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Jobs and technology in general equilibrium: A three-elasticities approach

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Abstract:
The impact of technological progress on jobs and wages has been subject to much empirical and some theoretical work. However, most of this literature has not addressed the general equilibrium interplay between the productive factors that are affected, the sectors in which these factors are used, and the consequent changes in the structure of employment and factor returns. This paper draws on tools from general equilibrium trade theory to provide an integrated approach to these issues. The analysis centres around three key elasticities linking technological change to jobs – the jobs-displacing substitution effect, the job-creating demand effect, and the general-equilibrium effects, through which factors are reallocated between sectors. The results highlight the role of relative factor intensities and the importance of openness in determining the effects of technology on jobs, wages, and structural change. The implications of interaction between non-tradable and tradable sectors are analysed.

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1. Introduction.

Technological progress has been a driver of aggregate prosperity, but also of labour market disruption. Adverse labour outcomes ranging from stagnant wages and the polarisation of employment patterns, to destruction of middleclass jobs and rising inequality, have been linked to technical change. Rapid advances in digital technology, machine learning, and advanced robotics raise further concerns about the future of work (Brynjolfsson and McAfee 2014, Ford 2015). Research on these issues to date has been dominated by empirical work, with less theoretical research on the economic mechanisms underpinning the technology-labour link. Much of the theory has focused on the nature of the technological change and its interaction with different types of workers. The baseline settings assumed tend to be partial equilibrium, or general equilibrium setups with limited feedback possibilities. This paper seeks to broaden the range of economic mechanisms considered in technology-labour research by analysing a general equilibrium model that allows a full range of interactions. This yields new insights, and points to further mechanisms which need to be taken into account in empirical work.

Investigation of the effects of technical progress on employment patterns and wages requires analysis at several different levels. The first is the interaction between factors within the technology of the firm or sector. The second is cross-sector effects arising through product market interaction; technical progress changes costs and prices causing expenditure switching by consumers. The third is also cross-sector, but through interactions in factor markets rather than goods markets. The employment and wage effects of technical change depend as much on the characteristics of sectors that factors move to (or are drawn from) as they do on the characteristics of sectors where technical change occurs. All three levels interact in determining outcomes. The extensive literature on the effects of technical change has devoted less attention to the third of these mechanisms than to the first two, and the contribution of this paper is to provide a relatively rich modelling of the general equilibrium of the entire economy, and thereby extend (and in some cases reverse) results that come from models with a simpler general equilibrium structure.

The first level – modelling the direct impact – is done most simply by assuming that technical change is factor augmenting, raising the efficiency of a particular factor or factors in one or more sectors. Effects depend on the elasticity of substitution, σ, between inputs in production. Recent work has looked at richer modelling of technical change, for example developing models of task production (Acemoglu and Restrepo 2018), and task production together with the use of more efficient 'robots' (Graetz and Michaels 2018). The present paper works with factor augmentation and we discuss its relationship to these alternative approaches in section 2.

The second level – demand switching – brings a further elasticity into play, the price elasticity of demand, ε, for affected activities. This effect is absent in single sector models such as the 'canonical model' of Acemoglu and Autor (2011). The interaction between these two levels – the elasticity of substitution and elasticity of demand – is shown with particular clarity in Graetz and Michaels (2018) where technical progress creates or destroys jobs in the innovating sector according to the sign of the difference between the two elasticities, $\varepsilon - \sigma$, creating jobs where $\varepsilon > \sigma$.

The third level is cross-sector through factor market interaction. If the economy has factors that move between sectors, then the impact of technical change on employment and on factor prices depends on the patterns of factor use in each sector. For example, suppose that demand is extremely elastic, so technical change tends to expand output and employment. If technical change augments one factor (in all sectors) then the relative price of this factor increases, and we show that the movement of factors between sectors depends on sectors' relative factor intensities. If the technical change is in one sector (all factors), then employment of factors in the sector increases, but the change in relative factor prices depends on the relative factor-intensities of sectors where employment is expanding or contracting.

These factor market interactions are analysed in Heckscher-Ohlin (HO) trade theory where they are represented by the dual Rybczynski and Stolper-Samuelson relationships. In the present context, these relationships underpin the 'third elasticity' essential to understanding the factor market implications of technical change. The Rybczynski theorem of trade theory says that (in well-defined circumstances) an increase in the supply of a factor expands the output of the sector intensive in its use. Moreover, the expansion is more than proportional, while other sectors contract. In the present context the basic insight is that technical progress has similarities with changes in factor endowments.[1]

This paper provides a systematic analysis of the effect of factor augmenting technical change on employment and relative wages when all three of these effects are present. We present results for cases in which one or both factors may be augmented in one or both sectors of a two-factor, two-sector economy, deriving explicit expressions for price, wage, and factor reallocation effects. Whereas most of the trade literature's analysis of the HO model is undertaken with fixed product prices (the small open economy assumption) we generalise this to include the product prices changes caused by technical change.[2] We also discuss the importance of the trade regime – the pattern of trade, terms of trade effects and the presence of a non-tradables sector – for understanding the impact of technical change.

The next section of the paper, Section 2, introduces elements of the full model and links with existing literature on modelling technical change. Section 3 sets up the general equilibrium and its comparative statics, and derives our main results. We look at the structure of employment and factor prices, pointing to the key role of the third elasticity, and to the range of qualitative and quantitative outcomes that are possible. Clearer insight into these results is possible for technical change that is sector specific (augmenting both factors in one sector) or factor specific (a single factor in both sectors), and results for these cases are presented in section 4. In section 5 we discuss various open-economy issues, while section 6 concludes.

2. Model preview and literature context.

We start with the simplest model in order to introduce building blocks of our approach, derive some benchmark results, and connect with existing literature. Technical progress occurs in a sector (which we call sector 1) which uses two factors of production, A and B, to produce an output, X_1. The production function has constant returns to scale and takes the form $X_1 = F(\alpha L_{A1}, L_{B1})$ where L_{A1}, L_{B1} are quantities of each factor. Technical progress is, for the moment, assumed to just augment factor-A, with α giving the efficiency of L_{A1}. The corresponding unit cost is, in equilibrium, equal to

[1] The observation dates back to Jones (1965) and is sometimes referred to as Jones' equivalence.

[2] Xu (2000) signs many of the relative wage effects, but does not look at factor reallocation or present expressions for the effects. Dixit and Norman (1980) show how technical change can be analysed in their dual general equilibrium approach, focussing on trade and welfare effects. Jones (2000), building on Jones (1965), uses a model similar to ours; however, he models technical change differently, and does not focus on employment effects. Haskel and Slaughter (2002) focus on the effects of skill biased technical change (SBTC) on skill premia in a HO model similar to ours, but do not provide a full analysis of the interaction between effects.

price, so $p_1 = c(w_A/\alpha, w_B)$, where w_A and w_B are wages of each factor. Totally differentiating the production and cost functions gives (with proportionate changes denoted ^),

$$\hat{X}_1 = \omega_1(\hat{L}_{A1} + \hat{\alpha}) + (1 - \omega_1)\hat{L}_{B1}, \tag{1}$$

$$\hat{p}_1 = \omega_1(\hat{w}_A - \hat{\alpha}) + (1 - \omega_1)\hat{w}_B, \tag{2}$$

where ω_1 is the cost share of factor-A in this sector. The definition of the elasticity of substitution between factors, $\sigma \geq 0$, is

$$\sigma \equiv -\left[\frac{\hat{L}_{A1} + \hat{\alpha} - \hat{L}_{B1}}{\hat{w}_A - \hat{\alpha} - \hat{w}_B}\right], \quad \text{so} \quad \left(1 - \frac{1}{\sigma}\right)\hat{\alpha} = \hat{w}_A - \hat{w}_B + \frac{(\hat{L}_{A1} - \hat{L}_{B1})}{\sigma}. \tag{3}$$

What are the wage and employment effects of this technical progress?

In the simplest case, the canonical model of Acemoglu and Autor (2011), there are two factors and a single sector (sector 1) which fully employs the economy's fixed endowment of each labour type. Thus $\hat{L}_{A1} = \hat{L}_{B1} = 0$ and it follows immediately, from (3), that the "sign test" for the relative wage change associated with technical change $\hat{\alpha} > 0$ depends on a single elasticity, $sign\ (\hat{w}_A - \hat{w}_B) = sign\ (\sigma - 1)$. In words, factor augmentation brings a direct impact and a substitution effect on the wage of the factor experiencing the progress (factor-A), as the increased factor-A supply (in efficiency units) is combined with the fixed stock of factor-B. If the two factors are gross substitutes ($\sigma > 1$) the net effect on w_A/w_B is positive. If they are gross complements ($\sigma < 1$) it is negative.[3] The individual wage effects can be seen from (2) and (3) to be $\hat{w}_A = (\sigma + \omega_1 - 1)/\sigma)\hat{\alpha}$ and $\hat{w}_B = (\omega_1/\sigma)\hat{\alpha}$, holding p_1 constant. Thus B-type labour always gains from augmentation of A-type labour, and A-type labour gains as long as $\sigma + \omega_1 > 1$.

Intersectoral interactions can be added by supposing that a second sector competes for both sales and factors. Substitution in demand is captured most simply by quasi-linear preferences with price elasticity ε, such that sector 1 demand takes the form

$$\hat{X}_1 = -\varepsilon\hat{p}_1. \tag{4}$$

Suppose that the supply of factor-A to sector 1 is perfectly elastic so $\hat{w}_A = 0$, and that the elasticity of factor-B supply to sector 1 (denoted ζ) is simply a function of the price factor-B, so

$$\hat{L}_{B1} = \zeta\hat{w}_B. \tag{5}$$

The implications of technical change, $\hat{\alpha} > 0$, for wages and employment of factor-B are then

$$\hat{w}_B = \frac{\omega_1(\varepsilon - \sigma)\hat{\alpha}}{\omega_1\sigma + (1 - \omega_1)(\varepsilon + \zeta)}, \quad \hat{L}_{B1} = \frac{\zeta\omega_1(\varepsilon - \sigma)\hat{\alpha}}{\omega_1\sigma + (1 - \omega_1)(\varepsilon + \zeta)}. \tag{6}$$

These expressions depend on the share of factor-A in sector 1, ω_1, and on three elasticities: substitution between factors within a sector, σ, substitution in demand between sectors, ε, and supply of factor-B to sector 1, ζ. If $\zeta > 0$ then the sign of both the wage and employment effects for factor-

[3] This also illustrates the distinction between factor-saving and factor-using technical progress. Factor-A augmenting technical change ($\hat{\alpha} > 0$) is *factor-A saving* if, at unchanged factor prices, it reduces the relative use of factor-A in the sector, while it is *factor-A using* if it increases the relative use of factor-A. From (3) it follows that $\hat{\alpha}$ is factor-A saving for $\sigma < 1$, and factor-A using for $\sigma > 1$.

B depend on the relationship between demand and substitution elasticities, $\varepsilon - \sigma$. This is as in Graetz and Michaels (2018), in a model in which the factor experiencing the technical progress are 'robots' (our factor-A, perfectly elastically supplied), and the other factor are workers (our factor-B).[4]

The third elasticity is to do with factor supply ζ. In the model sketched above ζ has a quantitative effect but, as long as $\zeta \geq 0$, does not change the sign of responses. However, in a fully specified two-sector and two-factor model these signs can change. There is a critical interaction between the nature of the technological progress and characteristics of the sector in which it occurs, so the analogue of ζ can be positive or negative. In such cases, all three elasticities matter in determining the sign as well as the magnitude of the impact of technological progress.

The following sections set out and analyse such a model where both sectors are fully specified. Factor supply elasticities come from the Rybczynski elasticity of trade models, which itself depends on shares of each factor in each sector. We fully characterise the responses of employment and factor prices to each combination of factor and sector specific technical changes.

Before moving to this, we discuss two alternative approaches to modelling technical progress. Haskel and Slaughter (2002) study skill-biased technical change (SBTC) in a two-sector open economy model similar to ours. Their focus is on whether the sector-bias or factor-bias of technical change is more important for skill premia (relative wage effects). However, rather than to model technical progress as factor augmentation, they model SBTC as an exogenous change in the share of skilled labour in a CES production function in one or both sectors. While general equilibrium mechanisms are similar to the model we present in section 3, below, our focus is different, as we study relative employment as well as relative wage effects, and derive specific results focussing on three key elasticities in determining these effects.

