now need have two countries, domestic and foreign. Variable $Q_d(z)$ which will denote the "domestic" country's total production of sector z. In equilibrium, this will equal the sum of domestic purchases of domestic goods, $Q_{dd}(z)$, and foreign purchases of domestic goods, $Q_{fd}(z)$. $Q_{df}(z)$ denotes domestic purchases of foreign goods.

The model now becomes larger in terms of dimensions and features a lot of simultaneity. In math programming language, it is a non-linear complementarity problem, in which corner solutions (which firm types are active or inactive in which industries in which cities) is a crucial feature of the model. Because of this, we discretise the number of sectors: in the simulations to follow model development, there are 51 z sectors. The variables of the model are as follows (other are computed after model solution):

Non-negative variables

L_1, L_2	labor demand or employment in city i
w_1, w_2	wages in city i
$X_{A1}, X_{A2}, X_{B1}, X_{B2}$	output of function k = (A,B) in city j
$\lambda_{A1}, \lambda_{A2}, \lambda_{B1}, \lambda_{B2}$	labor requirements in function k in city j
$Q_d(z)$	total output of sector z (all firm types)
$Q_{df}(z)$	domestic demand for foreign goods
$n_1(z), n_2(z), n_F(z)$	number of firms of type l, 2, F in sector z
$p(z)$	price of (domestic) good z

With the dimension of z equal to 51, the model has 318 non-negative variables complementary to 318 weak inequalities. A strict inequality corresponds to a zero value for the complementary variable.

First, the supply-demand relationships for labor demand in the two cities are given as follows, where ⊥ denotes complementarity between the inequality and a variable.

$$L_1 \geq \sum_z n_1(z)(a(z)\lambda_{A1} + b(z)\lambda_{B1}) + n_F(z)(a(z)\lambda_{A1}) \quad \perp \quad L_1 \tag{13}$$

$$L_2 \geq \sum_z n_2(z)(a(z)\lambda_{A2} + b(z)\lambda_{B2}) + n_F(z)(b(z)\lambda_{B2}) \quad \perp \quad L_2 \tag{14}$$

Second, wages are given from (11)

$$(w_1 - w_0)K/c \geq L_1 \quad \perp \quad w_1 \tag{15}$$

$$(w_2 - w_0)K/c \geq L_2 \quad \perp \quad w_2 \tag{16}$$

Third, output levels of the two functions in the two cities are given by

$$X_{A1} \geq \sum_z a(z)(n_1(z) + n_F(z)) \quad \perp \quad X_{A1} \tag{17}$$

$$X_{A2} \geq \sum_z a(z)n_2(z) \quad \perp \quad X_{A2} \tag{18}$$

$$X_{B1} \geq \sum_z b(z)n_1(z) \quad \perp \quad X_{B1} \tag{19}$$

$$X_{B2} \geq \sum_z b(z)(n_2(z) + n_F(z)) \quad \perp \quad X_{B2} \tag{20}$$

Fourth, the labor input coefficients (inverse productivity) are given by

$$\lambda_{A1} \geq \Lambda_{A1} - \sigma_A X_{A1} \quad \perp \quad \lambda_{A1} \tag{21}$$

$$\lambda_{A2} \geq \Lambda_{A2} - \sigma_A X_{A2} \quad \perp \quad \lambda_{A2} \tag{22}$$

$$\lambda_{B1} \geq \Lambda_{B1} - \sigma_B X_{B1} \quad \perp \quad \lambda_{B1} \tag{23}$$

$$\lambda_{B2} \geq \Lambda_{B2} - \sigma_B X_{B2} \quad \perp \quad \lambda_{B2} \tag{24}$$

The number of active firms of each type in each sectors is complementary to a zero-profit condition, that unit cost is greater than or equal to price. We use a simple formulation of the fragmentation cost: $t(w_1 + w_2)/2$. Note that all inequalities are homogeneous of degree 1 in wages and prices.

$$w_1(a(z)\lambda_{A1} + b(z)\lambda_{B1}) \geq p(z) \quad \perp \quad n_1(z) \tag{25}$$

$$w_2(a(z)\lambda_{A2} + b(z)\lambda_{B2}) \geq p(z) \quad \perp \quad n_2(z) \tag{26}$$

$$w_1 a(z)\lambda_{A1} + w_2 b(z)\lambda_{B2} + t(w_1 + w_2)/2 \geq p(z) \quad \perp \quad n_F(z) \tag{27}$$

Total output of good z is given by the sum the outputs across firm types.

$$Q_d(z) \;\geq\; n_1(z) + n_2(z) + n_z(z) \qquad \perp \qquad Q_d(z) \tag{28}$$

The final element is to specify the demand size of the model, which links outputs, prices, and the external foreign market. We give the demand functions here in order to complete specification of the model and then return to the utility and budget constraint which generate these demand functions in a short appendix.

The domestic country is assumed small as an importer, and so all foreign prices for the z sectors are given by an exogenous value $\bar{p}$, common across all sectors. Domestic and foreign goods within a sector are CES substitutes with an elasticity of substitution $\varepsilon > 1$. Sectoral composites (domestic and foreign varieties) are Cobb-Douglas substitutes. The agricultural good *R* is treated as a numeraire. It is additively separable with a constant marginal utility and hence income does not appear in the demand functions for the *Q* goods (though we will introduce a demand shifter later).

