O-Ring Production Networks

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Abstract

We study a production network where quality choices are interconnected across firms. High-quality firms are skill intensive and trade more with other high-quality firms. Using data from Turkish firms, we document strong assortative matching of skills in the production network. A firm-specific export demand shock from a rich country increases the firm's skill intensity and shifts the firm toward skill-intensive domestic partners. We develop a quantitative model with heterogeneous firms, endogenous quality choices, and network formation. An economy-wide export demand shock of 5 percent induces exporters and non-exporters to upgrade quality, raising the average wage by 1.2 percent. This effect is about nine times the effect in a special case of the model with no interconnection of quality choices.

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1 Introduction

The space shuttle *Challenger* exploded because one of its innumerable components, the O-rings, malfunctioned during launch. Using this as a leading example, Kremer (1993) studies production processes in which the value of output dramatically decreases if a single task fails. In his model, just one mistake of an unskilled worker is enough to destroy a product. Thus, firms that produce complex, higher-quality products hire skilled workers for all their tasks.

If we extend this rationale across firm boundaries, a high-quality, skill-intensive firm sources its inputs from other high-quality firms and sells more to high-quality firms that value its output. In addition, a firm's decision to upgrade quality depends critically on the willingness of its trading partners to also upgrade or on its ability to find new higher-quality partners. This mechanism applies to the quality of products as well as to the quality of inventory controls, research and development, and internal communications. Improvements in these areas generally allow for a wider product scope and render the firm more flexible to respond to shocks. A firm profits from these improvements if its suppliers also offer scope and flexibility and if its customers value these characteristics.

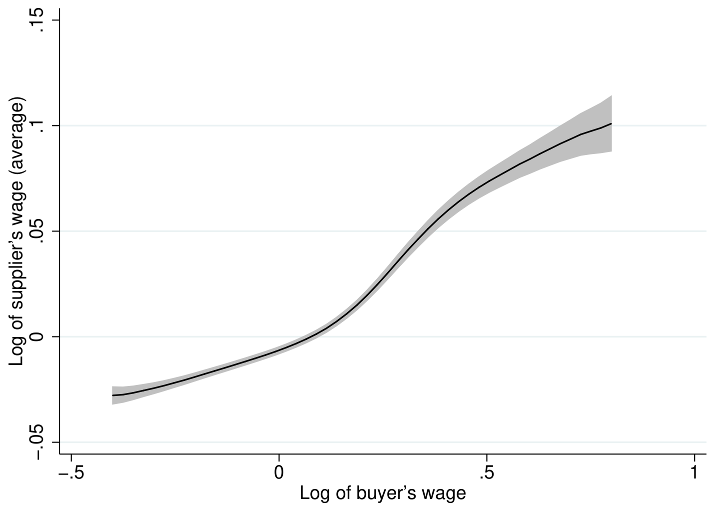
We study this interconnection in firms' quality choices theoretically and empirically. Our data cover all formal Turkish manufacturing firms from 2011 to 2015. We merge value-added tax (VAT) data with matched employer-employee and customs data. We observe the value of trade for each buyer-seller pair of firms; exports by firm, product and destination, and the occupation and wage of each worker in each firm.

We document a novel strong assortative matching of skills in the network. As an example, Figure 1 graphs firms' average wage (adjusted for industry-region) against the average wage of their suppliers.[1] A 10 percent increase in a firm's wage is associated with a 2.5 percent increase in its suppliers' wages. This number is large given that firms have on average eleven suppliers. This increasing relationship between buyer and supplier wages may arise from the extensive margin—with high-wage firms matching more with each other—or from the intensive margin—with high-wage firms spending relatively more on their high-wage suppliers. A decomposition indicates that the extensive margin accounts for 59 percent of the relationship and the intensive margin accounts for 41 percent.

We use shift-share regressions to evaluate firms' responses to shocks and movements along the schedule in Figure 1. Consider a Turkish firm that in 2011 exported a particular product category to a high-income country, say Germany. An increase in German imports

[1]The figure has only manufacturing firms, which are later used in our structural estimation, but an equally strong pattern holds if we include all sectors. See Table 1, column (4).