A task-based approach is adopted by Acemoglu and Restrepo (2018a, 2018b, 2019) in which technical progress takes two main forms. One is that tasks that were performed by labour become automated, principally requiring capital. The other is that new labour-using tasks emerge. The net effect on wages depends on the balance between these forces. The base-line modelling of this involves shifting weights on automated (capital using) and non-automated (labour using) tasks in a CES production function[5]. This is similar to a model with augmentation of the two factors, where augmentation terms are weights on the factor inputs, and therefore provides an attractive interpretation of factor augmentation. Their approach involves a single sector, so does not address the cross-sectoral interactions that are the focus of this paper.

These examples illustrate various ways in which the modelling of technical progress can be enriched to accommodate automation and the effects of more efficient robots. However, our main focus will be on general-equilibrium goods and factor market interactions, and to highlight that, we work with factor-augmenting technical progress in the sections below.

[4] Graetz and Michaels (2018) develop a model with a continuum of industries each of which uses multiple tasks. Some industries are robot-using, meaning that a fraction of tasks can be undertaken by robots, the remainder by labour. Technical progress takes the form of a reduction in the price of robots, and they derive the result that employment in robot-using industries increases if $\varepsilon - \sigma > 0$, where σ is the elasticity of substitution between tasks (and hence between robots and labour).

[5] Hence, the approach could be seen as providing a micro foundation for the the modelling of SBTC in Haskel and Slaughter (2002).

3. General equilibrium: the three elasticities

We now develop this approach into a fully specified 2x2 general equilibrium model, familiar from HO trade theory, and derive the effects of technical progress on employment and wages. We allow technical progress to augment efficiency in different combinations of factors and sectors, and establish the critical role of the 'third elasticity', the Rybczynski elasticity which gives the relationship between the supply of goods and the endowment of factors of production, and its dual, the Stolper-Samuelson elasticity giving the relationship between goods prices and factor prices.

There are two sectors, s =1, 2, and two factors, f = A, B, which can be thought of as different types of labour. Production has constant returns to scale and is described by unit cost functions $c_s = C_s(\, w_A/\alpha_{As}\,, w_B/\alpha_{Bs})$ and production functions $X_s = F_s(\alpha_{As}L_{As}, \alpha_{Bs}L_{Bs})$, these containing factor and sector specific efficiency levels α_{fs}. The economy's endowment of factor f is denoted L_f and full employment of factors means that factor inputs in each sector, L_{fs}, satisfy $L_{f1} + L_{f2} = L_f\,,\; f = A, B$. Demand for goods is derived from the homothetic utility function of a representative consumer, so relative demand is a function of relative prices, $X_1/X_2 = (p_1/p_2)^{-\varepsilon}$. With perfect competition and constant returns, price equals unit costs, so $p_s = c_s$.

We note that the demand elasticity facing a sector depends on whether this is a closed or an open economy, the extreme case being a small open economy with infinitely elastic demand. Generally, the smaller the country's market share on world markets, the more elastic is the demand it faces. It also matters greatly whether the progress takes place in only one country or in all countries simultaneously. We return to these issues in section 5.

We are interested in the effects of exogenous technical changes, $\hat{\alpha}_{fs}$. Differentiating the relationships above gives the following equations, where $\omega_s \equiv w_A L_{As}/p_s X_s$ is the cost share of factor-A in industry s, (hence $1-\omega_s$ the cost share of factor-B), and $\upsilon_f \equiv L_{f1}/L_f$ is the share of factor f used sector-1, $(1-\upsilon_f$ the share of f in sector-2).[6]

$$\hat{p}_s = \omega_s(\,\hat{w}_A - \hat{\alpha}_{As}) + (1-\omega_s)(\hat{w}_B - \hat{\alpha}_{Bs}), \qquad s = 1,2, \tag{7}$$

$$\hat{X}_s = \omega_s\big(\,\hat{L}_{As} + \hat{\alpha}_{As}\big) + (1-\omega_s)\big(\hat{L}_{Bs} + \hat{\alpha}_{Bs}\big), \qquad s = 1,2, \tag{8}$$

$$\upsilon_f \hat{L}_{f1} + \big(1-\upsilon_f\big)\hat{L}_{f2} = \hat{L}_f, \qquad f = A, B, \tag{9}$$

$$\hat{X}_1 - \hat{X}_2 = -\varepsilon(\hat{p}_1 - \hat{p}_2)\,, \tag{10}$$

$$\sigma_s \equiv -\left[\frac{\big(\,\hat{L}_{As} + \hat{\alpha}_{As}\big) - \big(\hat{L}_{Bs} + \hat{\alpha}_{Bs}\big)}{(\,\hat{w}_A - \hat{\alpha}_{As}) - (\hat{w}_B - \hat{\alpha}_{Bs})}\right], \qquad s = 1,2. \tag{11}$$

The last of these equations is the definition of the elasticity of substitution between factors, σ_s. These nine equations, together with a numeraire, are linear in the ten endogenous variables $\hat{p}_s, \hat{X}_s, \hat{L}_{fs}$, and $\hat{w}_f$. Explicit solutions can be derived (appendix 1), and in what follows we draw out the central results. To simplify expressions, we assume that the elasticity of substitution is the same in both

[6] The shares υ_f and ω_s are linked by the relative sizes of the sectors and the factor intensity of each. The relationship takes the form $\upsilon_A = s\omega_1/\varpi,\;\; \upsilon_B = s(1-\omega_1)/(1-\varpi)$ where s is the share of sector 1 in GDP, and $\varpi \equiv s\omega_1 + (1-s)\omega_2$ is the average cost share of factor-A in the economy.

sectors, $\sigma_1 = \sigma_2 = \sigma$, and we focus on relative effects. Thus, we look at the differences between sectors in the proportionate changes in prices and in outputs, which we denote $\Delta_{12}\hat{p} \equiv \hat{p}_1 - \hat{p}_2$ and $\Delta_{12}\hat{X} \equiv \hat{X}_1 - \hat{X}_2$. Similarly, changes in relative wages across the two factors are $\Delta_{AB}\widehat{w} \equiv \widehat{w}_A - \widehat{w}_B$ and changes in relative endowments of the two factors, $\Delta_{AB}\hat{L} \equiv \hat{L}_A - \hat{L}_B$.

To draw out comparative static properties of this system we utilise insights from HO-models. As is well-known, HO results hinge on the Stolper-Samuelson elasticity which measures the elasticity of relative wages with respect to relative prices, $\beta_{SS} \equiv \Delta_{AB}\widehat{w}/\Delta_{12}\hat{p}$, and its dual, the Rybczynski elasticity giving the elasticity of relative outputs with respect to relative endowments, $\beta_{RY} \equiv \Delta_{12}\hat{X}/\Delta_{AB}\hat{L}$. These take the form,

$$\beta_{SS} = 1/(\omega_1 - \omega_2), \qquad \beta_{RY} = 1/(\upsilon_A - \upsilon_B). \tag{12}$$

The two expressions in (12) differ only because they are expressed as elasticities rather than derivatives and together constitute our 'third elasticity'.[7] Their signs are determined by the relative factor intensity of the two sectors, so if industry 1 is relatively factor A-intensive, $\omega_1 > \omega_2$ (which also implies $\upsilon_A > \upsilon_B$) then $\beta_{SS} > 1,\ \beta_{RY} > 1$; conversely, if $\omega_1 < \omega_2$ then $\beta_{SS} < -1,\ \beta_{RY} < -1$. They necessarily have the same sign, and $\beta_{RY}\beta_{SS} > 1$.

It is convenient to define the supply elasticity of relative production w.r.t relative goods prices, η, so $\eta \equiv \Delta_{12}\hat{X}/\Delta_{12}\hat{p}$ and in appendix 1 it is shown that this takes the form

$$\eta = \sigma(\beta_{RY}\beta_{SS} - 1) \geq 0. \tag{13}$$

If factor intensities in each sector are very similar (i.e. $|\omega_1 - \omega_2|$ and $|\upsilon_A - \upsilon_B|$ are very small) then $|\beta_{RY}|, |\beta_{SS}| \to \infty$ so, providing $\sigma > 0$, the elasticity of supply goes to infinity, i.e. the production possibility frontier tends to linearity. If factor intensities are very different (tending to factor-sector specificity) then $|\beta_{RY}|, |\beta_{SS}| \to 1$ and η goes to zero.[8]

The full simultaneous system is given by eqns (7) - (11), and for expositional clarify we proceed in stages. First, the relative wage effect of technical progress follows from (7) as

$$\Delta_{AB}\widehat{w} = \beta_{SS}\{\Delta_{12}\hat{p} + \Delta_{12}\hat{\chi}\}. \tag{14}$$

The first term is the standard Stolper-Samuelson relationship between goods prices and factor prices. The second term adds the direct effect of technological change on costs in each sector. This cost-saving effect is the share weighted average of factor augmentation in each sector, denoted $\hat{\chi}_s$, so from eqn. (7), $\hat{\chi}_s \equiv \omega_s\hat{\alpha}_{As} + (1-\omega_s)\hat{\alpha}_{Bs},\ s = 1, 2$. The relative cost saving effect, across sectors, is $\Delta_{12}\hat{\chi} \equiv \hat{\chi}_1 - \hat{\chi}_2$. Eqn. (14) indicates that cost-saving effects change relative factor prices in the same way as do goods prices changes.

Second, output supply changes are derived using (8), (9) and (11), (appendix 1), giving the change in relative supply as

$$\Delta_{12}\hat{X} = \eta\Delta_{12}\hat{p} + \beta_{RY}(\Delta_{AB}\hat{L} + \Delta_{AB}V), \tag{15}$$

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[7] The Stolper-Samuelson and Rybczynski derivatives are cross-partial derivatives of the GNP function, equal by Young's theorem. The relationship between the two elasticities is $\beta_{RY} = \beta_{SS}\bar{\omega}(1-\bar{\omega})/s(1-s)$.

[8] The term η will be used to simplify algebra, allowing relatively compact and insightful exposition. But it is a function of underlying elasticities, so thought experiments should not, e.g., change σ without also changing η.

$$\text{where} \quad \Delta_{AB}V \equiv \Delta_{AB}\hat{\lambda} - \sigma\left(\Delta_{AB}\hat{\lambda} - \beta_{SS}\Delta_{12}\hat{\chi}\right). \tag{16}$$

In this expression the direct effect of technical change is given by $\Delta_{AB}V$, but for interpretation suppose first that $\Delta_{AB}V = 0$. Equation (15) then says that the change in relative supply is the relative price change times the price elasticity of supply, plus the relative endowment change times the Rybczynski elasticity.

Technical change augments the endowment change, and we refer to $\Delta_{AB}V$ as the "(relative)-factor-endowment-equivalent" (FEE) of technical change, so $\Delta_{AB}V > 0$ is relatively factor-A saving technical progress. $\Delta_{AB}V$ is made up of several elements, first of which is a direct factor-saving effect. This is the average augmentation of factor *f*, weighted by sector in which *f* is employed; it is denoted $\hat{\lambda}_f$, so from (9), $\hat{\lambda}_f \equiv \upsilon_f\hat{\alpha}_{f1} + \left(1 - \upsilon_f\right)\hat{\alpha}_{f2},\ f = A, B$; the relative change is $\Delta_{AB}\hat{\lambda} \equiv \hat{\lambda}_A - \hat{\lambda}_B$. This direct effect is exactly like an increase in relative endowments. Remaining terms in $\Delta_{AB}V$ come from movement around the production possibility frontier. They depend on the elasticity of substitution between factors, and on both the cost-saving and factor-saving impacts.[9]

To close the model, we combine the demand effect in (10) with the supply effect in (15) to get

$$\Delta_{12}\hat{p} = \frac{-\beta_{Ry}}{\varepsilon + \eta}\Delta_{AB}V, \qquad \Delta_{12}\hat{X} = \frac{\varepsilon\beta_{Ry}}{\varepsilon + \eta}\Delta_{AB}V. \tag{17}$$

In these expressions, and until section 5, we assume that factor endowments are unchanging, $\Delta_{AB}\hat{L} = 0$. The sign of the relative price and output changes causes by technical change depend on the FEE representation of the change, $\Delta_{AB}V$, and on the sign of the Rybczynski elasticity, β_{Ry}. If technical change acts like a relative increase in the supply of factor-A, and sector 1 is relatively A-intensive, then the relative price of good 1 will fall. The magnitude of this general equilibrium effect depends on the sum of the general equilibrium demand and supply elasticities $\varepsilon + \eta$.