The market clearing equation for the domestic good z is that supply equal the sum of domestic and foreign demand. α_d and α_f are "short hand" scaling parameters for domestic and foreign, that could depend on the relative market sizes for example (see appendix). θ_d and θ_f are the weights on the domestic and foreign varieties in the nest for each sector z.

$$\begin{aligned} Q_d(z) \;=\; Q_{dd}(z) + Q_{fd}(z) \;=\; & \theta_d(p(z))^{-\varepsilon}(\theta_d p(z)^{1-\varepsilon} + \theta_f \bar{p}^{1-\varepsilon})^{-1}\alpha_d \\ & + \theta_d(p(z))^{-\varepsilon}(\theta p_d(z)^{1-\varepsilon} + \theta_f \bar{p}^{1-\varepsilon})^{-1}\alpha_f \qquad \perp\; p(z) \end{aligned} \tag{29}$$

Domestic demand for foreign goods is not needed to solve the core model, but is needed for welfare calculations after solution. These are given by

$$Q_{df}(z) \;=\; \theta_f(\bar{p})^{-\varepsilon}(\theta_d(z)^{1-\varepsilon} + \theta_f \bar{p}^{1-\varepsilon})^{-1}\alpha_d \qquad \perp\; Q_{df}(z) \tag{30}$$

As noted above, the core model is then 318 weak inequalities complementary with 318 non-negative unknowns.

4.3 Symmetric Ricardian and spillovers cases in general equilibrium

Figures A1 to A7 present simulation results that develop economic implications of the model. Figure A1 presents the symmetric Ricardian case, with fragmentation costs *t* on the horizontal axis. Each column of the figure is a solution to the model for that value of *t*, as will be the case in the following figures (the jagged line is a consequence of the discreteness of sectors). The results naturally qualitatively resemble Figure 2 earlier in the paper. With all firms integrated, the middle sector (there is an odd number of sectors, 51) is produced in both countries.

Figure A2 shows further results for this case in four panels. The upper left panel gives Herfindahl concentration indices for sector concentration and function concentration for each level of fragmentation costs. The sector concentration is the sum over sectors of the squared share of industry output located in city 1 plus the squared share in city 2 divided by the number of sectors (a normalization such that the index equals 1 if each sector produces only in one city). Output of the fragmented sectors is divided according to the task intensities of those sectors.

The function concentration index is the sum over tasks and cities of the share of task i produced in city j squared, divided by 2 (a normalization such that the index equals 1 if each task is produced in only one city). The upper left panel illustrated a principal result of the paper, that cities become more specialized in functions and less in sectors as fragmentation costs fall.

The upper right panel of Figure A2 illustrated an effect which was not discussed in previous sections. The fall in fragmentation costs improves the competitiveness of the urban (manufacturing and services) sectors relative to the rural (agricultural) good. With trade balance in urban sectors calibrated to zero at zero fragmentation costs, the trade balance with the foreign world is negatively related to fragmentation costs. Ease of internal transport and communications is a source of comparative advantage.

The bottom left panel of Figure A2 graphs the producer wage and welfare (recall all workers earn a wage of w_0 after commuting costs and land rent). Note from equations (15) and (16) that the producer wage is proportional to urban population or city size. Thus the very flat producer wage shown in the bottom left of Figure A2 indicates that a lowering a fragmentation costs doesn't have a big effect on city size: increased outputs depress product prices some and so from the free-entry conditions, producer wages (city populations) don't change much. The increase in welfare as fragmentation costs fall is small. Part of potential welfare gains is dissipated by falling prices (worsening terms of trade) due to the increased domestic productivity. Average Q prices are 2.5% lower with full fragmentation than under fully integrated production. This fall in prices also holds down urbanization (producer wages and employment) as fragmentation costs fall.

The bottom right-hand panel of Figure A2 relates to the findings of A. Markusen and Barbour (2003) mentioned earlier. The graph shows results (arbitrarily) for city 1 and function A. Across sectors, we use the results to get the "function intensity" of actual employment; that is, what is function A's share of employment in sector z. Then the column of these employment intensities are correlated with the function intensity *a(z)* of integrated production for each sector. So, for example, if all sectors are fragmented, then the employment share of function A is 1.0 in all sectors in city 1, and hence the correlation of these with the overall A-function intensity of sector A is zero. If all sectors are integrated, the employment intensity is the same as the sector function intensity for all sectors in which there is employment. Since there is no employment in some sectors, the correlation when all sectors are integrated is somewhat less than one.

This correlation in the bottom right-hand panel of Figure A3 is something that we can examine in the data. The theoretical results indicate that as fragmentation costs fall, the function (occupational) specialization of cities becomes less correlated with sectoral (industrial) specialization.

Turning to the spillovers case, Figure A3a shows results confirming those in Figure 3 earlier.

There is a region of multiple equilibria: one in which all sectors are integrated and one in which some (middle) sectors are fragmented. Results corresponding to those in Figure A2 for the Ricardian case are qualitatively the same as for the Ricardian case, and thus we won't show them here.

One thing that is qualitatively different between the Ricardian and spillovers cases is the effect of increasing demand (increases in α_d α_f in (29 and (30)) on the equilibrium regime. In the Ricardian case in which the λ's are constants, a symmetric situation ($w_1 = w_2$) means that the boundaries between the integrated and fragmented sectors do not depend on demand (also true in the partial-equilibrium case as seen in (3)).

However, in (7) and here in (21)-(24) we see that increases in total market demand will affect the λ's and hence will affect regime boundaries in the spillovers case. Figure A3b shows the effect on the regime boundaries following a 50 percent increase in α_d and α_f. For middle levels of *t*, additional sectors will now fragment as shown, which implies increases function specialization and lower sectoral specialization for a given level of fragmentation costs.