Figure 1: Assortative Matching on Wages



Notes: We define the wage as the firm's wage bill divided by the number of workers. The supplier wage is the average wage across the firm's manufacturing suppliers, weighted by the firm's spending on each supplier. Both the x- and y-axis variables are demeaned by 4-digit NACE industry and region. The fitted curve is a local polynomial regression with an Epanechnikov kernel. The shaded area shows the 95 percent confidence intervals. The regression corresponding to this figure is in Table 1, column (2).

of that product category from countries other than Turkey from 2011 to 2015 is associated with an increase in the Turkish firm's wage and in the average wage of its suppliers and customers. The new employees, suppliers and customers that the firm adds over the period, from 2011 to 2015, had on average higher wages in 2011 than the firm's original employees and partners. Our proposed mechanism combined with evidence from the literature that high-income countries demand relatively more skill-intensive goods explains these patterns:[2] An increase in the relative demand for high-quality goods increases a firm's quality and skill intensity. The firm shifts toward skill-intensive trading partners and may prod its existing partners to upgrade.

The interconnection in firms' quality choices implies that a shock that is common to a significant share of firms may have a larger effect than the sum of idiosyncratic, firm-specific shocks. We develop a model to study these types of shocks. The model is in the spirit of Kremer (1993), but to allow a quantitative analysis, we base it on Melitz's (2003) model of heterogeneous firms. We add to Melitz (2003) the assumptions on quality from Verhoogen (2008) and Kugler and Verhoogen (2011), and an endogenous network formed through search and matching, similar to models of labor.[3] Firms post costly ads to

[2] See Hallak (2006), Brambilla et al. (2012), Manova and Zhang (2012), Feenstra and Romalis (2014), and Bastos et al. (2018).

[3] See Mortensen (1986) and Rogerson et al. (2005) for surveys.

search for other firms. More productive firms post more ads and have more customers and suppliers. A firm's quality determines its production function. We assume that higher-quality firms are skill intensive, and we allow them to be intensive in high-quality inputs. When posting ads, firms imperfectly target other firms with similar quality levels.

We estimate the model for Turkish manufacturing firms using the method of simulated moments. We focus on manufacturing firms because the shift-share regressions above apply only to them. The moments describe assortative matching on wages and the joint distribution of firm revenue, wages, and number of customers and suppliers. Targeted search in the model captures differences in matching across firms with different wages (the extensive margin of assortative matching). Only 8 percent of the ads posted by buyers in the lowest quintile of wages are directed at suppliers in the highest wage quintile and vice-versa. Differences in marginal productivity capture the spending patterns (the intensive margin). The marginal product of an input in the top quintile of the quality distribution is always larger than that of an input in the bottom quintile. But it is 46 percent larger for the production of output in the top quintile of quality and 10 percent larger for the production of output in the bottom quintile.

In the data and in the model, exporters are large and skill intensive and have many network connections, especially connections to other large, skill-intensive firms. Export intensity generally increases with exporter wage. This pattern holds in the estimated model because the relative demand for higher quality is higher abroad. A firm-specific export demand shock in the model increases the firm's quality and skill intensity. The responsiveness of firms' quality choices to these idiosyncratic shocks in the model is estimated to match the shift-share regressions. In the data and in the model, a 5 percent increase in export demand increases the firm's wages by 0.21 percent.

We use a counterfactual to study the general equilibrium effect of an export shock of the same magnitude, but applied to all exporters instead of individual firms. The probability that any firm matches with a high-quality firm in the network increases with the shock. Matching with a high-quality supplier decreases the relative cost of producing high-quality output, and matching with a high-quality customer increases the demand for high-quality inputs. This demand effect accounts for about two-thirds of the counterfactual increase in profit from producing high- relative to low-quality goods, and the cost effect accounts for one-third. Non-exporting firms not directly impacted by the shock upgrade quality and increase their wages by 1.0 percent on average. The wages of exporters increase by 1.92 percent, almost an order of magnitude larger than the effect of firm-specific shocks.

To highlight the importance of assortative matching, we consider a special case of the model in which all firms equally value the quality of their inputs. The same counterfactual in this special case increases the wages of exporters by 0.23 percent, almost the same as the 0.21 percent response to the firm-specific shocks. In contrast, manufacturing output responds similarly in the special case and in the general model. The predicted increase of about six percent is larger than the prediction in Hulten (1978) but in line with Baqaee and Farhi (2019a) because the elasticity of substitution between varieties is larger than one.

The network literature has focused on Hicks-neutral shocks, while quality in our model changes the types of material and labor inputs that firms use. We relax Hicks neutrality through log-supermodular shifters. We follow Teulings (1995) and Costinot and Vogel (2010) for labor and Fieler et al. (2018) for material inputs, and we apply these functions anew to search.[4] Our novel search-and-matching set up is tractable and yields a closed-form solution in the special case of the model with only one quality level. We abstract, however, from the following aspects of the network highlighted in the literature: Dynamics in Lim (2018) and Huneeus (2018), asymmetries in network centrality in Acemoglu et al. (2012), and market distortions in Baqaee and Farhi (2019b), Bigio and La'O (2020), Jones (2011), and Liu (2019). The model features roundabout production and technologies with constant elasticities of substitution, and each firm has a continuum of suppliers and customers. Some of these theoretical elements and the study of shocks to international trade appear in Lim (2018), Dhyne et al. (2018), Bernard et al. (2019a,b), Eaton et al. (2018), Huneeus (2018), and Lenoir et al. (2019).