We are now able to derive explicit closed form solutions for further variables of interest. First, the movement of factors between sectors in response to technical change. For each factor, the change in the division of employment between sectors can be captured by relative change, $\Delta_{12}\hat{L}_f \equiv \hat{L}_{f1} - \hat{L}_{f2}, f = A, B$. We derive

$$\begin{aligned}\Delta_{12}\hat{L}_f &= \Delta_{12}\hat{X} + \sigma\Delta_{12}\hat{p} + (\sigma - 1)\left(\hat{\alpha}_{f1} - \hat{\alpha}_{f2}\right)\\ &= \frac{(\varepsilon - \sigma)\beta_{Ry}}{\varepsilon + \eta}\Delta_{AB}V + (\sigma - 1)\left(\hat{\alpha}_{f1} - \hat{\alpha}_{f2}\right).\end{aligned} \tag{18}$$

[9] Detailed in appendix 1. To see this, think of how relative factor use in a single sector changes when technology changes. If technology is CES then labour demand is $L_{fs} = \omega_s \frac{x_s}{\alpha_{fs}}\left(\frac{p_s}{w_f/\alpha_{fs}}\right)^{\sigma}$. Hence, $\hat{L}_{As} - \hat{L}_{Bs} = -(\hat{\alpha}_{As} - \hat{\alpha}_{Bs}) - \sigma[(\hat{w}_A - \hat{\alpha}_{As}) - (\hat{w}_B - \hat{\alpha}_{Bs})] = -(\hat{\alpha}_{As} - \hat{\alpha}_{Bs}) + \sigma[(\hat{\alpha}_{As} - \hat{\alpha}_{Bs}) - (\hat{w}_A - \hat{w}_B)]$. Technical progress has direct impact, $(\hat{\alpha}_{As} - \hat{\alpha}_{Bs})$, and a substitution effect due to the change in relative (efficiency) wages, $\left(\frac{w_A/\alpha_{As}}{w_B/\alpha_{Bs}}\right)$. $\Delta_{AB}V$ is the multi-sector equivalent of this, measured as relative factor saving rather than relative factor use, and using $(\hat{w}_A - \hat{w}_B) = \beta_{SS}\Delta_{12}\hat{\chi}$.

Since total employment of each factor is constant, the signs of these relative changes also give us the signs of absolute changes, i.e. $sign(\Delta_{12}\hat{L}_f) = sign(\hat{L}_{f1}) = -sign(\hat{L}_{f2}),\ f = A, B$. In section 4 we interpret this for special cases,

Second, using (17) in (14) the change in relative wages of the two factors is

$$\Delta_{AB}\hat{w} = \beta_{SS}\{\Delta_{12}\hat{p} + \Delta_{12}\hat{\chi}\} = \frac{-\beta_{SS}\beta_{Ry}}{\varepsilon + \eta}\Delta_{AB}V + \beta_{SS}\Delta_{12}\hat{\chi}. \tag{19}$$

The impact through goods prices is determined by the sign of $\Delta_{AB}V$ and hence negative for any combination of technical change that amounts to an increase in the FEE of factor-A. The impact of the cost-saving effect depends on relative factor intensities, through β_{SS}. As $\varepsilon \to \infty$, the former effect vanishes and the Stolper-Samuelson effect of the relative cost-saving, $\Delta_{12}\hat{\chi}$, dominates.

4. Factor-bias and sector-bias

To this point we have carried through four distinct possible forms of factor augmentation, α_{fs}, f = A, B, s =1, 2. Clearer, although less general, results are derived if we collapse this to two dimensions of technical change, splitting factor augmentation into a factor- and sector-specific element so $\alpha_{fs} = \alpha_f\alpha_s$, f = A, B, s =1, 2. Technical progress is now changes $\hat{\alpha}_f, \hat{\alpha}_s$, so e.g. $\hat{\alpha}_{A1} = \hat{\alpha}_A + \hat{\alpha}_1$, and so on. We define the sector bias of technical change, $\Delta_{12}\Sigma$, and the factor bias, $\Delta_{AB}\Phi$, as

$$\Delta_{12}\Sigma \equiv \hat{\alpha}_1 - \hat{\alpha}_2 = \hat{\alpha}_{A1} - \hat{\alpha}_{A2} = \hat{\alpha}_{B1} - \hat{\alpha}_{B2}\,, \quad \Delta_{AB}\Phi \equiv \hat{\alpha}_A - \hat{\alpha}_B = \hat{\alpha}_{A1} - \hat{\alpha}_{B1} = \hat{\alpha}_{A2} - \hat{\alpha}_{B2}. \tag{20}$$

Technical change is biased to sector 1 if $\Delta_{12}\Sigma > 0$, and to factor-A if $\Delta_{AB}\Phi > 0$. This is less general than the formulation in section 3, excluding for example change that is specific to a single sector and single factor; appendix 2 gives results for these cases.

Sector-bias and factor-bias are related to the relative cost-saving ($\Delta_{12}\hat{\chi}$) and relative factor-saving ($\Delta_{AB}\hat{\lambda}$) measures by

$$\Delta_{12}\hat{\chi} = \Delta_{12}\Sigma + \Delta_{AB}\Phi/\beta_{SS}, \qquad \Delta_{AB}\hat{\lambda} = \Delta_{12}\Sigma/\beta_{Ry} + \Delta_{AB}\Phi. \tag{21}$$

Thus, a pure sector-biased shock ($\Delta_{12}\Sigma \neq 0,\ \Delta_{AB}\Phi = 0$) has an equal cost-saving impact, plus a factor-saving impact that depends on the relative factor intensities of the sectors, positive for factor-A if sector 1 is A-intensive. A pure factor-biased shock ($\Delta_{12}\Sigma = 0, \Delta_{AB}\Phi \neq 0$) has an equal factor-saving effect, and cost-saving effect depending on factor intensities. Using (21) in (16), the FEE can be expressed as $\Delta_{AB}V = (1 + \eta)\,\Delta_{12}\Sigma/\beta_{RY} + \Delta_{AB}\Phi$, once again a combination of the direct factor-saving effect and cost changes moving around the production possibility frontier.

This formulation applied in (17), (19) and (18) gives the following statement of results:

$$\Delta_{12}\hat{p} = -\frac{(1 + \eta)\Delta_{12}\Sigma + \beta_{RY}\Delta_{AB}\Phi}{\varepsilon + \eta}, \tag{22}$$

$$\Delta_{AB}\hat{w} = \frac{(\varepsilon - 1)\beta_{SS}\Delta_{12}\Sigma + [\varepsilon - \sigma + (\sigma - 1)\beta_{RY}\beta_{SS}]\Delta_{AB}\Phi}{\varepsilon + \eta}, \tag{23}$$

$$\Delta_{12}\hat{L}_A = \Delta_{12}\hat{L}_B = \frac{(\varepsilon - 1)(\sigma + \eta)\Delta_{12}\Sigma + (\varepsilon - \sigma)\beta_{RY}\Delta_{AB}\Phi}{\varepsilon + \eta}. \tag{24}$$

All of these expressions depend on all three elasticities, σ, ε, and the inter-sectoral supply response expressed through η, β_{RY}, and β_{SS}. The signs of β_{RY} and β_{SS} depend on factor-intensity differences (although the product $\beta_{RY}\beta_{SS}$ is positive), and we see that this sign matters for all the mappings between relative sector and relative factor effects (i.e. between relative changes Δ_{12} and Δ_{AB}). Thus, relative price changes depend on the sector bias of technical change, and the factor bias times the Rybczynski elasticity.

Wage and employment effects exhibit dual responses to factor intensity differences, in the following sense. The wage response to sector-biased technical change depends on the factor-intensity of the sector in which it occurs (through the sign of β_{SS}), but the price, output and inter-sectoral movement of employment do not. The converse statement holds for factor-biased technical change, i.e. inter-sectoral employment changes depend on relative factor intensities (through β_{RY}), relative wage changes do not.

Elasticities σ and ε enter in complex, although sometimes familiar looking, ways. Intra-sectoral substitution between factors now occurs both because of the direct effect of factor augmentation, and via equilibrium wage changes. Final expenditure switching occurs through the direct impact of technical change on costs, and via equilibrium changes in factor prices and hence goods prices. We see that the term $(\varepsilon - \sigma)$ crops up again, for factor biased but not sector biased technical change, and now its employment impacts depend on factor intensities (β_{RY} in (24)), and its wage impacts are combined with the term $(\sigma - 1)\beta_{RY}\beta_{SS}$ in equation (23).

Factor movements between sectors are in general given by (18), where the first term captures the employment effects of general equilibrium reallocation of production, while the second term captures factor-specific changes. In the special cases we look at here, the second term is always the same for both factors, and given by $\left(\hat{\alpha}_{f1} - \hat{\alpha}_{f2}\right) = (\hat{\alpha}_1 - \hat{\alpha}_2) = \Delta_{12}\Sigma$.

An important point concerns the relationship between the signs of factor movements and of factor price changes. If technical change is entirely sector-biased (e.g. $\Delta_{12}\Sigma > 0,\ \Delta_{AB}\Phi = 0$), the relationship between the signs of $\Delta_{12}\hat{L}_f$ and $\Delta_{AB}\hat{w}$ depends on factor intensities in an intuitive way. As might be expected, when sector 1 bias raises employment in sector 1 ($\varepsilon > 1$), then it also raises the price of the factor used intensively in sector 1, via the sign of β_{SS}.

The analogous statement is not true for factor-bias, as the difference in signs of $\Delta_{12}\hat{L}_f$ and $\Delta_{AB}\hat{w}$ depends on factor intensities and on the relative magnitude of terms $(\varepsilon - \sigma + (\sigma - 1)\beta_{RY}\beta_{SS})$ and $(\varepsilon - \sigma)$, parts of the coefficients on $\Delta_{AB}\Phi$ in equations (23) and (24) respectively. For example, suppose that change is biased to factor-A ($\Delta_{12}\Sigma = 0,\ \Delta_{AB}\Phi > 0$), that sector 1 is A-intensive (so $\beta_{RY} > 0$), and that $\varepsilon > \sigma$. Then, as expected, employment in sector 1 expands ($\Delta_{12}\hat{L}_A = \Delta_{12}\hat{L}_B > 0$). However, $\Delta_{AB}\hat{w}$, may be positive or negative. Thus, if $(\sigma - 1)\beta_{RY}\beta_{SS}$ is large enough negative, then factor-A biased technical change which expands employment in the A-intensive sector may also reduce the relative wage of factor-A. Conversely, if $\varepsilon < \sigma \gg 1$, then A-biased technical change may increase the relative wage of factor A, despite reducing employment in the A-intensive sector (sector 1). Another way of seeing this, is by realising that the relative wage effect depends on the combined change in the relative demand for factor-A in both sectors, while the relative employment effect captures whether the factor demand effect (for both factors) is stronger in the A-intensive sector than in the other sector.

Finally, we look at changes in the nominal and real wages of each factor separately. Directly from (7), and setting good 2 as numeraire (so $\Delta_{12}\hat{p} = \hat{p}_1$) the wage change for each factor is

$$\begin{aligned} \widehat{w}_A &= \beta_{SS}[(1-\omega_2)(\hat{p}_1 + \hat{\chi}_1) - (1-\omega_1)\hat{\chi}_2], \\ \widehat{w}_B &= -\beta_{SS}[\omega_2(\hat{p}_1 + \hat{\chi}_1) - \omega_1\hat{\chi}_2]. \end{aligned} \tag{25}$$

Suppose that technical occurs only in sector 1 (both factors) so is given by $\hat{\alpha}_1$, and hence $\Delta_{12}\Sigma \equiv \hat{\alpha}_1$, $\Delta_{AB}\Phi = 0$, $\hat{\chi}_1 = \hat{\alpha}_1$, and $\hat{\chi}_2 = 0$. Then,

$$\hat{p}_1 = -\frac{(1+\eta)\hat{\alpha}_1}{\varepsilon + \eta}, \quad \widehat{w}_A = (1-\omega_2)\beta_{SS}\frac{(\varepsilon - 1)\hat{\alpha}_1}{\varepsilon + \eta}, \quad \widehat{w}_B = -\omega_2\beta_{SS}\frac{(\varepsilon - 1)\hat{\alpha}_1}{\varepsilon + \eta}. \tag{26}$$

Regardless of consumption shares, if $\varepsilon > 1$, then real wages increase for the factor intensive in sector ($\widehat{w}_A > \hat{p}_1$ if $\beta_{SS} > 0$). Nominal wages always move in opposite directions and, for the factor unintensive in sector 1, the real wage certainly increases if $\varepsilon < 1$, as the goods price falls and nominal wage rises.[10]

Alternatively, if technical occurs only in factor-A (both sectors) so is given by $\hat{\alpha}_A$, and hence $\Delta_{12}\Sigma = 0$, $\Delta_{AB}\Phi = \hat{\alpha}_A$, $\hat{\chi}_1 = \omega_1\hat{\alpha}_A$, and $\hat{\chi}_2 = \omega_2\hat{\alpha}_A$. Then,

$$\hat{p}_1 = -\frac{\beta_{RY}\hat{\alpha}_A}{\varepsilon + \eta}, \quad \widehat{w}_A = \left\{1 - \frac{(1-\omega_2)\beta_{SS}\beta_{RY}}{\varepsilon + \eta}\right\}\hat{\alpha}_A, \quad \widehat{w}_B = \frac{\omega_2\beta_{SS}\beta_{RY}\hat{\alpha}_A}{\varepsilon + \eta}. \tag{27}$$

If sector 1 is A-intensive, then the real wage of factor-B unambiguously increases, while that of factor-A increase if ε and $\sigma$are sufficiently large.[11]

So, if A and B are unskilled and skilled labour respectively, then unskilled-biased technical progress will always give real wage gains for skilled workers, while the real wage for unskilled workers depends on elasticities – high ε and σ implies real gains for the workers that become more productive. If ε is high, there is a strong demand effect for the goods using unskilled labour intensively. If σ is high, there is a strong substitution effects as the productivity advantage for unskilled workers imply that they replace some of the skilled workers. Hence, with high elasticities, both unskilled and skilled workers gain, and more so for the unskilled ones. With low elasticities, unskilled workers may lose, while the gain for skilled workers is positive and higher the lower the elasticities.