Although the level of demand does not affect the integration / specialization pattern in the Ricardian case, an increase in demand does lead to large cities in both the Ricardian and spillovers cases. These results are shown in Figure A4. Higher demand shift up the employment / city size curves as shown. So urbanization follows from higher demand. Although we have not modeled income elasticities or demand here, we can think of this as a parable for a world in which the urban sectors have a high income-elasticity of demand such that rising per-capita incomes (for whatever reason) shift demand toward the income-elastic urban goods, thereby increasing urbanization.

4.4 Asymmetric cases

Figures A5 and A6 consider some asymmetry between the sectors/cities. Figure A5 assumes that $\lambda_{A1} < \lambda_{A2} = \lambda_{B1} = \lambda_{B2}$. That is, city 1 has a comparative and absolute advantage in function A, while city 2 has a comparative advantage in function B, but no absolute advantage. For intermediate or high levels of fragmentation costs, the result in Figure A5 is that city 1 will have a larger range of integrated industries. The intuition follows from a simple argument by contradiction. Suppose that the solution was symmetric across cities. Then if sector (1-*z*) ($z >$ 0.5) is just breaking even in city 2, there would be positive profits for sector *z* in city 1.

Figure A6 shows a similar result for the spillovers case: here only function A has spillovers, but in both cities (in contrast to the Ricardian case where only λ_A is smaller in city 1 only). In equilibrium however, the spillovers case is similar: city 1 will have an a comparative and absolute advantage in function A, while city 2 has a comparative but not absolute advantage in function B.

These results show up as differences in city size/employment (which in turn translate into producer wages), shown in the right-hand panels of Figures A5 and A6. The city size difference is large when all industries are integrated and small when all are fragmented (though largest in the middle for the spillovers case). Again, the intuition follows from a simple argument by contradiction. If city sizes (employment) were the same, then producer wages would be the same, in which case there must be positive profit opportunities in city 1 and/or losses incurred in city 2.

The convergence in city sizes as fragmentation costs become small seems to be in large part a terms-of-trade effect: as fragmentation costs fall, the relative prices of goods with low sector indices (located in city 1) fall a lot more in general equilibrium than the prices of the high index goods. An alternative way to think about this is that the high productivity of city 1 workers in the A function means that less workers are required to produce those tasks at given output prices and hence city 1's employment falls some in response to that increased productivity.

4.5 Multi-function example

Figure A7 presents results for a multi-function case, the analysis of which is very preliminary as of this draft of the paper. The simulation is for a symmetric Ricardian case with 12 sectors and six functions. The difficulty with a multi-function case is that there is no unambiguous way to think about function intensities of sectors in a multi-function case (an old problem characterizing the n-good, n-factor Heckscher-Ohlin analysis of the 1970s). Factor intensity is inherently a binary concept. What we have done in this simulation is shown in the matrix of function intensities in the upper right-hand panel of Figure A7. Middle sectors 6 and 7 have relatively even function intensities across functions. As we move up or down the list of industries, factor intensities move toward being more uneven, with low-index sectors being intensive in functions 1-3 and high-index sectors being intensive in functions 4-6. This is obviously a very simple case where intensity rankings are pretty clear. The lower right-hand panel of A7 show the matrix of λs, with city 1 having a comparative advantage in low function index functions. Combining these two right-hand panels, city 1 will have an unambiguous comparative advantage in low index sectors.

Another problem is how to define fragmented firms. There are potentially a great many firm types, where "type" is defined by the number of functions in city 1 with the rest in city 2. In addition, some potential firm types might do each of two functions in separate cities and some in both cities. The count of potential firm types is large. This problem is quite familiar to those of us working in the multinationals' literature. What we have done in the simulations for Figure A7 is simply assume a single fragmented firm type, with has functions 1-3 in city 1 and 4-6 in city 2.

With only three firm types, the top left panel of Figure A7 looks similar to earlier results. The advantage of multiple functions is that we can now talk more meaningfully about city function concentration across functions (with only two functions, the concentration of A and B across cities is the same in the symmetric case). In the middle panel of Figure A7, we show Ellison-Glaeser indices for the employment concentration of each industry across cities. Similar to our earlier results on the Herfindahl index of sector concentration, here we see results by sector and see that sector concentration falls with falling fragmentation costs, and falls most markedly for the middle sectors. Low fragmentation costs produce heterogeneous concentration figures across industries while they are homogeneous with high fragmentation costs.

The bottom left panel of Figure A7 show Ellison-Glaeser indices for employment concentration by function. As per our earlier results, these increase as fragmentation costs fall and increase more for the "fringe" functions. The indices of function specialization become more homogeneous as fragmentation costs fall.

This analysis of a multi-function case is extremely preliminary in this draft as noted earlier. However, any more complete analysis is going to run into the difficulties here in defining function intensities and in somehow limiting the number of firm types.

5. Toward empirical analysis

We have just begun an empirical investigation into the issues raised in this paper as of the writing of this draft. In order to show where we are going and to solicit comments, we have attached some data plots to the end of the paper. We will try to work with two details data sources.

First, data on the concentration/dispersion of employment by industry (sector) has been obtained from the US Census Bureau's *Country Business Patterns*. This gives us 965 5-digit NAICS industry codes. For each of these, we calculate an Ellison-Glaeser employment concentration index for each industry across states. These are plotted in Figure B1, grouped according to the broad 1-digit classification. The plot shows a lot of dispersion within the broad catagories.