The estimated model is consistent with well-established facts in the quality literature. Higher-quality production is intensive in skilled labor as in Schott (2004), Verhoogen (2008), and Khandelwal (2010) and in higher-quality inputs as in Kugler and Verhoogen (2011), Manova and Zhang (2012), and Bastos et al. (2018). Fieler et al. (2018) combine these elements to study, as we do, the general equilibrium effect of international trade on demand for skills and quality. These papers all use data on prices. We complement this work with direct information on the extent to which skill-intensive firms trade with each other. Our main finding on assortative matching is akin to the finding in Voigtländer (2014) that skill-intensive sectors use intensively inputs from other skill-intensive sectors in the United States.[5]

The paper is organized as follows. Section 2 describes the data sources and facts. We

[4]The production function in Dingel (2017) aggregates workers with heterogeneous skills in the same manner that our production function aggregates material inputs with heterogeneous qualities. See also Milgrom and Roberts (1990) and Costinot (2009) for earlier applications of log-supermodular functions to economics and international trade.

[5]A related finding in Carvalho and Voigtländer (2014) is that firms are more likely to match with the suppliers of their suppliers. They interpret this finding in terms of information frictions.

present a closed economy version of the model in Section 3 and a small open economy model in Section 4. The estimation procedure is in Section 5. Section 6 reports the estimation results and connects them to the empirical facts from Section 2. In Section 7, we experiment with counterfactual export shocks. Alternative counterfactual specifications guide a policy discussion in Section 8. Section 9 concludes.

2 Data and Empirical Facts

2.1 Data Sources

We combine five data sets from Turkey: (1) VAT data on domestic firm-to-firm trade, (2) data on firms' balance sheets and income statements, (3) the firm registry, (4) customs data, and (5) linked employer-employee data. These data sets are all maintained by the Ministry of Industry and Technology. They all use the same firm identifier and cover all formal firms in Turkey from 2011 through 2015.

The VAT data report all domestic firm-to-firm transactions whenever the total value of transactions for a seller-buyer pair exceeds 5,000 Turkish liras (about US$1,800 in 2015) in a given year. From the balance sheet and income statement data, we use information on each firm's gross domestic and foreign sales. From the firm registry, we extract the firm's location (province) and industry. The industry classification is the 4-digit NACE code, the standard in the European Union. From the customs data, we use information on annual exports by firm, destination country, and 4-digit Harmonized System (HS) product code.

The employer-employee data are collected by the Turkish social security administration. We observe the quarterly wage of each worker in each firm. We also observe the worker's occupation (4-digit ISCO classification), age, and gender. The worker identifier is unique, allowing us to track workers across firms and over time.

We restrict most of the analysis to the more tradable manufacturing sector. Unless otherwise noted, facts about the network refer to trade between firms within manufacturing. We drop firms that do not report their balance sheet or income statement. These are usually very small firms that use a single-entry bookkeeping system. The cross-sectional facts refer to the year 2015. The final sample has data on 77,418 manufacturing firms from 2015.

Section 2.2 describes the assortative matching in the firm-to-firm network. Section 2.3 associates firm-specific trade shocks with systematic changes in firm outcomes, including wages and network connections. To estimate these trade shocks, we use annual bilateral trade data from BACI, disaggregated at the four-digit HS product code level.[6] Section 2.4 describes other salient features of the data. These features are not novel, but they justify some elements of the model.

2.2 Assortative Matching in the Cross-Section

Kremer's (1993) O-ring theory, when applied to interfirm production chains, yields the prediction that skill-intensive firms disproportionately buy from and sell goods to other skill-intensive firms. We use a firm's average wage as a proxy for its skill intensity, under the assumption that firms observe skills better than we econometricians and that wages reflect differences in skills. We use other measures of skills for robustness in Section 2.2.1.

Define $wage_f$ as firm f's total monthly wage bill divided by its number of workers. Define the wage of firm f's suppliers as:

$$\log wage_f^S = \sum_{\omega \in \Omega_f^S} s_{\omega f} \log wage_\omega \tag{1}$$

where Ω_f^S is the set of suppliers to firm f and $s_{\omega f}$ is the share of supplier ω in firm f's total spending on inputs.