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[10] For other cases, like factor-B when $\varepsilon > 1$, the real wage effect depends on elasticities and cost shares in production and consumption, as $\widehat{w}_B < 0$ and $\hat{p}_1 < 0$.

[11] For factor-B we always have $\widehat{w}_B > 0$ while $\hat{p}_1 < 0$ so the real wage must go up. For factor-A we know that $\widehat{w}_A > 0$ for sufficiently high $\varepsilon + \eta = \varepsilon + \sigma(1 - \beta_{SS}\beta_{RY})$.

5. Open economy issues

We have so far worked with a closed economy, either a single country, or a group of integrated countries closed to 'outside'. Extending this to an open economy requires specifying the country or countries in which technical change occurs and the linkages between countries. We address these issues in the next subsection, and turn to the distinction between tradable and non-tradable sectors within a country in section 5.2.

5.1 Goods trade and the terms-of-trade

A country (or group of integrated countries) experiences technical change and trades goods with the rest of the world. Factors and technologies are internationally immobile, so goods trade is the only interaction, and prices are set on the world market. Output by the country under consideration (that experiencing the technical change) is X, and the quantity of this good supplied by the rest of the world is $S^*p^{\eta^*}$; world demand, from both of the countries, is $Dp^{-\varepsilon^*}$, where η^* and ε^* are supply and demand elasticities of firms and consumers in each country. Market clearing is $X + S^*p^{\eta^*} = Dp^{-\varepsilon^*}$. Differentiating, the demand elasticity faced by the country producing X is $\varepsilon = \hat{X}/\hat{p} = -\{(1-\delta)\eta^* + \varepsilon^*\}/\delta$ where δ is the share of this country in world supply. Thus, if the price falls, final demand increases in all countries, and supply falls in countries that produce the good but have not experienced the technical change. Clearly, if $\delta = 1$ then the elasticity is simply $\varepsilon = \varepsilon^*$, the elasticity of demand from consumers in each country, and as δ goes to zero so the small open economy case of infinite demand elasticity is approached.

As we saw in preceding sections, the magnitude of ε is critical for the response to technical change. Thus, and unsurprisingly, employment is more likely to expand if countries experiencing the technical change have opportunities for export growth to (or import substitution from) countries that do not experience the change.[12] If, on the other hand, technical progress takes place in parallel in all countries then the closed-economy framework applies, and the relevant elasticity takes a much lower value, as it is that of local consumers.

The international context also shapes the real income effects of technical change. For finite ε technical progress changes relative prices, this benefiting the country if there is a decrease in the price of its imports, harming it if the price of its exports fall. The rest of the world gains (or loses) as the effect of innovation is transmitted through these terms-of-trade changes.

5.2 Tradable and non-tradable sectors

It is sometimes suggested that non-tradable sectors – predominantly services – are likely to be the principal source of employment for workers displaced by technical progress. How does the presence of such a sector change our analysis, and can it in any sense cushion the impact of technical change?

We capture this extra margin of adjustment by adding a third sector to the model. Goods 1 and 2 are tradeable, and there is a further sector, N, which produces non-tradeable output and, we assume, does not experience technical progress. The analysis is simplified by three facts. First, the size of the N-sector and its factor usage depend on domestic demand. Second, the supply of factors available to tradeable (T) sectors is the economy's endowment minus factor use in the N-sector. Third, if there are two factors of production and two traded goods, then factor prices are fixed by technology and world

[12] See Krugman (2000) for a forceful statement of this point.

prices of the two traded goods, as long as there is diversified production. It follows that the only way that the presence of the N-sector changes outcomes is by changing the amount of the endowment available to traded goods production.

A full analysis of the model with an N-sector is given in Appendix 3. Key insights can be obtained by focusing on how adjustments in the N-sector affect factor supply to the T-sectors. There are two steps. First, technical progress in the T-sectors affects demand for N-sector output via income and price effects. Second, the factor usage this creates in the N-sector shifts the endowment available for use in T-sectors, and can be captured as shift in $\Delta_{AB}V$. As above, the full implications of this are transmitted through Rybczynski effects, and depend on the factor intensity of N relative T-sectors.

For the first step, we assume demand for N-output is Cobb-Douglas in which case $\hat{X}_N = \hat{Y} - \hat{p}_N$.[13] Using ω_{N} for the cost share of factor-A in N-production, ω_T the share in overall T-production, and $\bar{\omega}$ as the economy-wide share, income and price changes are $\hat{Y} = \bar{\omega}\hat{w}_A + (1-\bar{\omega})\hat{w}_B$, and $\hat{p}_N = \omega_N \hat{w}_A + (1-\omega_N)\hat{w}_B$ and hence,[14]

$$\hat{X}_N = (\bar{\omega} - \omega_N)(\hat{w}_A - \hat{w}_B). \tag{28}$$

One immediate insight follows from (28). If the factor intensity of the N-sector coincides with that of the combined T-sectors (and hence that of the economy as a whole), then demand for non-tradables remains unchanged.[15] If intensities differ, then the N-effects are determined by the interplay of relative intensities and relative wage effects.

For the second step we assume fixed coefficients in N-production, so that a proportionate change in output raises sector N's factor usage equi-proportionately, $\hat{L}_{AN} = \hat{L}_{BN} = \hat{X}_N$. The endowment of each factor available for use by the T-sectors is $L_f^T = L_f - L_{fN}, f = A, B$, and the relative endowment change takes the form

$$\Delta_{AB}\,\hat{L}^T \equiv \hat{L}_A^T - \hat{L}_B^T = -\hat{X}_N(L_{AN}/L_A^T - L_{BN}/L_B^T) = \frac{s_N(\omega_T-\omega_N)^2\Delta_{AB}\hat{w}}{\omega_T(1-\omega_T)}. \tag{29}$$

The last equation uses (28), $(L_{AN}/L_A^T - L_{BN}/L_B^T) = s_N(\omega_N - \omega_T)/[\omega_T(1-\omega_T)(1-s_N)]$, and $(\bar{\omega} - \omega_N) = (1-s_N)(\omega_T - \omega_N)$, (appendix 3).

The change in the factor endowment available for the T-sectors is then, extending the FEE in (16), $\Delta_{AB}V^T \equiv \Delta_{AB}V + \varphi\Delta_{AB}\hat{w}$, where the term $\varphi\Delta_{AB}\hat{w}$ captures the added impact due to the N-sector, and $\varphi \equiv s_{\mathrm{N}}\{\omega_T - \omega_N\}^2/\omega_T(1-\omega_T)$. This is non-negative, and strictly positive if the factor-intensity of the N-sector differs (in either direction) from that of the T-sector.

Implications for wage, price and employment changes can now be readily established. For relative wages, the additional term means that eqn. (19) becomes

[13] A two-level utility function with Cobb-Douglas at the top level between non-traded and a nest of traded goods, and CES with elasticity ε between the traded goods at the lower level.
[14] s_N is the share of N in GDP, and $\bar{\omega} = s_N\omega_{\mathrm{N}} + (1-s_N)\omega_{\mathrm{T}}$. We take p_2 as numeraire.
[15] The income and price effects cancel each other. If upper level preferences are not Cobb-Douglas then income and price elasticities enter this expression.

$$\Delta_{AB}\widehat{w}\left\{1+\frac{\beta_{SS}\beta_{Ry}\varphi}{\varepsilon+\eta}\right\}=\frac{-\beta_{SS}\beta_{Ry}}{\varepsilon+\eta}\Delta_{AB}V+\beta_{SS}\Delta_{12}\hat{\chi}\,. \tag{30}$$

The right-hand side of this is as before, and the term is curly brackets is greater than or equal to unity. The sign of relative wage changes is therefore as before, but the magnitude of these change is reduced if the factor-intensity of the N-sector differs from that of the T-sector ($\varphi > 0$).

Using this in the price equation (17), it works as if there is a larger $\Delta_{AB}V$ if $\Delta_{AB}\widehat{w} > 0$ and a smaller $\Delta_{AB}V$ if $\Delta_{AB}\widehat{w} < 0$. For production and employment effects, (17) shows that for $\Delta_{AB}\widehat{w} > 0$ the N-sector will tend to increase A-intensive traded production, for $\Delta_{AB}\widehat{w} < 0$ to increase B-intensive production. This is the Rybczynski effect again. However, since total employment in T-sectors now may go up or down, there is not a direct correspondence to employment in each sector.

To illustrate these effects, think of factor-A as unskilled labour and assume that N-production is more A-intensive than the combined T-sectors. What are the consequences of a technical shock favouring unskilled labour in the T-sectors (sector-1 biased or factor-A biased)? From section 4 we know that the sign of $\Delta_{AB}\widehat{w}$ depends on elasticities; so with sufficiently high ε we have $\Delta_{AB}\widehat{w} > 0,$ and in increase of employment in the A-intensive T-sector. From (30) it is clear that the N-sector will reinforce that change. So for an open economy with high ε, unskilled-biased technical progress leads to increased production and exports of unskilled-intensive products, and the N-sector reinforces the shift by releasing factor. The reason being that the price of N-goods goes up when $\Delta_{AB}\widehat{w} > 0$.

For a closed economy with low ε, we have $\Delta_{AB}\widehat{w} < 0$ and a change in relative employment away from the A-intensive T-sector. In this case, N-production will increase due to its lower relative price, and thus absorb more of both factors, but relatively more of factor-A (unskilled labour); thus, again reinforcing the relative employment changes. In both cases, the N-sector will dampen the relative wage changes.

6. Concluding remarks

The impact of technology on labour market outcomes has been subject to an intense empirical research effort in recent years. The theoretical underpinnings of this effort are less well developed as existing studies have focused on partial equilibrium settings or general equilibrium settings with limited feedback channels. Our paper contributes to the theory by studying the labour impact of factor augmenting technological progress in a setting that allows for full general equilibrium and open-economy considerations.

A key result in the existing literature is that two elasticities are important in determining the jobs impact of a technological shock. When labour becomes more productive in a sector there is an anti-jobs effects (fewer workers needed per unit produced) that is governed by the elasticity of substitution among productive factors. There is also a pro-jobs effect (productivity lowers prices and thus raises units sold) that is governed by the elasticity of demand. For example, Graetz and Michaels (2018) show that a technological improvement in one sector raises employment in that sector if and only if the elasticity of demand for the goods exceeds the elasticity of substitution in production.

Allowing for full general equilibrium feedback effects, as we do, brings to the fore an additional elasticity that affects the sign of wage and jobs effects caused by a given technology shock, namely the

Rybczynski elasticity. In neoclassical trade theory, this elasticity captures the proportional change in sectors' relative outputs in response to a change in the relative supply of productive factors. As we show, a factor-augmenting technology shock triggers an output response that involves the Rybczynski elasticity, and that this response feeds through to changes in relative prices, wages, and the employment pattern. An additional insight from this result is that the nature of the labour market change depends not only on the nature of the technology shock (say its skill bias), but also on an interaction between the nature of the shock and the factor intensity of the sector in which it occurs.

Additionally we show that the wage and employment effects must be considered separately since, in a full general equilibrium setting, it is possible that a particular technology shock raises the wage of, say, the factor used intensively in a given sector while simultaneously reducing employment of that factor in the same sector. Much of the existing literature focuses either on wage effects or jobs effects since in partial equilibrium settings the two tend to be positively correlated.