Second, data on the concentration/dispersion of employment by occupation (function) has been obtained from the Bureau of Labor Statistics' *Occupational Employment Statistics*. This gives 568 catagories. For each of these, we calculate an Ellison-Glaeser employment concentration index for each occupation across states. These are plotted in Figure B2, grouped by broad classification. The plot shows a lot of dispersion within the broad categories as in the case of the industry employment concentration indices.

Figures B3 and B4 give a time-series picture, plotting 2010 figures again 2000 values. Somewhat to our disappointment at this early stage, there is no strong evidence of increased dispersion or concentration over this time period. Obviously, a lot more work needs to be done.

6. Summary and conclusions

Under construction. Comments and suggestions most welcome.

Appendix 2: specification of utility and income.

The specification of utility (welfare) is quite standard for trade models. The Q goods are a two-level ces nest. Domestic and foreign varieties for any z sector have an elasticity of substitution of $\varepsilon > 1$ whereas goods from different z sectors are Cobb-Douglas substitutes. R is the agricultural good, giving a standard quasi-linear utility function

$$U = \beta \ln\left(\prod_z \left(\theta_d \left(\frac{Q_{dd}(z)}{\theta_d} \right)^{\frac{\varepsilon - 1}{\varepsilon}} + \theta_f \left(\frac{Q_{df}(z)}{\theta_f} \right)^{\frac{\varepsilon - 1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon - 1}} \right) + R \qquad \text{(A1)}$$

where β is a scaling parameter. Income (Y) is given the sum of wages (net of commuting costs and rents = w_0) for all rural and urban workers ($\bar{L}$) plus land rents H_1 and H_2 from (12).

$$Y = w_0 \bar{L} + H_1 + H_2 \qquad \text{(A2)}$$

The domestic economy's budget constraint is that Y is spend on R (used as numeraire) plus domestic and foreign urban goods.

$$Y = R + \sum_z p(z) Q_{dd} + \sum_z \bar{p} Q_{fd} \qquad \text{(A3)}$$

(A3) can be substituted into (A1) to replace R.

$$U = \beta \ln\left(\prod_z \left(\theta_d \left(\frac{Q_{dd}(z)}{\theta_d} \right)^{\frac{\varepsilon - 1}{\varepsilon}} + \theta_f \left(\frac{Q_{df}(z)}{\theta_f} \right)^{\frac{\varepsilon - 1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon - 1}} \right) + Y - \sum_z p(z) Q_{dd} - \sum_z \bar{p} Q_{fd}$$

(A4)

Maximization of (A4) with respect to the Q's (and equivalently for foreign) yields the demand functions in the body of the paper, which do not depend directly on Y as is the usual result in quasi-linear preferences. Domestic demand for domestic good z for example is:

$$Q_{dd}(z) = \theta_d (p(z))^{-\varepsilon} (\theta_d p(z)^{1-\varepsilon} + \theta_f \bar{p}^{1-\varepsilon})^{-1} \alpha_d \qquad \text{(A5)}$$

where α_d is a scaling parameter that is increasing in β (β_d which could differ from the foreign β_f). Suppose $\theta_d = \theta_f = 0.5$ and all $p(z) = \bar{p} = 1$. Then $\alpha = 2$ in the demand functions implies $\beta = 2^{1/\varepsilon}$ and $Q_{ij} = 1$. Parameters α_d and α_f in the demand functions in section 2 are increasing in the β of the domestic or foreign economy, and increases in the α's or β's can represent increases in or differences in market size.[1]

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[1] [1]My algebra indicates that the relationship between the β in (A1) and the α in the demand functions above are related by $\alpha = 2(\beta/2)^{\frac{\varepsilon}{1+2\varepsilon}}$. Because of the concavity of the log formulation of utility, β must more than double to double market demand (α) at constant prices.

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Figure A1: Symmetric Ricardian Case

Ricardian comparative advantage, free entry, no spillovers

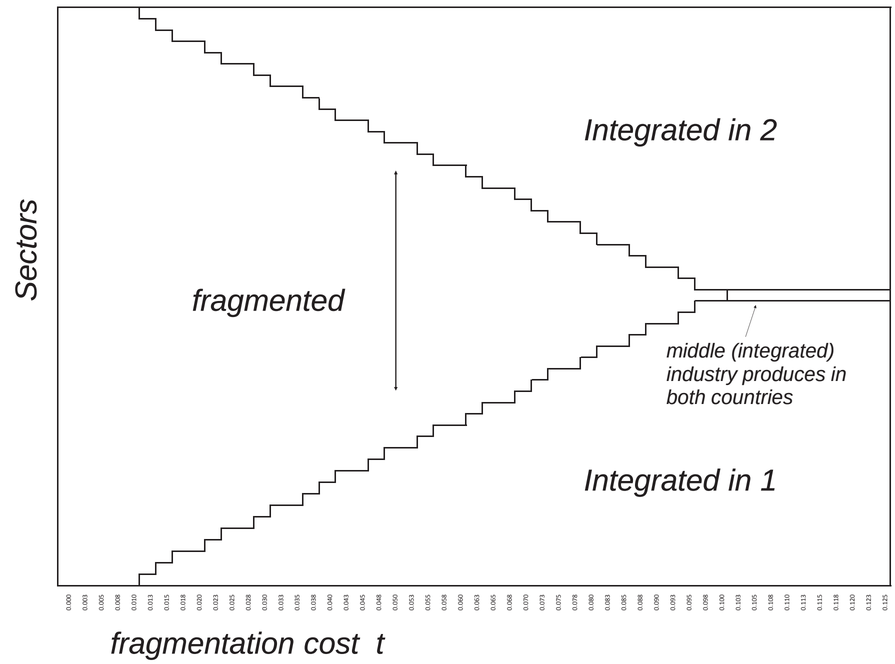


Figure A2: Symmetric Ricardian Case (fragmentation cost t on horizontal axes)