Table 1 reports the results from the regression:

$$\log wage_f^S = \beta \log wage_f + \gamma X_f + e_f \tag{2}$$

where e_f is the residual and X_f are control variables that vary across columns. Columns (1) through (3) contain only the manufacturing sub-sample. Column (1) has no control variables. Column (2) includes fixed effects for each industry-province pair. The coefficient decreases from column (1) because firms match more within province and industry and some industry-province pairs have higher skill shares. Still, the decrease is small, from 0.294 to 0.259, suggesting that most of the variation across firms occurs within industry-province. A 10 percent increase in the average buyer wage is associated with a 2.5 percent increase in the average supplier wage.

Column (3) controls for the buying firm's employment. Since employment and wages are correlated, the coefficient on wages decreases. But its magnitude is comparable to other columns. Column (4) repeats specification (2) with the sample of all firms.[7] The

[6] We aggregate these data from 6- to 4-digit HS codes for two reasons. First, it is less likely for any single country to have significant market power in a given destination at the 4-digit product level than at the 6-digit level. Second, the value of trade at the country-product level is excessively volatile at the 6-digit product level.

[7] We exclude finance, insurance, utilities and public services firms.

Table 1: Assortative Matching on Wages

Dependent variable: $\log wage_f^S$

	Manufacturing firms			All firms
	(1)	(2)	(3)	(4)
$\log wage_f$	0.294	0.259	0.188	0.241
	(0.013)	(0.012)	(0.009)	(0.013)
$\log employment_f$			0.044	
			(0.003)	
R^2	0.095	0.173	0.199	0.150
N	77,418	77,418	77,418	410,608
Fixed effects		ind-prov	ind-prov	ind-prov

Notes: The wage is defined as the average value of monthly payments per worker. The suppliers' average wage $\log wage_f^S$ is defined in equation (1). *Ind* and *prov* refer to 4-digit NACE industries and provinces, respectively. Robust standard errors are clustered at the 4-digit NACE industry level.

coefficient of 0.241 is similar to 0.259 in specification (2).

Decomposition into Margins The positive coefficients on Table 1 could be driven by high-wage firms having more high-wage suppliers—the extensive margin—or by such firms spending relatively more on their high-wage suppliers given the same matches—the intensive margin. We decompose the coefficient of our preferred specification (2) into these margins.

Define the extensive margin as the unweighted average wage of firm f's suppliers:

$$EM_f^S = \sum_{\omega \in \Omega_f} \frac{1}{|\Omega_f|} \log wage_\omega. \tag{3}$$

Define the intensive margin as the difference between $\log wage_f^S$ in (1) and the extensive margin:

$$\begin{aligned} IM_f^S &= \log wage_f^S - EM_f^S \\ &= \sum_{\omega \in \Omega_f} (s_{\omega f} - 1/|\Omega_f|)(\log wage_\omega - \sum_{\omega' \in \Omega_f} (1/|\Omega_f|) \log wage_{\omega'}). \end{aligned} \tag{4}$$

The intensive margin is large if firm f's spending shares $s_{\omega f}$ are particularly large for high-wage suppliers ω.

One at a time, we regress $\log wage_f^S$, EM_f^S and IM_f^S on the wage of firm f and on industry-province fixed effects. The results are in Table 2. The first regression is the same as in column (2), Table 1. By construction, the coefficients in the second and third columns add up to the total, 0.259, in the first column. The extensive margin accounts