Our paper also highlights novel open-economy considerations. The importance of the demand elasticity stems from the way that additional production depresses prices. In the extreme of a small open economy, demand is, in effect, limitless, so the demand elasticity plays less of a role. In less extreme settings, we show that the size of the domestic market and the share of nations experiencing the same technological shock mitigate the importance of the demand elasticity in determining the sign of labour market effects. Additionally, we show that the existence of a non-trade sector that does not experience technological progress (say, a non-traded services sector) dampens the wage impacts since some of the adjustment occurs via changes in non-traded employment.

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Appendix 1. Complete 2x2 model

In this appendix the complete 2-sector model is developed, assuming CES cost (and production) functions and exogenously given factor endowments. The linear system (7) – (11) is at the outset more general, but as long as the elasticity of substitution is assumed to be the same and constant for both sectors, it coincides with the equilibrium conditions developed with CES functions.

Cost functions:

$$C_s\left(\frac{w_A}{\alpha_{As}},\frac{w_B}{\alpha_{Bs}}\right)=\left[\Omega_s\left(\frac{w_A}{\alpha_{As}}\right)^{1-\sigma}+(1-\Omega_s)\left(\frac{w_B}{\alpha_{Bs}}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}=p_s,\qquad s=1,2$$

We evaluate equilibrium around a point where the share of factor-A in costs of sector *s* are denoted $\omega_s \equiv w_A L_{As}/p_s X_s$, so that

$$\omega_s\widehat{w}_A+(1-\omega_s)\widehat{w}_B=\hat{p}_s+\omega_s\hat{\alpha}_{As}+(1-\omega_s)\hat{\alpha}_{Bs}=\hat{p}_s+\hat{\chi}_s,\qquad s=1,2,$$

where $\hat{\chi}_s=\omega_s\hat{\alpha}_{As}+(1-\omega_s)\hat{\alpha}_{Bs}$ is the cost-saving impact of factor augmenting technical progress in sector *s*. These 2 equations yield:

$$\widehat{w}_A=\frac{1}{\omega_1-\omega_2}\{(1-\omega_2)(\hat{p}_1+\hat{\chi}_1)-(1-\omega_1)(\hat{p}_2+\hat{\chi}_2)\}$$

$$\widehat{w}_B=\frac{-1}{\omega_1-\omega_2}\{\omega_2(\hat{p}_1+\hat{\chi}_1)-\omega_1(\hat{p}_2+\hat{\chi}_2)\}$$

Hence, $\widehat{w}_A-\widehat{w}_B=\frac{1}{\omega_1-\omega_2}\{(\hat{p}_1+\hat{\chi}_1)-(\hat{p}_2+\hat{\chi}_2)\}$.

Using $\Delta_{AB}\widehat{w}\equiv\widehat{w}_{\mathrm{A}}-\widehat{w}_{\mathrm{B}}$, $\Delta_{12}\hat{p}\equiv\hat{p}_1-\hat{p}_2$, $\Delta_{12}\hat{\chi}\equiv\hat{\chi}_1-\hat{\chi}_2$, and $\beta_{SS}\equiv 1/(\omega_1-\omega_2)$ (where β_{SS} captures the Stolper-Samuelson effect as an elasticity) this is

$$\Delta_{AB}\widehat{w}=\beta_{SS}\{\Delta_{12}\hat{p}+\Delta_{12}\hat{\chi}\}.\qquad (A1.1)$$

From the cost function we get the labour demand

$$L_{fs}=X_s\frac{\partial C_s}{\partial w_f}=\Omega_s\frac{X_s}{\alpha_{fs}}\left(\frac{p_s}{w_f/\alpha_{fs}}\right)^{\sigma},\qquad s=1,2,\qquad f=A,B$$

Differentiating these and using $L_{f1}+L_{f2}=L_f$, $\upsilon_f\equiv L_{f1}/L_f$ as the share of factor *f* used in sector *s*, $\hat{L}_f$ as any exogenous change in the endowment of factor *f*, and $\hat{\lambda}_f\equiv\upsilon_f\hat{\alpha}_{f1}+(1-\upsilon_f)\hat{\alpha}_{f2}$, as a summary measure of the factor-augmenting impact for factor *f*, we get

$$\begin{bmatrix}\hat{X}_1\\ \hat{X}_2\end{bmatrix}=\frac{1}{\upsilon_A-\upsilon_B}\begin{bmatrix}(1-\upsilon_B) & -(1-\upsilon_A)\\ -\upsilon_B & \upsilon_A\end{bmatrix}\begin{bmatrix}\hat{L}_A+(1-\sigma)\,\hat{\lambda}_A-\sigma[\upsilon_A\hat{p}_1+(1-\upsilon_A)\hat{p}_2-\widehat{w}_A]\\ \hat{L}_B+(1-\sigma)\,\hat{\lambda}_B-\sigma[\upsilon_B\hat{p}_1+(1-\upsilon_B)\hat{p}_2-\widehat{w}_B]\end{bmatrix}$$

Using $\beta_{Ry}\equiv 1/(\upsilon_A-\upsilon_B)$ (where β_{Ry} is the Rybczynski elasticity) we then have

$$\Delta_{12}\hat{X}\equiv\hat{X}_1-\hat{X}_2=\beta_{Ry}\{\hat{L}_A-\hat{L}_B+(1-\sigma)\left(\hat{\lambda}_A-\hat{\lambda}_B\right)+\sigma(\widehat{w}_A-\widehat{w}_B)\}-\sigma(\hat{p}_1-\hat{p}_2),$$

which, using (A1.1), can be written

$$\Delta_{12}\hat{X}=\beta_{Ry}\{\Delta_{AB}\hat{L}+(1-\sigma)\,\Delta\hat{\lambda}+\sigma\beta_{SS}\Delta_{12}\hat{\chi}\}+\sigma\left(\beta_{Ry}\beta_{SS}-1\right)\Delta_{12}\hat{p}$$

where $\Delta_{AB}\hat{L} \equiv \hat{L}_A - \hat{L}_B$ and $\Delta_{AB}\hat{\lambda} \equiv \hat{\lambda}_A - \hat{\lambda}_B$. Define $\eta \equiv \sigma(\beta_{Ry}\beta_{SS} - 1)$ as the supply elasticity of relative production with respect to relative price, i.e. movement around the production possibility frontier. Note that β_{Ry} and β_{SS} always have the same sign, and their absolute magnitudes exceed 1. The sign depends on relative factor intensities; if sector 1 is relatively A-labour intensive then $\omega_1 > \omega_2$, $\upsilon_A > \upsilon_B$ and thus $\beta_{SS} > 1$ and $\beta_{Ry} > 1$.

Further, define $\Delta_{AB}\mathrm{V} \equiv (1-\sigma)\Delta_{AB}\hat{\lambda} + \sigma\beta_{SS}\Delta\hat{\chi}$ as the "relative factor-endowment equivalent" (FEE) of the technical change. See section 3 in the main text for a discussion of FEE. We can then write

$$\Delta_{12}\hat{X} = \beta_{Ry}(\Delta_{AB}\hat{L} + \Delta_{AB}\mathrm{V}) + \eta\Delta_{12}\hat{p} \qquad (A1.2)$$

Together with the CES demand, $\Delta_{12}\hat{X} = -\varepsilon\Delta_{12}\hat{p}$, we have

$$\beta_{Ry}(\Delta_{AB}\hat{L} + \Delta_{AB}\mathrm{V}) + \eta\Delta_{12}\hat{p} = -\varepsilon\Delta_{12}\hat{p}$$

$$\Delta_{12}\hat{p} = \frac{-\beta_{Ry}}{\varepsilon + \eta}(\Delta_{AB}\hat{L} + \Delta_{AB}\mathrm{V}) \qquad (A1.3)$$

Note first that if we disregard technical progress, i.e. $\Delta_{AB}\mathrm{V} = 0$, then (A1.2) gives the familiar Rybczynski effect of factor endowment changes, and (A1.3) is the accompanying price effect. Second, note that $\Delta_{AB}\mathrm{V} \neq 0$ has exactly the same effect as a change in factor endowments. In the remainder of this appendix, we will assume no changes in factor endowments, and focus on technical progress. Finally, employment effects follow from the labour demand functions, and can be expressed as:

$$\Delta_{12}\hat{L}_f \equiv \hat{L}_{f1} - \hat{L}_{f2} = \Delta_{12}\hat{X} + \sigma\Delta_{12}\hat{p} - (1-\sigma)(\hat{\alpha}_{f1} - \hat{\alpha}_{f2}) = (\varepsilon - \sigma)(-\Delta_{12}\hat{p}) - (1-\sigma)(\hat{\alpha}_{f1} - \hat{\alpha}_{f2}) \qquad (A1.4)$$

Equations (A1.1) - (A1.4) give the comparative static effects of technical shocks, $\hat{\alpha}_{fs}$, using

$$\Delta_{AB}\hat{\lambda} = \hat{\lambda}_A - \hat{\lambda}_B = \upsilon_A\hat{\alpha}_{A1} + (1-\upsilon_A)\hat{\alpha}_{A2} - [\upsilon_B\hat{\alpha}_{B1} + (1-\upsilon_B)\hat{\alpha}_{B2}] \qquad (A1.5)$$

$$\Delta_{12}\hat{\chi} = \hat{\chi}_1 - \hat{\chi}_2 = \omega_1\hat{\alpha}_{A1} + (1-\omega_1)\hat{\alpha}_{B1} - [\omega_2\hat{\alpha}_{A2} + (1-\omega_2)\hat{\alpha}_{B2}]$$

In section 4, we assume $\alpha_{fs} = \alpha_f\alpha_s$ and thus $\hat{\alpha}_{fs} = \hat{\alpha}_f + \hat{\alpha}_s$, and report results in terms of

- factor-biased technical progress $\Delta_{AB}\Phi \equiv \hat{\alpha}_A - \hat{\alpha}_B = \hat{\alpha}_{A1} - \hat{\alpha}_{B1} = \hat{\alpha}_{A2} - \hat{\alpha}_{B2}$
- sector-biased technical progress $\Delta_{12}\Sigma \equiv \hat{\alpha}_1 - \hat{\alpha}_2 = \hat{\alpha}_{A1} - \hat{\alpha}_{A2} = \hat{\alpha}_{B1} - \hat{\alpha}_{B2}$

In appendix 2 we give examples of more general cases by looking at cases where any α_{fs} can change individually, keeping all other technology parameters constant.

Some useful observations

Small open economy: For a small, open economy, the relevant equations are (since $\Delta_{12}\hat{p} = 0$)

$\Delta_{12}\hat{X} = \beta_{Ry}\Delta_{AB}\mathrm{V}$; $\Delta_{AB}\hat{w} = \beta_{SS}\Delta_{12}\hat{\chi}$, and $\Delta\hat{L}_f = \Delta_{12}\hat{X} - (1-\sigma)(\hat{\alpha}_{f1} - \hat{\alpha}_{f2})$.

Factor augmenting vs factor saving impact: The factor saving effect of a technological change $\hat{\alpha}_{fs}$ is defined as the change in the use of the relevant factor per unit of output at unaltered factor prices. Hence, in our CES-model the factor-saving effect of $\hat{\alpha}_{fs}$ is $(1-\sigma)\hat{\alpha}_{fs}$. For this to be positive, we need to assume $\sigma < 1$; otherwise e.g. $\hat{\alpha}_{A1} > 0$ leads to increased used of factor-A per unit of output in sector 1, due to the substitution from factor-B to factor-A, and hence a negative factor-saving effect (or positive factor-using effect). However, it is important to notice that in our context, the decisive term is not the factor-saving effect defined in this way, but rather $\Delta_{AB}V \equiv (1-\sigma)\,\Delta_{AB}\hat{\lambda} + \sigma\beta_{SS}\Delta_{12}\hat{\chi}$. The difference being the factor substitution taking place due to a change in relative factor prices (at given goods prices) when technology changes.

Relationship between various elasticities and variables: If s denotes the share of sector 1 in GDP, and $\bar{\omega} = \frac{w_A L_A}{w_A L_A + w_B L_B} = s\omega_1 + (1-s)\omega_2$ is the average cost share of factor-A in the economy, then

$$\upsilon_A - \upsilon_B = \frac{L_{A1}}{L_A} - \frac{L_{B1}}{L_B} = \frac{w_A L_{A1}}{w_A L_A} - \frac{w_B L_{B1}}{w_B L_B} = \frac{s\omega_1}{\bar{\omega}} - \frac{s(1-\omega_1)}{1-\bar{\omega}} = \frac{s(1-s)(\omega_1-\omega_2)}{\bar{\omega}(1-\bar{\omega})}$$

. Since $\beta_{SS} \equiv 1/(\omega_1 - \omega_2)$ and $\beta_{Ry} \equiv 1/(\upsilon_A - \upsilon_B)$ we have:

$$\beta_{Ry} = \frac{1}{\upsilon_A - \upsilon_B} = \frac{\bar{\omega}(1-\bar{\omega})}{s(1-s)(\omega_1-\omega_2)} = \frac{\bar{\omega}(1-\bar{\omega})}{s(1-s)}\beta_{SS}$$

In a ***symmetric economy***, with initial situation $w_A L_A = w_B L_B$ and $s = 0.5$, we have $\beta_{Ry} = \beta_{SS}$.