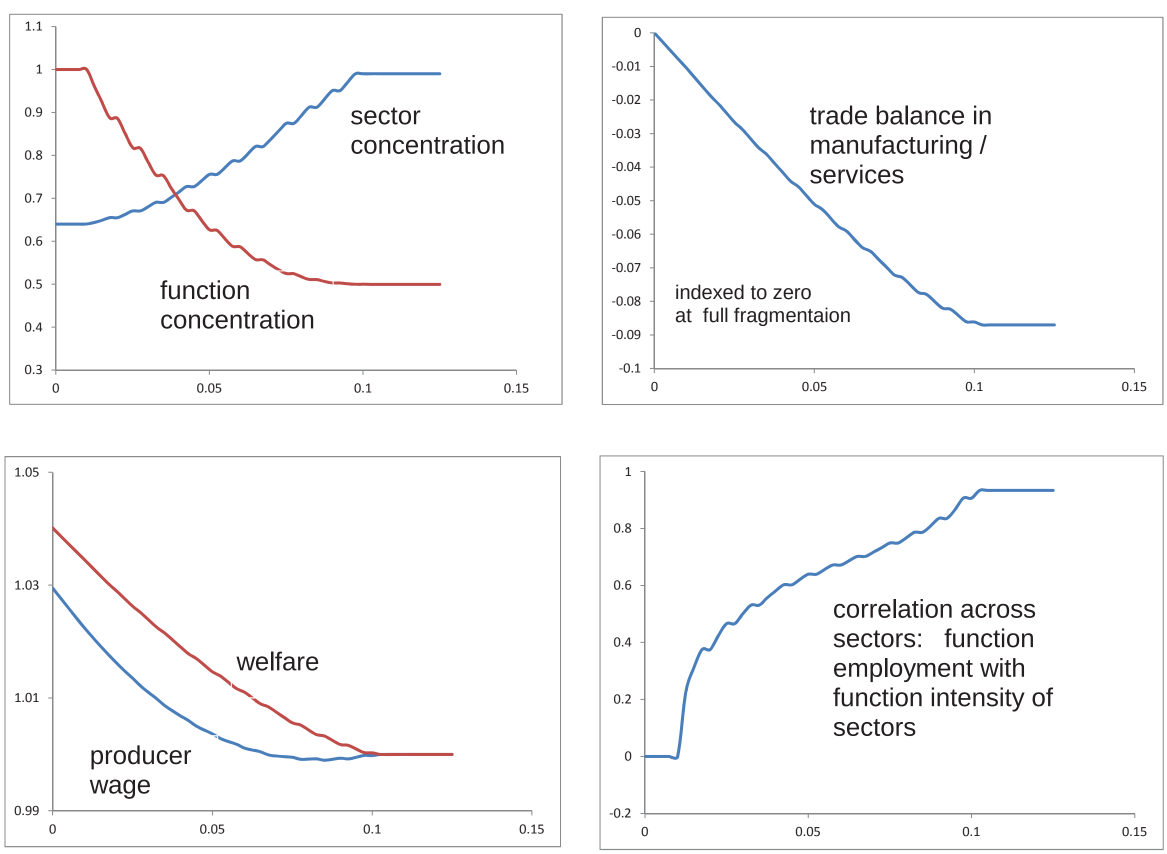
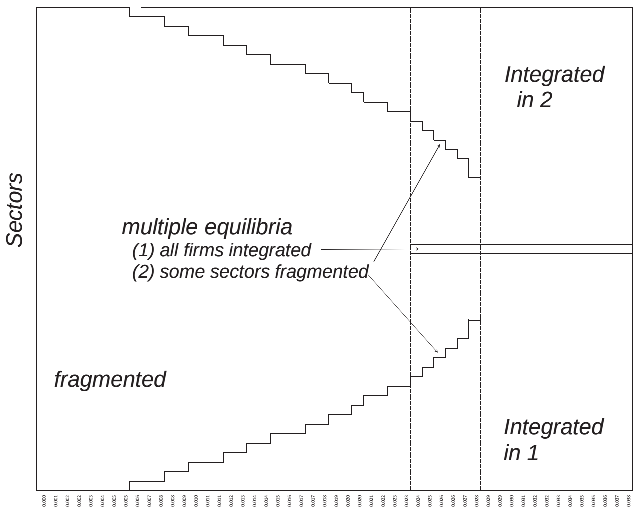