Table 2: Assortative Matching on Wages: Decomposition

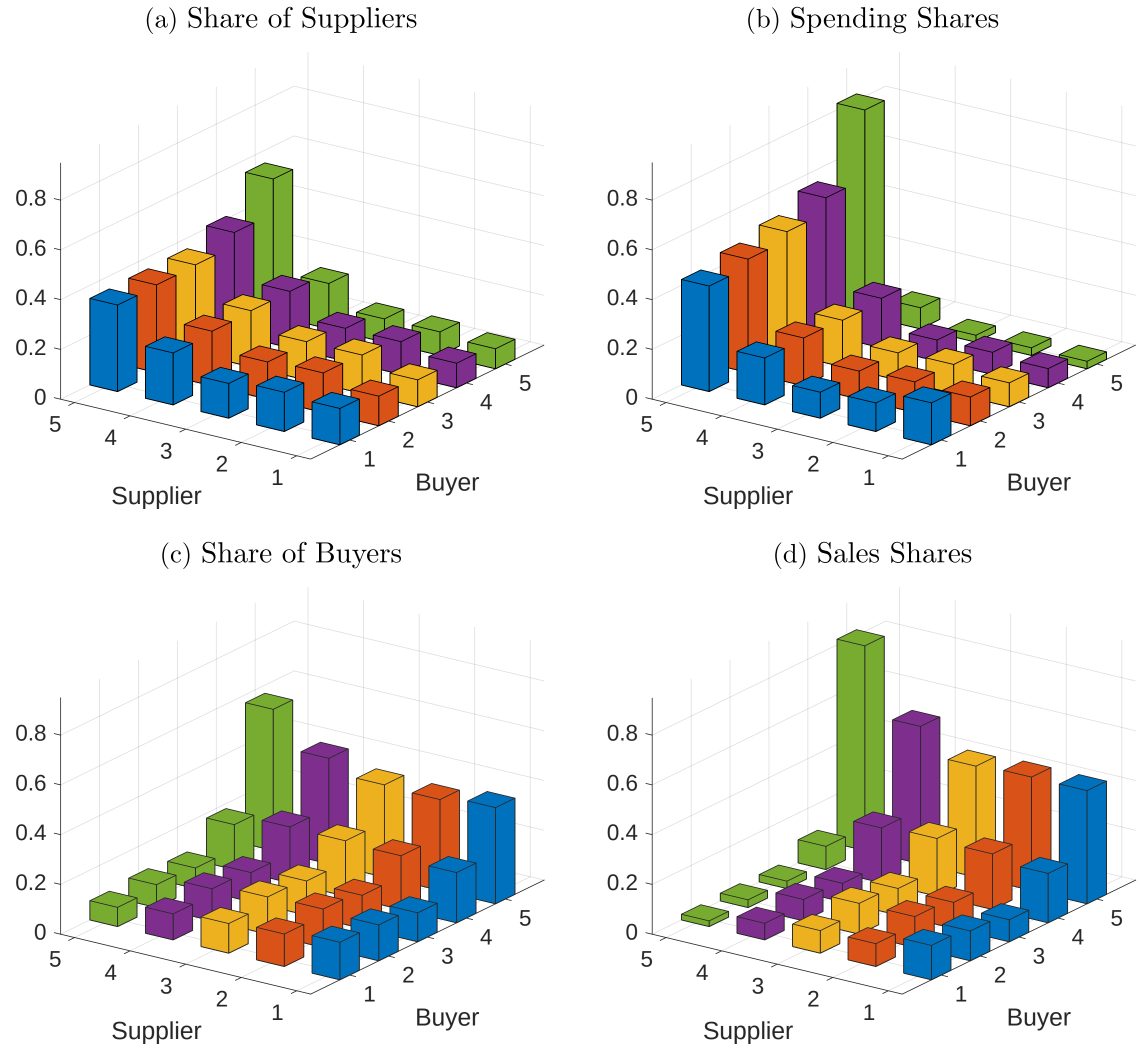
	total $\log wage_f^S$ (A)	extensive margin EM_f^S	intensive margin IM_f^S
$\log wage_f$	0.259	0.152	0.107
	(0.012)	(0.007)	(0.007)
coeff. / coeff in (A)		*59%*	*41%*
R^2	0.173	0.150	0.089
N	77,418	77,418	77,418
Fixed effects	ind-prov	ind-prov	ind-prov

Notes: The wage is defined as the average value of monthly payments per worker. The suppliers' average wage $\log wage_f^S$ is defined in equation (1). *Ind* and *prov* refer to 4-digit NACE industries and provinces, respectively. Equations (3) and (4) define the extensive (EM_f^S) and intensive margins (IM_f^S). They capture, respectively, the extent to which firm f matches with high-wage suppliers or tilts its spending toward high-wage suppliers. Robust standard errors are clustered at the 4-digit NACE industry level.

for 59 percent (= 0.152/0.259) of the partial correlation between the firm's wage and its suppliers' wages, while the intensive margin accounts for 41 percent. Since these margins are both large, the model allows for both.

Figure 2 illustrates assortative matching using the raw data. We split firms into quintiles of $wage_f$. Panels (a) and (b) describe firms' upstream links. The height of the bars in panel (a) is the supplier quintile's share in the number of suppliers to firms in each buyer quintile. The height in panel (b) is the supplier quintile's share in the spending of firms in each buyer quintile. Thus, by construction, the sum of bars of the same color across supplier quintiles is one for each buyer quintile. Suppliers in the highest quintile of wages generally have larger sales and more buyers. Their shares are hence larger for all buyer quintiles. However, in both panels, the difference between sellers in quintiles 1 and 5 is much larger when the buyer has a high wage. In addition, due to the intensive margin, these differences are more pronounced in panel (b) than in panel (a). In panel (a), high-wage suppliers account for 35 percent of links to buyers in the lowest quintile of wages and 55 percent of links to buyers in the highest quintile. In panel (b), the corresponding numbers for spending are 43 and 83 percent. Panels (c) and (d) describe the corresponding patterns for firms' downstream links. The shares across buyers now add up to one for each supplier quintile. Panels (c) and (d) are almost the mirror images of panels (a) and (b).