Appendix 2. Factor- and sector-specific technical progress

The system of equations (17) – (19) in the main text can be applied for any individual or combination of technical augmentations, $\hat{\alpha}_{fs}$. For the four cases of individual augmentation (only one factor being augmented in only one sector), i.e. $\hat{\alpha}_{fs} > 0$, and $\hat{\alpha}_{f's'} = 0$ for any $f' \neq f, and\ s' \neq s$ we have

Individual cases	$\Delta_{12}\hat{\chi}$	$\Delta_{AB}\hat{\lambda}$	$\Delta_{AB}\mathbf{V}$
$\hat{\alpha}_{A1} > 0$	$\omega_1\hat{\alpha}_{A1} > 0$	$\upsilon_A\hat{\alpha}_{A1} > 0$	$\{(1-\sigma)\upsilon_A + \sigma\beta_{SS}\omega_1\}\hat{\alpha}_{A1} > 0$
$\hat{\alpha}_{B1} > 0$	$(1-\omega_1)\hat{\alpha}_{B1} > 0$	$-\upsilon_B\hat{\alpha}_{B1} < 0$	$\{-(1-\sigma)\upsilon_B + \sigma\beta_{SS}(1-\omega_1)\}\hat{\alpha}_{B1}$
$\hat{\alpha}_{A2} > 0$	$-\omega_2\hat{\alpha}_{A2} < 0$	$(1-\upsilon_A)\hat{\alpha}_{A2} > 0$	$\{(1-\sigma)(1-\upsilon_A) - \sigma\beta_{SS}\omega_2\}\hat{\alpha}_{A2}$
$\hat{\alpha}_{B2} > 0$	$-(1-\omega_2)\hat{\alpha}_{B2} < 0$	$-(1-\upsilon_B)\hat{\alpha}_{B2} < 0$	$-\{(1-\sigma)(1-\upsilon_B) + \sigma\beta_{SS}(1-\omega_2)\}\hat{\alpha}_{B2} < 0$

If we assume (as we will do throughout this appendix) that sector 1 is A-intensive, it should be noted that for augmentation of the factor used intensively in a sector, i.e. factor-A augmentation in sector 1, and for factor-B augmentation in sector 2, the cost-saving and the factor-saving impacts draw in the same direction, thus also ensuring that the FEE, $\Delta_{AB}V$, has the same sign. For the other two cases, augmentation of the un-intensive factor, the cost-saving and the factor-saving impacts draw in opposite directions, and the sign of FEE depends on the relative size of these as well as on σ.

We will look at two of these cases to illustrate how factor- and sector-specific augmentation may work. The remaining two cases follow by symmetry. The general formulae of section 3 apply, and here we illustrate adjustment for augmentation that takes place in sector 1; we look first at the case of factor-A augmentation, $(\hat{\alpha}_{A1})$, and then at B-augmentation, $(\hat{\alpha}_{B1})$.

Augmentation of factor-A is $\widehat{\alpha}_{A1} > 0$, with all other $\widehat{\alpha}_{fs} = 0$. From the table above, it is clear that this gives $\Delta_{12}\hat{\chi} = \omega_1\hat{\alpha}_{A1} > 0$, $\Delta_{AB}\hat{\lambda} = \upsilon_A\hat{\alpha}_{A1} > 0$, and $\Delta_{AB}V = \{(1-\sigma)\upsilon_A + \sigma\beta_{SS}\omega_1\}\hat{\alpha}_{A1} > 0$; the latter effect follows from the fact that as long as sector 1 is A-intensive, we have $\beta_{SS}\omega_1 > 1 > \upsilon_A$. This is a good example of a case where the direct factor-saving effect, $(1-\sigma)\upsilon_A$, could be negative, while the overall factor-endowment equivalence of the technical change is still positive, due to the technology-induced changes in relative factor prices. Equations (17) – (19) then give

$$\Delta_{12}\hat{p} = -\frac{\beta_{RY}\,\Delta_{AB}V}{(\varepsilon+\eta)} = -\frac{\beta_{RY}\{(1-\sigma)\upsilon_A + \sigma\beta_{SS}\omega_1\}}{(\varepsilon+\eta)}\hat{\alpha}_{A1}$$

$$\Delta_{AB}\widehat{w} = \beta_{SS}\{\Delta_{12}\hat{p} + \omega_1\hat{\alpha}_{A1}\} = \frac{\beta_{SS}\{\omega_1(\varepsilon-1) + (\sigma-1)(\beta_{RY}\upsilon_A - \omega_1)\}}{(\varepsilon+\eta)} \qquad \text{(A2.1)}$$

$$\Delta_{12}\hat{L}_A = (\varepsilon-\sigma)(-\Delta\hat{p}) + (\sigma-1)\hat{\alpha}_{A1}, \quad \Delta_{12}\hat{L}_B = (\varepsilon-\sigma)(-\Delta\hat{p}).$$

The relative price of the A-intensive good (good 1) falls, as $\Delta_{AB}V$ and β_{RY} are both positive in this case. The wage change is ambiguous; positive for any combination of $\varepsilon > 1$ and $\sigma > 1$, negative for any combination of $\varepsilon < 1$ and $\sigma < 1$, and always more likely to be positive the more elastic demand is, since $-\Delta_{12}\hat{p}$ is falling in ε.

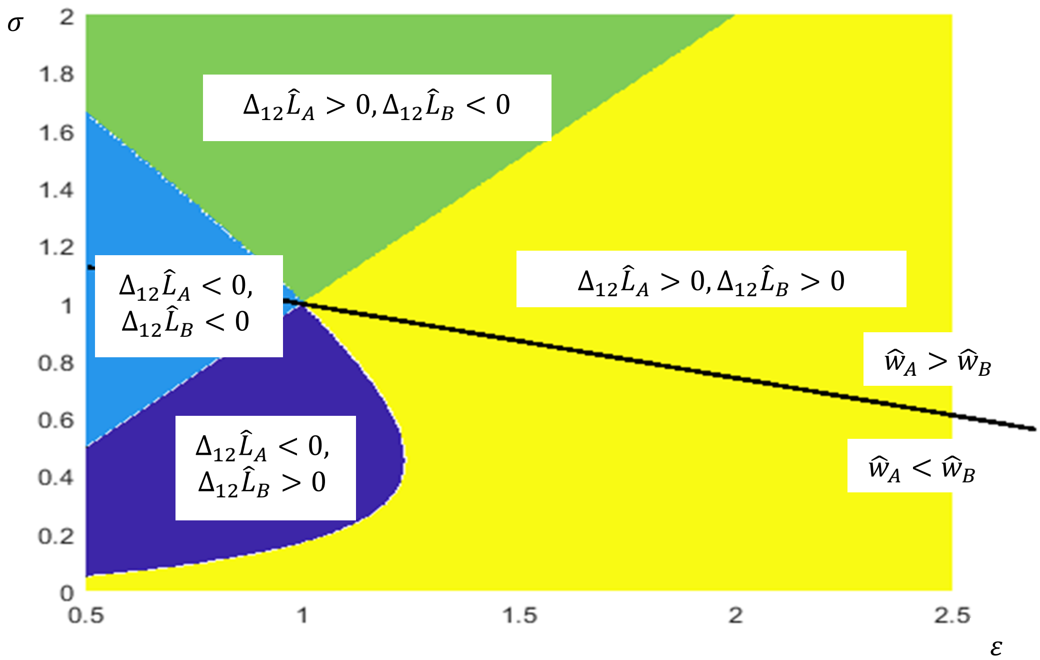
Employment effects (as well as relative wage effects) are illustrated in Figure 1a, which has parameters σ and ε on the vertical and horizontal axes respectively.[16] Sector 1 employment of factor-B increases if $\varepsilon > \sigma$, as is clear from eq. (A2.1), i.e. if the demand effect following from the price change is greater than the (general equilibrium) substitution effect. The sector's employment of factor-A combines direct effects with general equilibrium effects. Employment increases for sufficiently large ε, as expected. At lower values of ε outcomes depend on σ, but in a non-monotonic way, with employment falling at intermediate values of σ. The intuition is seen from a closer inspection of the $\Delta_{12}\hat{L}_A$ equation in (A2.1): at $\varepsilon = \sigma = 1$ we have $\Delta_{12}\hat{L}_A = 0$. A small reduction in σ from that point, keeping $\varepsilon = 1$, yields $\Delta_{12}\hat{L}_A < 0$, while as $\sigma \to 0$ we get $\Delta_{12}\hat{L}_A > 0$. Hence, as σ falls below 1, to restore $\Delta_{12}\hat{L}_A = 0$ first requires an increase and then a reduction in the demand elasticity. Similarly, keeping $\sigma = 1$, yields $\Delta_{12}\hat{L}_A < 0$ for $\varepsilon < 1$ and $\Delta_{12}\hat{L}_A > 0$ for $\varepsilon > 1$.

Finally, it should be observed that for any combination of $\varepsilon > 1$ and $\sigma > 1$, we have $\Delta_{12}\hat{L}_A > 0$ and $\Delta_{AB}\widehat{w} > 0$. Hence, biased technical progress for the factor used intensively in a sector yields increased employment of the factor in that sector and increased relative wage for the factor, as long as both elasticities exceed 1. For $\varepsilon > 1$ and $\sigma < 1$ it is interesting to see that there are areas in which the sector's relative employment of the factor increases, yet the relative factor price falls.[17]

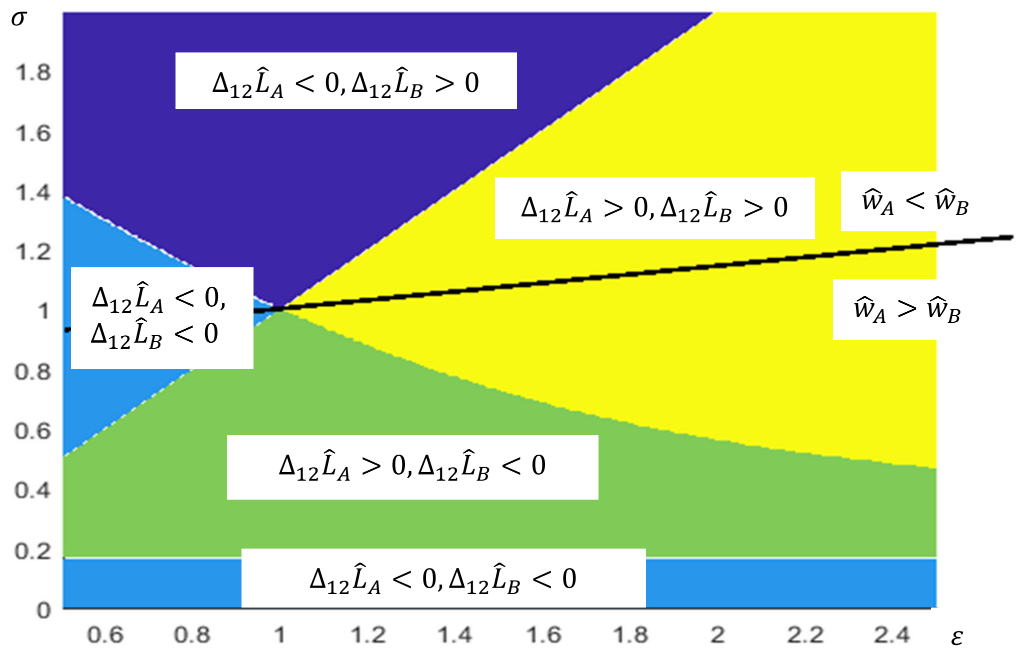
[16] Computed with the two sectors symmetric, s = ½, $\omega_1 = 1 - \omega_2$. If $\sigma \neq 1$ then ω_1, ω_2 are endogenous. The figure uses equations from section 3 for local variations around equilibrium points at which $\omega_1 = 1 - \omega_2 = 0.6$.
[17] Similarly, for $\varepsilon < 1$ and $\sigma > 1$ there are areas with $\Delta_{12}\hat{L}_A < 0$ and $\Delta_{AB}\widehat{w} > 0$. This illustrates clearly the fact that there is not a one-to-one correspondence between relative wage effects and relative employment effects in the sector experiencing technical progress.