Figure A3a: Symmetric Spillovers Case

Spillovers, free entry, no Ricardian comparative advantage

Base Case: region of multiple equilbria

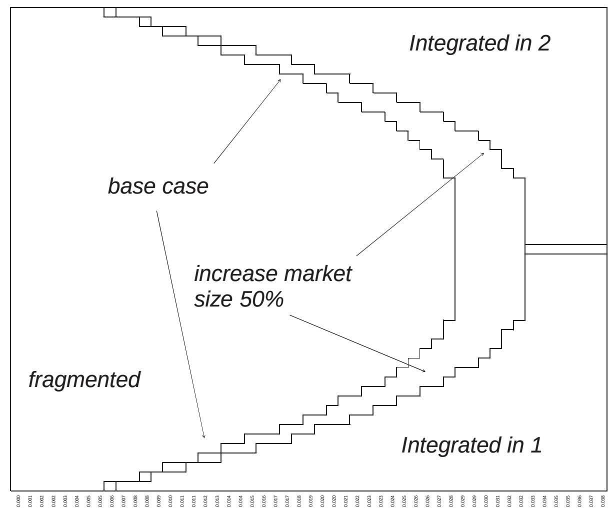


fragmentation cost *t*

Figure A3b: Symmetric Spillovers Case

Spillovers, free entry, no Ricardian comparative advantage

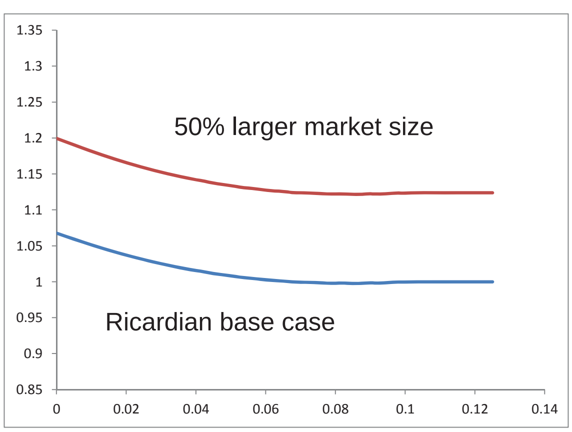
Base case, + 50% increase in market size



fragmentation cost *t*

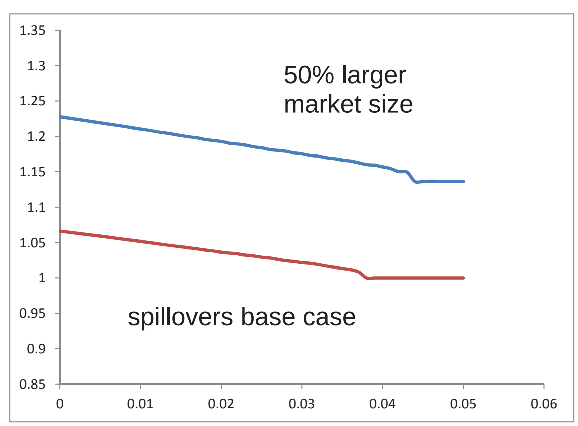
Figure A4: fragmentation cost versus market size and city employment (both cities have same employment)

Ricardian case city employment (size)



fragmentation cost t

spillover case city employment (size)



fragmentation cost t

Figure A5: Asymmetric Ricardian Case
City 1: comparative and absolute advantage in function A