Figure 2: Firm-to-Firm Trade Links and Values by Quintile



Notes: The sample includes manufacturing buyers and suppliers. Firms are sorted according to the average value of their monthly payments per worker and grouped into five equal-sized groups. The buyer and supplier quintiles are shown on the x- and y-axis, while the z-axis shows the corresponding shares. Panels (a) and (b) illustrate buyers' upstream connections. In panel (a), the values on the z-axis is the share of suppliers that belong to the wage quintiles on the y-axis for each buyer quintile on the x-axis. In panel (b) the shares in the z-axis are spending shares. Panels (c) and (d) illustrate suppliers' downstream connections. In panel (c) the values on the z-axis is the share of buyers that belong to the wage quintiles on the y-axis for each supplier quintile on the x-axis. Panel (d) the shares on the z-axis are sales shares.

2.2.1 Robustness of Assortative Matching

Other Measures of Skill Intensity In addition to differences in skills, wages may contain rents and differences in profit-sharing policies across firms. To address this concern, in Appendix A.1, we decompose the variation in wages into firm and worker components as in Abowd et al. (1999) using our employer-employee data from 2014 to 2016. Following Bombardini et al. (2019), we then take a firm's skill intensity to be the average fixed effect of its workers. When we repeat the regressions in Table 2 with this measure of skill intensity, the coefficients are about half the size of the originals. This decrease is not surprising since the measure excludes the firm fixed effect and the skills of workers who never left the firm. Still, the coefficient is highly significant, and the decomposition into the extensive and intensive margins remains close to Table 2.

We do not observe worker education in our data. But we observe the share of workers with tertiary education in the EU15 countries for each one-digit ISCO occupation code. Using this share as a measure of occupational skill intensity, Appendix A.2 confirms that firms with relatively more workers in skill-intensive occupations buy and sell more inputs to other firms with skill-intensive occupations.

Geography In Appendix Table A3, we investigate whether positive assortative matching on wages arises because firms trade more with other firms physically close to them and some labor markets are more skill abundant than others. We conduct three exercises. In panel A, we control for firm location at a finer level, i.e., the district instead of the province level as in the baseline.[8] In panel B, we construct average supplier wages in equation (1) excluding suppliers located in the same province as the firm. In panel C, we use a subsample of single-establishment firms. Our VAT data aggregate transactions at the firm (instead of establishment) level, limiting our ability to control for the location of firms with establishments in multiple provinces. The positive assortative matching and decomposition into the extensive and intensive margins in Table 2 are robust to all three tests, although the total coefficient decreases from 0.259 in Table 2 to 0.214 in panel B and to 0.161 in panel C of Table A3. In panel A, it is 0.245.

Other Firm Characteristics Appendix Table A4 repeats the regression from column (2) in Table 1 substituting wages with other firm characteristics. Assortative matching on sales is positive but less pronounced than that on wages, and the sorting is insignificant

[8]Turkey is divided into 81 provinces. Each province is further divided into districts, the total number of which is close to 1,000. We use provinces in our baseline results because a province better represents a local labor market.

on the number of network links.[9] To evaluate the relative importance of sales vis-à-vis wages in sorting, Appendix A.5 conducts a horse-race between sales and wages following the empirical approach in Johnson and Wichern (1988), in the spirit of Becker (1973). Both wages and sales matter for the positive assortative matching, but wages are about 3 times more important than sales for a firm's downstream linkages and 8.5 times more important for its upstream linkages.[10]

2.3 Trade Shocks

We use shift-share regressions to show that firms respond to firm-specific trade shocks by changing their skill intensity and network connections.[11]

Define two shifters associated with country c and product category k:

$$\begin{aligned} Z^u_{ck} &= \Delta \log \text{Imports}_{ck} \\ Z^a_{ck} &= (\Delta \log \text{Imports}_{ck}) * \log(\text{GDP per capita}_{c,2010}) \end{aligned} \tag{5}$$

where $\Delta \log \text{Imports}_{ck}$ is the log change between 2011-2012 and 2014-2015 in the total imports of country c in product category k from all countries other than Turkey and GDP per $\text{capita}_{c,2010}$ is the income per capita of country c in 2010.