Figure 1: ***Factor- and sector-specific technical progress.***

Panel a: Augmentation of factor-A in the A-intensive sector: $\hat{\alpha}_{A1} > 0$



Panel b: Augmentation of factor-B used in A-intensive sector: $\hat{\alpha}_{B1} > 0$



If technical change augments factor-B in sector 1, ($\hat{\alpha}_{B1} > 0$, augmentation of the factor un-intensive in the sector, all other $\hat{\alpha}_{fs} = 0$), then $\Delta_{AB}V = \{-(1-\sigma)\upsilon_B + \sigma\beta_{SS}(1-\omega_1)\}\hat{\alpha}_{B1}$, which can be positive or negative, as the factor-saving (for factor B) and the cost-saving (for sector 1) impact draw in opposite directions. For low values of σ the factor-saving impact dominates, and $\Delta_{AB}V < 0$; for higher values of σ the cost-saving impact dominates, and $\Delta_{AB}V > 0$. For the endogenous variables we have

$$\Delta_{12}\hat{p} = -\frac{\beta_{RY}\,\Delta_{AB}V}{(\varepsilon+\eta)} = \frac{\beta_{RY}\{(1-\sigma)\,\upsilon_B - \sigma\beta_{SS}(1-\omega_1)\}}{(\varepsilon+\eta)}\hat{\alpha}_{B1}$$

$$\Delta_{AB}\hat{w} = \beta_{SS}\{\Delta_{12}\hat{p} + (1-\omega_1)\hat{\alpha}_{B1}\}, \qquad \text{(A2.2)}$$

$$\Delta_{12}\hat{L}_A = (\varepsilon-\sigma)(-\Delta\hat{p}), \qquad \Delta_{12}\hat{L}_B = (\varepsilon-\sigma)(-\Delta\hat{p}) + (\sigma-1)\hat{\alpha}_{B1}.$$

Relative employment and wage changes are illustrated in Figure 1b.

In this case, the price may go either way, reflecting the sign of $(-\Delta_{AB}V)$. Hence for very low values of σ (less than 0.167 in the example of Figure 1b), the relative price of good 1 increases, and the production of good 1 falls. For higher values of σ the price falls and, as usual, high ε tends to raise employment of both factors in the sector. For factor-A (the now non-augmenting factor), the sign is determined from $\varepsilon - \sigma$ alone, while for factor-B the negative factor-saving effect implies that factor-B may be reallocated from sector 1 to sector 2 even for higher values of ε when $\sigma < 1$.

When demand is inelastic ($\varepsilon < 1$), it should be observed that there is a range of values of σ for which there is reduced employment of both factors in the sector that experiences technical progress. This is true whether the factor augmentation is for the intensive or the un-intensive factor.

Appendix 3. Model with non-traded sector

In this appendix we will introduce a non-traded sector in the model in a simple way. Labelling non-traded sector as N, the labour market equilibrium conditions are, in general, $L_{f1} + L_{f2} + L_{f\mathrm{N}} = L_f$, $f = A, B$, which we can rewrite as:

$$L_{f1} + L_{f2} = L_f - L_{fN} \equiv L_f^T, \quad f = A, B$$

Where L_f^T is the "factor supply" to the two traded sectors. Hence, an increase in the production of non-traded goods will lower the factor supply to traded sectors, while reduced non-traded production releases more factors. The production and employment effects depend on relative factor intensities.

To stay as close to the previous analysis as possible, we will now define $v_f^T = L_{f1}/L_f^T$ as sector 1's share of traded sector employment of factor *f*, and let $v_f^N = L_{fN}/L_f^T$ be the non-traded employment relative to the traded sector employment of factor *f*. For factor market equilibrium, the equivalence of (9) in section 3 becomes (assuming that there are no changes in the factor endowments, L_f)

$$v_f^T\hat{L}_{f1} + \left(1-v_f^T\right)\hat{L}_{f2} = \left(dL_f - dL_{f\mathrm{N}}\right)/L_f^T = -v_f^0\hat{L}_{f\mathrm{N}}, \qquad f = A, B. \qquad \text{(A3.1)}$$

For non-traded production we will, to simplify, assume no technical progress and fixed coefficients ($\sigma_N = 0$), so we have $\hat{L}_{jN} = \hat{X}_N$.[18] Given that the technical progress is assumed to take place only in the traded sectors, our summary measures of factor-saving progress become $\hat{\lambda}_f^T \equiv v_f^T\hat{\alpha}_{f1} +$

[18] We could allow for sector-specific technical progress in the non-traded sector by writing $\hat{L}_{fN} = \hat{X}_N - \hat{\alpha}_N$.

$\left(1 - v_f^T\right)\hat{\alpha}_{f2}$ and $\Delta\hat{\lambda}^T \equiv \hat{\lambda}_A^T - \hat{\lambda}_B^T$. For the cost-saving measures there is no difference, since the cost shares in each sector are unchanged. The system of equations (see Appendix 1) then becomes

$$\begin{bmatrix}\hat{X}_1\\ \hat{X}_2\end{bmatrix} = \frac{1}{v_A^T - v_B^T}\begin{bmatrix}(1-v_B^T) & -(1-v_A^T)\\ -v_B^T & v_A^T\end{bmatrix}\begin{bmatrix}(1-\sigma)\,\hat{\lambda}_A^T - \sigma[v_A^T\hat{p}_1 + (1-v_A^T)\hat{p}_2 - \hat{w}_A] - v_A^{\mathrm{N}}\hat{X}_{\mathrm{N}}\\ (1-\sigma)\hat{\lambda}_B^T - \sigma[v_B^T\,\hat{p}_1 + (1-v_B^T)\hat{p}_2 - \hat{w}_B] - v_B^{\mathrm{N}}\hat{X}_{\mathrm{N}}\end{bmatrix}$$

Solving this for differences in relative production changes we get

$$\Delta_{12}\hat{X} = \hat{X}_1 - \hat{X}_2 = \frac{1}{v_A^T - v_B^T}\left\{(1-\sigma)\Delta\hat{\lambda}^T - \left(v_A^{\mathrm{N}} - v_B^{\mathrm{N}}\right)\hat{X}_{\mathrm{N}} + \sigma[\hat{w}_A - \hat{w}_B]\right\} - \sigma\Delta_{12}\hat{p}$$

Define $s_T \equiv s_1 + s_2 = 1 - s_{\mathrm{N}}$ to be the total share of GDP coming from the traded sectors, and $s_i^T = s_i/s_T$ to be the share of sector *s* in traded GDP. Let $\omega_T \equiv s_1^T\omega_1 + s_2^T\omega_2 = s_1^T\omega_1 + (1-s_1^T)\omega_2$ be the average cost share of A-factor in traded goods production. Then we can write $v_A^{\mathrm{N}} = \frac{\omega_{\mathrm{N}}s_{\mathrm{N}}}{\omega_T(1-s_{\mathrm{N}})}$, $v_B^{\mathrm{N}} = \frac{(1-\omega_{\mathrm{N}})s_{\mathrm{N}}}{(1-\omega_T)(1-s_{\mathrm{N}})}$, and

$$v_A^{\mathrm{N}} - v_B^{\mathrm{N}} = \frac{(1-\omega_{\mathrm{N}})s_{\mathrm{N}}}{(1-\omega_T)(1-s_{\mathrm{N}})}\left(\frac{\omega_{\mathrm{N}}}{1-\omega_{\mathrm{N}}}\Big/\frac{\omega_T}{1-\omega_T} - 1\right) = \frac{s_{\mathrm{N}}}{(1-s_{\mathrm{N}})}\frac{(\omega_{\mathrm{N}} - \omega_T)}{(1-\omega_T)\omega_T}$$

Using $\beta_{Ry}^T \equiv 1/(v_A^T - v_B^T)$ as the Rybczynski elasticity in this case, it has the standard property, but the magnitude may differ from the case without a non-traded sector. Then we have

$$\Delta_{12}\hat{X} = \beta_{Ry}^T\left\{(1-\sigma)\Delta_{AB}\hat{\lambda}^T - \frac{s_{\mathrm{N}}}{(1-s_{\mathrm{N}})}\frac{(\omega_0 - \omega_T)}{(1-\omega_T)\omega_T}\hat{X}_{\mathrm{N}} + \sigma\Delta_{AB}\hat{w}\right\} - \sigma\Delta_{12}\hat{p} \qquad (A3.2)$$

So far we have only used the assumption that there are fixed coefficients in the non-traded sector. To proceed, we need to add a demand side, since $\hat{X}_N$ is determined from the demand effect for non-traded goods and services. We will do this in the simplest possible way, by assuming that the demand for non-tradables is determined from Cobb-Douglas preferences, such that the income share going to non-traded goods is given.[19] Then we have $\hat{X}_N = \hat{Y} - \hat{p}_N$.

For income and prices, the following applies: Factor prices are determined from the $C_1(w_A/\alpha_{A1}, w_B/\alpha_{B1}) = p_1$ and $C_2(w_A/\alpha_{A2}, w_B/\alpha_{B2}) = p_2$ as long as there is positive production of both traded goods. Then (19) from section 3, $\Delta_{AB}\hat{w} = \beta_{SS}\{\Delta_{12}\hat{p} + \Delta_{12}\hat{\chi}\}$, still applies. For income and the price of non-traded goods, we have

$$\hat{Y} = \bar{\omega}\hat{w}_A + (1-\bar{\omega})\hat{w}_B; \quad \hat{p}_{\mathrm{N}} = \omega_{\mathrm{N}}\hat{w}_A + (1-\omega_{\mathrm{N}})\hat{w}_B \qquad (A3.3)$$

Where $\bar{\omega}$ is the average cost share of factor A in the economy, and we can write $\bar{\omega} = (1-s_N)\omega_T + s_N\omega_N$. Then demand effects for non-traded goods can be written

$$\hat{X}_{\mathrm{N}} = \hat{Y} - \hat{p}_{\mathrm{N}} = \bar{\omega}\hat{w}_A + (1-\bar{\omega})\hat{w}_B - [\omega_{\mathrm{N}}\hat{w}_A + (1-\omega_{\mathrm{N}})\hat{w}_B] = (\bar{\omega} - \omega_{\mathrm{N}})(\hat{w}_A - \hat{w}_B)$$

Which gives $\hat{X}_N = (1-s_N)(\omega_T - \omega_N)\Delta_{AB}\hat{w}$. Using all this in (A3.2) yields

[19] This could e.g. come from a two-level utility function where at the top-level there is Cobb-Douglas between non-traded and a nest of traded goods, and at the next level CES with elasticity ε between the traded goods.

$$\Delta_{12}\hat{X} = \beta_{Ry}^T\left\{(1-\sigma)\Delta_{AB}\hat{\lambda}^T + \left[\frac{s_0(\omega_0-\omega_T)^2}{(1-\omega_T)\omega_T} + \sigma\right]\beta_{SS}\{\Delta_{12}\hat{p} + \Delta_{12}\hat{\chi}\}\right\} - \sigma\Delta_{12}\hat{p} \qquad (A3.2')$$

Finally, define the following elasticities

$$\eta_T \equiv \sigma\left(\beta_{Ry}^T\beta_{SS} - 1\right) \text{ and } \eta_{\mathrm{N}} \equiv \beta_{Ry}^T\beta_{SS}\frac{s_{\mathrm{N}}(\omega_{\mathrm{N}}-\omega_T)^2}{(1-\omega_T)\omega_T}$$

It is sometimes convenient to write $\eta_{\mathrm{N}} = \beta_{Ry}^T\beta_{SS}\,\varphi$ with $\varphi \equiv \frac{s_{\mathrm{N}}(\omega_{\mathrm{N}}-\omega_T)^2}{(1-\omega_T)\omega_T} \geq 0$. We then get:

$$\Delta_{12}\hat{X} = \beta_{Ry}^T\left\{(1-\sigma)\Delta\hat{\lambda}^T + \beta_{SS}(\sigma+\varphi)\Delta_{12}\hat{\chi}\right\} + (\eta_T+\eta_{\mathrm{N}})\Delta_{12}\hat{p} \qquad (A3.2'')$$

Note that $\left\{(1-\sigma)\Delta\hat{\lambda}^T + \beta_{SS}(\sigma+\varphi)\Delta_{12}\hat{\chi}\right\}$ is the equivalent of $\Delta_{AB}V$ in the 2-sector model. If we, to simplify even further, assume that the relative distribution of factors between the traded sectors is the same with and without a non-traded sector (i.e. $v_f^T = v_f, f = A, B$), then $\Delta_{AB}\hat{\lambda}^T = \Delta_{AB}\hat{\lambda}$, $\beta_{Ry}^T = \beta_{RY}$ and $\eta_T = \eta$ and we can write[20]

$$\Delta_{12}\hat{X} = \beta_{Ry}^T\{\Delta_{AB}V + \beta_{SS}\varphi\Delta_{12}\hat{\chi}\} + (\eta+\eta_{\mathrm{N}})\Delta_{12}\hat{p} = \beta_{Ry}^T\Delta_{AB}V + \eta_{\mathrm{N}} + (\eta+\eta_{\mathrm{N}})\Delta_{12}\hat{p}$$

Hence, the non-traded sector works as if both FEE and η increase. We then have

$$\Delta_{12}\hat{p} = -\frac{\beta_{RY}\Delta_{AB}V + \eta_{\mathrm{N}}}{\varepsilon + \eta + \eta_{\mathrm{N}}}. \qquad (A3.4)$$

Comparing with the 2-sector model, the only differences are that the supply elasticity η is replaced by $\eta + \eta_N$ and that $\Delta_{AB}V$ is replaced by $\Delta_{AB}V + \eta_{\mathrm{N}}/\beta_{RY}$. η captures structural changes through factor substitution between the two traded goods sectors; recall that it is = 0 if $\sigma = 0$ and positive for any $\sigma > 0$. η_N captures structural changes appearing through changes in factor use in the non-traded sector. Note that we have $\eta_N \geq 0$ and always positive as long as $s_N > 0$ and $\omega_N \neq \omega_T$ (the same applies for φ). η_N is higher the larger the share of the non-traded sector is, and the more its factor intensity differs from the average traded sector intensity.