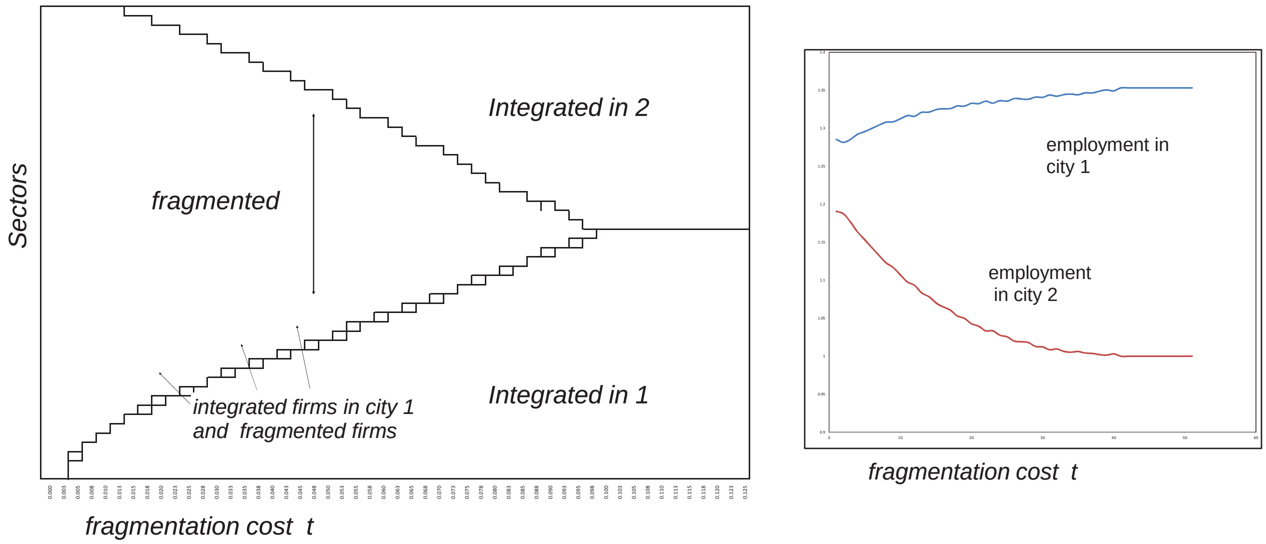


Figure A6: Asymmetric Spillovers Case
spillovers in function A only

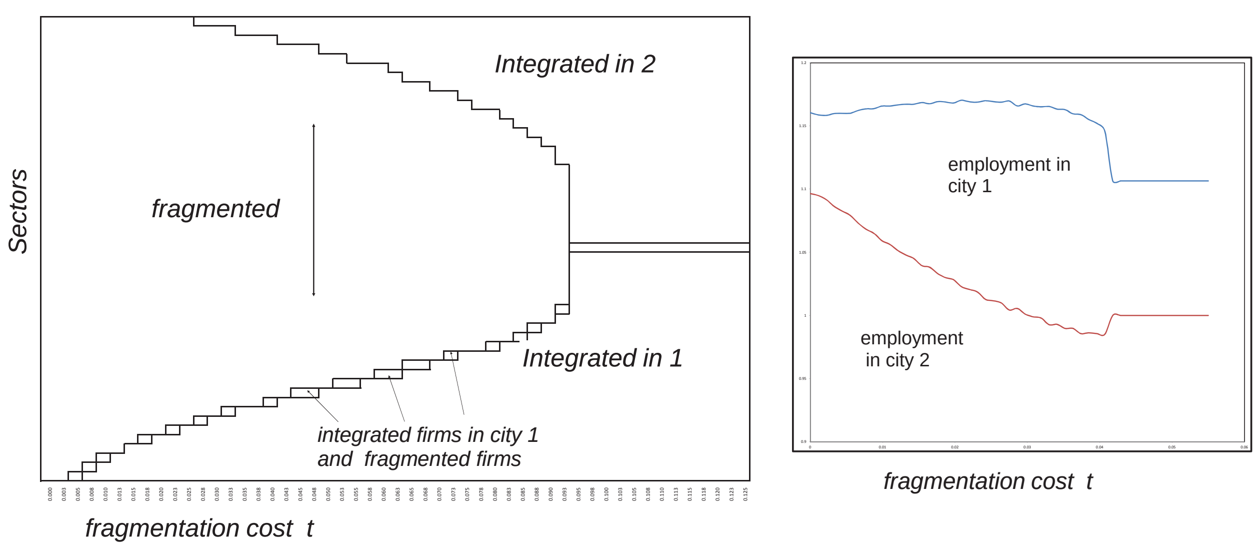
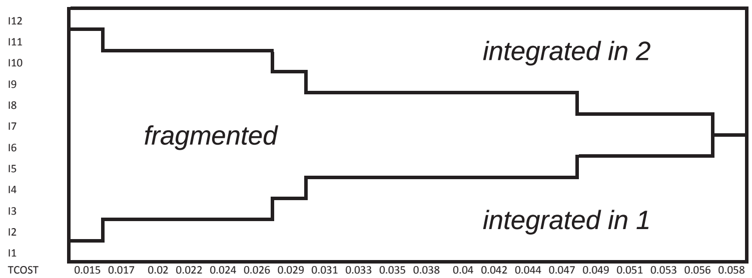


Figure A7: Multi-task Ricardian Model

12 sectors 6 functions



Matrix of function intensities

	K1	K2	K3	K4	K5	K6
I12	0	0	0	0.333	0.333	0.333
I11	0.083	0	0	0.333	0.333	0.250
I10	0.083	0.083	0	0.333	0.250	0.250
I9	0.083	0.083	0.083	0.250	0.250	0.250
I8	0.167	0.083	0.083	0.250	0.250	0.167
I7	0.167	0.167	0.083	0.250	0.167	0.167
I6	0.167	0.167	0.250	0.083	0.167	0.167
I5	0.167	0.250	0.250	0.083	0.083	0.167
I4	0.250	0.250	0.250	0.083	0.083	0.083
I3	0.250	0.250	0.333	0	0.083	0.083
I2	0.250	0.333	0.333	0	0	0.083
I1	0.333	0.333	0.333	0	0	0

Ellison- Glaeser type index for sector concentation

I12	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
I11	0.706	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
I10	0.454	0.454	0.454	0.454	0.454	0.454	1	1	1	1	1	1	1	1	1	1	1	1	1	1
I9	0.250	0.250	0.250	0.250	0.250	0.250	0.250	1	1	1	1	1	1	1	1	1	1	1	1	1
I8	0.116	0.116	0.116	0.116	0.116	0.116	0.116	0.116	0.116	0.116	0.116	0.116	0.116	0.116	0.116	1	1	1	1	1
I7	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030	1
I6	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030	1
I5	0.116	0.116	0.116	0.116	0.116	0.116	0.116	0.116	0.116	0.116	0.116	0.116	0.116	0.116	0.116	1	1	1	1	1
I4	0.250	0.250	0.250	0.250	0.250	0.250	0.250	1	1	1	1	1	1	1	1	1	1	1	1	1
I3	0.454	0.454	0.454	0.454	0.454	0.454	1	1	1	1	1	1	1	1	1	1	1	1	1	1
I2	0.706	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
I1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
TCOST	0.015	0.017	0.02	0.022	0.024	0.026	0.029	0.031	0.033	0.035	0.038	0.04	0.042	0.044	0.047	0.049	0.051	0.053	0.056	0.058

Matrix of Lambdas

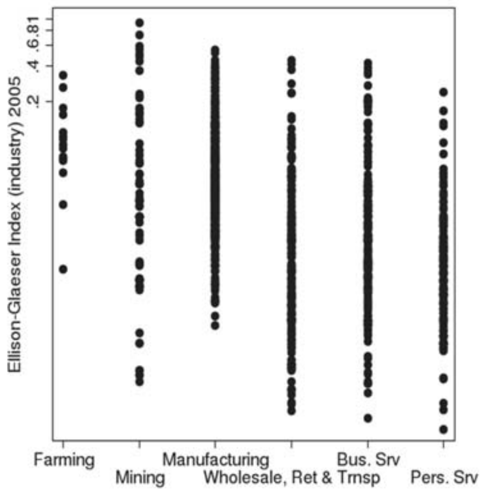
	K1	K2	K3	K4	K5	K6
LAM1	0.9	0.94	0.98	1.02	1.06	1.1
LAM2	1.1	1.06	1.02	0.98	0.94	0.9