We measure the export shock to firm f during the period of our data as:

$$\begin{aligned} \text{ExportShock}^u_f &= \sum_{ck} x_{ckf} Z^u_{ck} \\ \text{ExportShock}^a_f &= \sum_{ck} x_{ckf} Z^a_{ck} \end{aligned} \tag{6}$$

where x_{ckf} is the share of firm f's revenue in 2010 that is exported to country c in product category k. We interpret Z^u_{ck} as a change in the demand for product category k in country c. The underlying assumption is that shocks to imports of product k by country c from countries other than Turkey are uncorrelated with other unobserved shocks to Turkish firms that export k to c. Under this assumption, ExportShock^u_f is a standard shift-share shock that captures the increased demand for firm f's exports. But we are interested in shocks that increase the incentives for firm f to upgrade its quality, and it is well

[9] Lim (2018) also finds assortative matching on sales using data on large firms in the United States (Compustat). This pattern arises in our estimated model because of a positive correlation between firm sales and wages.

[10] These numbers are from a canonical correlation analysis. This method is often used in marriage markets to evaluate which individual characteristics are most relevant for matching.

[11] See Bartik (1991) for an early application of these regressions and Borusyak et al. (2018), Goldsmith-Pinkham et al. (2020), Adão et al. (2019) for statistical properties in general setups.

Table 3: Effects of Export Shock

	$\Delta \log \text{wage}_f$	$\Delta \log \text{wage}_f$ (first stage)	$\Delta \log$ domestic sales$_f$	Δexport intensity$_f$	$\Delta \log wage^S_f$ OLS	$\Delta \log wage^S_f$ IV
	(1)	(2)	(3)	(4)	(5)	(6)
ExportShock^u_f (unadjusted)	0.021 (0.033)					
ExportShock^a_f (adjusted)		0.042 (0.006)	-0.026 (0.022)	0.0146 (0.0023)		
$\Delta \log \text{wage}_f$ (IV = ExportShock^a_f)					0.085 (0.008)	0.434 (0.185)
F-Stat	0.404	43.6	1.409			
N	33,157	33,157	33,157	33,157	33,157	33,157
Fixed effects	ind-prov	ind-prov	ind-prov	ind-prov	ind-prov	ind-prov

Notes: Wage$_f$ is the average value of monthly payments per worker in firm f. The suppliers' average wage $\log wage^S_f$ is defined in equation (1). The Δ operator denotes changes between 2011-2012 and 2014-2015. ExportShock^u_f is a weighted average of changes in imports at the country (c) and 4-digit HS product (k) level between 2011-2012 and 2014-2015, where the weights are constructed as the share of firm f's exports of product k to importer c in its total sales in 2010. ExportShock^a_f adjusts these shocks by giving higher weights to rich destinations. See equation (6). *Ind* and *prov* refer to 4-digit NACE industries and provinces, respectively. Robust standard errors are clustered at the 4-digit NACE industry level.

documented that the relative demand for higher-quality, skill-intensive goods is higher in rich countries.[12] Then, export shocks that originate in rich countries should induce larger changes in quality. ExportShock^a_f is an adjusted measure that gives higher weights to rich countries.

To compare these two measures, we separately use them in the regression:

$$\Delta \log wage_f = \delta \text{ExportShock}_f + \alpha_{sr} + \epsilon_f$$

where α_{sr} is industry-province fixed effects.

Columns (1) and (2) of Table 3 report the results. The unadjusted ExportShock^u_f has an insignificant effect on firm wages, while the adjusted ExportShock^a_f has a positive and significant effect.[13] Thus, as anticipated, increased demand for a firm's exports increases the firm's skill intensity only if the demand originates in rich countries.[14]

The mean of ExportShock^a_f is 0.12. To understand the magnitude of the coefficient 0.042 in column (2), consider two firms. They both export a quarter of their sales (the mean export intensity among exporters in the data). One firm exports to a country at the 90$^{\text{th}}$ percentile of the per capita GDP distribution (US$41.3 thousand, France), and the

[12]See footnote 2 for references.

[13]These results hold when both shocks are in the same regression in Appendix Table A7(1).

[14]A related finding is in De Loecker (2007). For Slovenian firms, the productivity gains from exporting are larger when the firm exports to high-income destinations than to low-income destinations.

other firm exports to a country at the 10th percentile (US$766, Benin). For the average change in imports over the sample period, $Z^u_{ck} = 5$ percent, the implied ExportShock^a_f for the two firms is 13.3 percent ($= 0.25 \times 0.05 \times \log(41,300)$) and 8.3 percent, respectively, and the estimated wage increase is 0.56 percent ($= 0.042 \times 0.133$) and 0.35 percent.