It is clear from the equation above that the existence of a non-traded sector (i.e. $\eta_{\mathrm{N}} > 0$) affects the magnitude of $|\Delta_{12}\hat{p}|$: if $\beta_{RY}\Delta_{AB}V < \varepsilon + \eta$ then $|\Delta_{12}\hat{p}|$ increases with η_0; if $\beta_{RY}\Delta_{AB}V > \varepsilon + \eta$ then $|\Delta_{12}\hat{p}|$ falls.

To study the relative wage effect, it is convenient to rewrite (A3.2') and (A3.4) as

$$\Delta_{12}\hat{X} = \beta_{Ry}^T\{\Delta_{AB}V + \beta_{SS}\varphi\Delta_{AB}\hat{w}\} + \eta\Delta_{12}\hat{p}$$

$$\Delta_{12}\hat{p} = -\frac{\beta_{RY}\{\Delta_{AB}V + \beta_{SS}\varphi\Delta_{AB}\hat{w}\}}{\varepsilon + \eta}$$

Then from $\Delta_{AB}\hat{w} = \beta_{SS}\{\Delta_{12}\hat{p} + \Delta_{12}\hat{\chi}\}$ we get

[20] In the remainder of this appendix, we will use this simplified version; however, note that all the results hold in the more general case, using (A3.2").

$$\Delta_{AB}\widehat{w}\left\{1+\frac{\beta_{RY}\beta_{SS}\varphi\Delta_{AB}\widehat{w}}{\varepsilon+\eta}\right\}=\beta_{SS}\left\{-\frac{\beta_{RY}\Delta_{AB}V}{\varepsilon+\eta}+\Delta_{12}\widehat{\chi}\right\} \qquad (A3.5)$$

Since $\{1+\beta_{RY}\beta_{SS}\varphi/(\varepsilon+\eta)\}>1$ for $\varphi>0$ and the right-hand side is identical to the 2-sector model, the existence of a non-traded sector always dampens the relative wage effect, i.e. reduces $|\Delta_{AB}\widehat{w}|$.

Qualitatively, all the price effects from the 2-sector model apply, with the modification given in (A3.4), as do the results for relative employment in the two traded sectors. However, for absolute employment effects, we need to take into account the employment changes in the non-traded sector as well. For the general case, we have from (18)

$$\Delta_{12}\hat{L}_f\equiv\hat{L}_{f1}-\hat{L}_{f2}=\Delta_{12}\hat{X}_1+\sigma\Delta_{12}\hat{p}+(\sigma-1)\left(\hat{\alpha}_{f1}-\hat{\alpha}_{f2}\right)=(\varepsilon-\sigma)(-\Delta_{12}\hat{p})+(\sigma-1)\left(\hat{\alpha}_{f1}-\hat{\alpha}_{f2}\right)$$

Using the general equilibrium conditions $\upsilon_f^T\hat{L}_{f1}+\left(1-\upsilon_f^T\right)\hat{L}_{f2}=-\upsilon_f^{\mathrm{N}}\hat{L}_{f\mathrm{N}}$ we get

$$\hat{L}_{f1}=\left(1-\upsilon_f^T\right)\Delta_{12}\hat{L}_f-\upsilon_f^{\mathrm{N}}\hat{L}_{f\mathrm{N}} \quad \text{and} \quad \hat{L}_{f2}=-\left[\upsilon_f^T\Delta_{12}\hat{L}_f+\upsilon_f^{\mathrm{N}}\hat{L}_{f\mathrm{N}}\right] \qquad (A3.6)$$

Recall that in the 2-sector model, with $\hat{L}_{fN}=0$, the signs of $\hat{L}_{f1}$ and $\hat{L}_{f2}$ were always opposite and followed directly from $\Delta_{12}\hat{L}_f$. Now total employment in the traded sectors can change ($\hat{L}_{fN}\neq 0$) and we may thus have cases in which employment in both traded sectors may go up or down simultaneously.

To illustrate, we will work out some details for the same examples as in section 4 – a sector-biased and a factor-biased shock.

A3.1 Sector-biased technical change $\Delta\boldsymbol{\Sigma}>\mathbf{0},\Delta\boldsymbol{\Phi}=\mathbf{0}$

We have $\Delta_{12}\widehat{\chi}=\Delta_{12}\Sigma,\ \ \Delta_{AB}\hat{\lambda}=\Delta_{12}\Sigma/\beta_{Ry}^T$, and (A3.4), we get

$$\Delta_{12}\hat{p}=\frac{-(1+\eta+\eta_{\mathrm{N}})}{\varepsilon+\eta+\eta_{\mathrm{N}}}\Delta_{12}\Sigma$$

$$\Delta_{AB}\widehat{w}=\ \beta_{SS}\{\Delta_{12}\hat{p}+\Delta_{12}\widehat{\chi}\}=\beta_{SS}\frac{\varepsilon-1}{\varepsilon+\eta+\eta_{\mathrm{N}}}\Delta_{12}\Sigma$$

$$\Delta_{12}\hat{L}_f=(\varepsilon-\sigma)(-\Delta\hat{p})-(1-\sigma)\Delta_{12}\Sigma=\frac{\sigma+\eta+\eta_{\mathrm{N}}}{\varepsilon+\eta+\eta_{\mathrm{N}}}(\varepsilon-1)\Delta_{12}\Sigma$$

Hence, the non-traded sector reinforces the relative employment effect if $\varepsilon>\sigma$. For the individual employment effects, we have

$$\hat{L}_{f1}=\left(1-\upsilon_f^T\right)\Delta_{12}\hat{L}_f-\upsilon_f^{\mathrm{N}}\hat{X}_{\mathrm{N}}\ =\frac{(\varepsilon-1)\Delta_{12}\Sigma}{\varepsilon+\eta+\eta_{\mathrm{N}}}\left\{\left(1-\upsilon_f^T\right)(\sigma+\eta+\eta_{\mathrm{N}})-\upsilon_f^{\mathrm{N}}(1-s_N)(\omega_T-\omega_N)\beta_{SS}\right\}$$

$$\hat{L}_{f2}=-\left[\upsilon_f^T\Delta_{12}\hat{L}_f+\upsilon_f^{\mathrm{N}}\hat{L}_{f\mathrm{N}}\right]=\frac{-(\varepsilon-1)\Delta_{12}\Sigma}{\varepsilon+\eta+\eta_{\mathrm{N}}}\left\{\upsilon_f^T(\sigma+\eta+\eta_{\mathrm{N}})+\upsilon_f^{\mathrm{N}}(1-s_N)(\omega_T-\omega_N)\beta_{SS}\right\}$$

So if $\omega_N>\omega_T$ and $\varepsilon>1$ then we clearly have $\hat{L}_{f1}>0$ as the relative employment of both factors increase in sector 1, and the non-traded sector releases both factors. For sector 2, the relative change is negative, while the release of factors from non-traded sector is positive. If $\omega_N < \omega_T$ the reverse is true, i.e. employment in sector 2 goes down, while for sector 1 the two forces move in opposite directions.

A3.2 Factor-bias technical change $\Delta_{AB}\boldsymbol{\Phi} > \mathbf{0}, \Delta_{12}\boldsymbol{\Sigma} = \mathbf{0}$

Note that we assume technical progress only to apply in the traded goods sectors; technology in the non-traded sector is unaltered. We now have $\Delta_{12}\hat{\chi} = \Delta_{AB}\Phi/\beta_{SS}$, $\Delta_{AB}\hat{\lambda} = \Delta_{AB}\Phi$ and, using (A3.4)

$$\Delta_{12}\hat{p} = -\frac{\beta_{RY}\Delta_{AB}V + \eta_{\mathrm{N}}}{\varepsilon + \eta + \eta_{\mathrm{N}}} = \frac{-\beta_{RY}(1+\varphi)}{\varepsilon + \eta + \eta_{\mathrm{N}}}\Delta_{AB}\Phi$$

$$\Delta_{AB}\widehat{w} = \beta_{SS}\{\Delta\hat{p} + \Delta\hat{\chi}\} = \beta_{SS}\Delta\hat{p} + \Delta_{AB}\Phi = \left\{1 - \frac{\beta_{SS}\beta_{RY}(1+\varphi)}{\varepsilon + \eta + \eta_{\mathrm{N}}}\right\}\Delta_{AB}\Phi = \frac{\varepsilon + \eta - \beta_{SS}\beta_{RY}}{\varepsilon + \eta + \eta_{\mathrm{N}}}\Delta_{AB}\Phi$$

This verifies the general results that η_{N} dampens the magnitude of $\Delta_{AB}\widehat{w}$, but does not affect the sign. For employment effects, we have

$$\Delta_{12}\hat{L}_f = (\varepsilon - \sigma)(-\Delta_{12}\hat{p}) = (\varepsilon - \sigma)\frac{\beta_{RY}(1+\varphi)}{\varepsilon + \eta + \eta_{\mathrm{N}}}\Delta_{AB}\Phi.$$

Since $\eta_N = \beta_{SS}\beta_{RY}\varphi$, the non-traded sector reinforces the magnitude of the relative employment effects if $\varepsilon + \eta > \beta_{SS}\beta_{RY}$, i.e. if $\Delta_{AB}\widehat{w} > 0$. For the individual employment effects we have

$$\hat{L}_{f1} = \left(1 - \upsilon_f^T\right)\Delta_{12}\hat{L}_f - \upsilon_f^{\mathrm{N}}\hat{X}_{\mathrm{N}}$$

$$= \frac{\Delta_{AB}\Phi}{\varepsilon + \eta + \eta_{\mathrm{N}}}\left\{\left(1 - \upsilon_f^T\right)(\varepsilon - \sigma)\beta_{RY}(1+\varphi) - \upsilon_f^{\mathrm{N}}(1 - s_{\mathrm{N}})(\omega_T - \omega_{\mathrm{N}})(\varepsilon + \eta - \beta_{SS}\beta_{RY})\right\}$$

$$\hat{L}_{f2} = -\left[\upsilon_f^T\Delta_{12}\hat{L}_f + \upsilon_f^{\mathrm{N}}\hat{L}_{f\mathrm{N}}\right]$$

$$= \frac{-\Delta_{AB}\Phi}{\varepsilon + \eta + \eta_{\mathrm{N}}}\left\{\upsilon_f^T(\varepsilon - \sigma)\beta_{RY}(1+\varphi) - \upsilon_f^{\mathrm{N}}(1 - s_{\mathrm{N}})(\omega_T - \omega_{\mathrm{N}})(\varepsilon + \eta - \beta_{SS}\beta_{RY})\right\}$$

For $\varepsilon > \sigma$, both being sufficiently high to ensure $\Delta_{AB}\widehat{w} > 0$, and $\omega_{\mathrm{N}} > \omega_T$ we have, as in the sector-biased case, that employment of both factors will increase in the A-intensive sector, while the employment effects in the other sector are ambiguous. For $\varepsilon < \sigma$ and both being sufficiently low to give $\Delta_{AB}\widehat{w} < 0$, the employment of both factors will decrease in the A-intensive sector. For $\omega_{\mathrm{N}} < \omega_T$ the effects for sector 1 become ambiguous, while the sector-2 effects are clearly negative, in the case of $\varepsilon > \sigma$ and $\Delta_{AB}\widehat{w} > 0$, and positive in the case of $\varepsilon < \sigma$ and $\Delta_{AB}\widehat{w} < 0$.