Memory jog: lambdas are labor input requirements
inverse of productivity
City 1 has comparative advantage in low K functions

Ellison- Glaeser type index for function concentation

K1	1	0.810	0.809	0.808	0.807	0.806	0.640	0.496	0.495	0.495	0.494	0.493	0.492	0.491	0.490	0.271	0.271	0.270	0.270	0.123
K2	1	1	1	1	1	1	0.824	0.668	0.667	0.667	0.666	0.665	0.665	0.664	0.664	0.53	0.529	0.529	0.529	0.315
K3	1	1	1	1	1	1	1	0.838	0.838	0.837	0.837	0.837	0.836	0.836	0.836	0.693	0.693	0.692	0.692	0.567
K4	1	1	1	1	1	1	1	0.838	0.838	0.837	0.837	0.837	0.836	0.836	0.836	0.693	0.693	0.692	0.692	0.567
K5	1	1	1	1	1	1	0.824	0.668	0.667	0.667	0.666	0.665	0.665	0.664	0.664	0.53	0.529	0.529	0.529	0.315
K6	1	0.810	0.809	0.808	0.807	0.806	0.640	0.496	0.495	0.495	0.494	0.493	0.492	0.491	0.490	0.271	0.271	0.270	0.270	0.123
TCOST	0.015	0.017	0.02	0.022	0.024	0.026	0.029	0.031	0.033	0.035	0.038	0.04	0.042	0.044	0.047	0.049	0.051	0.053	0.056	0.058

Figure B1: Ellison-Glaeser indices of employment concentration by industry (sector)
965 industries, grouped by broad classification



Naics	Sector	mean	sd	number of industries
1	Farming	0.110	0.088	15
2	Mining	0.110	0.180	72
3	Manufacturing	0.073	0.086	365
4	WholesaleRetail	0.030	0.063	189
5	BusinessServices	0.039	0.071	180
6 -- 8	PersonalServices	0.016	0.030	144
	Total |	0.053	0.090	965

Source data: US Census Bureau
County Business Patterns

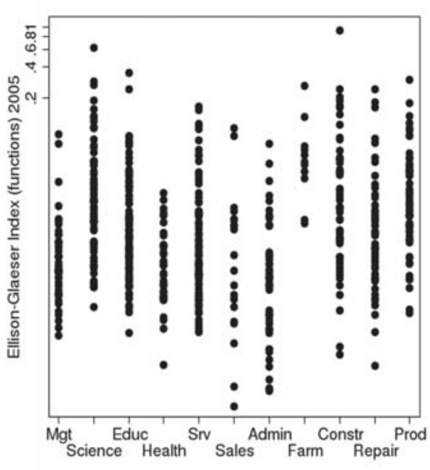
Results are derived from State level information on productionby industry

When EG index is high there is geographic concentration

Some industries within each broad sector are concentrated and others dispersed

Manufacturing is more concentrated on average

Figure B2: Ellison-Glaeser indices of employment concentration by occupation (function)
586 occupations, grouped by broad classification



	OCC1	mean	sd	No. of functions
1	Management, Business, and Financial Occupa	0.009	0.016	48
2	Computer, Engineering, and Science Occupat	0.049	0.090	70
3	Education, Legal, Community Service, Arts, ar	0.022	0.044	102
4	Healthcare Practitioners and Technical Occup	0.006	0.006	38
5	Service Occupations	0.019	0.032	81
6	Sales and Related Occupations	0.015	0.027	20
7	Office and Administrative Support Occupatio	0.007	0.012	52
8	Farming, Fishing, and Forestry Occupations	0.068	0.071	11
9	Construction and Extraction Occupations	0.057	0.130	53
10	Installation, Maintenance, and Repair Occupa	0.025	0.048	48
11	Production Occupations	0.034	0.048	63
12	Transportation and Material Moving Occupations			
13	Military Specific Occupations			
	Total	0.027	0.062	586

These are derived from State level information on employment by function

When EG index is high there is geographic concentration

Health, sales and administration functions are less concentrated geogrpahically

Some functions within each broad category are concentrated and others dispersed

Source data: Bureau of Labor Statistics,
Occuptional Employment Statisitics

Figure B3: Change in Ellison-Glaeser indices of employment concentration by industry (sector), 2000-2010

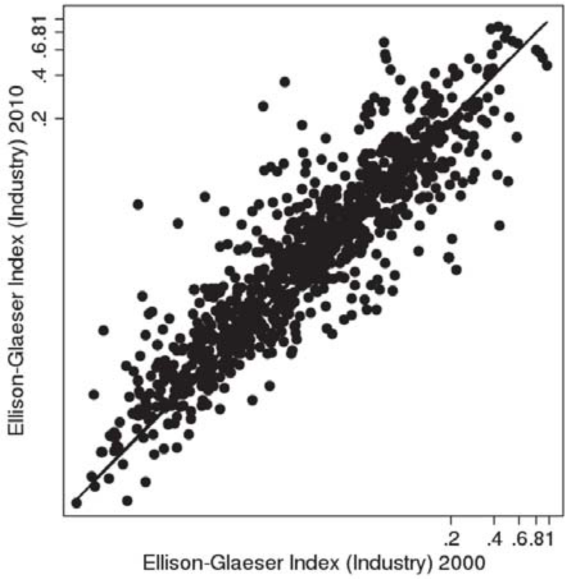


Figure B4: Change in Ellison-Glaeser indices of employment concentration by occupation (function), 2000-2010