Given these results, we henceforth use the adjusted export shock in all exercises. In column (3), we replace the dependent variable in column (2) with domestic sales. The insignificant coefficient is reassuring, since we assume that ExportShock^a_f is uncorrelated with domestic shocks to firm f. It is also reassuring that the shock is not spurious but associated with an increase in the firm's export intensity (export sales divided by total sales) in column (4).

Columns (5) and (6) regress the change in the wage of firm f's suppliers on the change in firm f's own wage:

$$\Delta \log wage^S_f = \delta \Delta \log wage_f + \alpha_{sr} + \epsilon_f$$

where α_{sr} is again industry-province fixed effects. In column (6), we instrument the change in the firm's wage $\Delta \log wage_f$ with the export shock.[15] The coefficient is 0.434 with standard error of 0.185. The interpretation is that when a firm's average wage increases by one log point relative to other firms in response to an export shock, then the average wage of its suppliers increases by 0.4 log points. The coefficient in the OLS regression in column (5) is smaller at 0.085. It is difficult to predict *ex ante* the direction of the bias. The OLS coefficient is confounded by unobserved shocks that affect the wage growth of firms in the same industry and province.

In sum, Table 3 suggests that demand for a firm's exports from rich countries increases the firm's own wage as well as that of its suppliers. Table 4 shows that these increases arise, at least in part, through new workers and network connections. Recall that the export shock is constructed from changes between 2011-2012 and 2014-2015. Take the workers that a firm f added between 2013 and 2015. Using matched employer-employee data, we regress the log difference between these new workers' wages in 2011-2012 (before they entered the firm) and firm f's average wage in 2011-2012 (before the shock) on ExportShock^a_f in the first column. The second and third columns repeat the exercise for the firm's new suppliers and new customers. The coefficients on all columns are positive and statistically significant.[16]

[15] This approach follows Hummels et al. (2014). To study the effect of exports on wages, they use a shift-share variable similar to ExportShock^u_f as an instrument for firm exports.

[16] We use the unweighted average in (3) because we cannot measure the weights $s_{\omega f}$ that the firm would have placed on new suppliers in the initial year or the equivalent weights of new customers on initial sales. In Appendix Table A9, we obtain similar results when we compare the the wages of new

Table 4: Effects of Export Shock on Composition of Inputs

Log of	Average wage of new workers relative to all workers at $t = 0$	Average wage paid by new suppliers relative to all suppliers at $t = 0$	Average wage paid by new buyers relative to all buyers at $t = 0$
ExportShock_f	0.0189	0.0241	0.0303
	(0.010)	(0.007)	(0.009)
R^2	0.0531	0.0439	0.0434
N	33157	33157	33157
Fixed effects	ind-prov	ind-prov	ind-prov

Notes: The wage is defined as the average value of monthly payments per worker. ExportShock_f is a weighted average of changes in (real per capita) income-adjusted imports at the country (c) and 4-digit HS product (k) level between 2011-2012 and 2014-2015, where the weights are constructed as the share of firm f's exports of product k to importer c in its total sales in 2010. Time $t = 0$ represents the period before the export shock, 2011-2012. *Ind* and *prov* refer to 4-digit NACE industries and provinces, respectively. Robust standard errors are clustered at the 4-digit NACE industry level.

Identification and Robustness Checks Recent papers discuss shift-share regressions similar to ours. Borusyak et al. (2018) and Goldsmith-Pinkham et al. (2020) propose methods to study, respectively, which shifts or shares matter most for consistency. Following the recommendation in Borusyak et al. (2018), we check three key conditions in Appendix B. First, shifts are numerous. To calculate Z^a_{ck}, we use 208 distinct destination countries c and 1,242 4-digit HS codes k, generating 153,186 ck pairs. Second, the shifts are dispersed within industries. The average Herfindahl-Hirschman index within industries is 5×10^{-5}. The standard deviation of Z^a_{ck} is 3.26 across all firms and industries and 3.24 across firms within industries. Third, the shifts are relevant. We obtain a coefficient close to zero when we substitute $ExportShock^a_z$ with a placebo $ExportShock^{\text{random}}_f$ generated from randomly drawn shifts Z^a_{ck}.

Appendix Table A7 presents additional checks to Table 3. The results in column (2) do not change when we add the export shares x_{ck} weighted by destination income per capita as a control. This exercise addresses the concern in Adão et al. (2019) that observations with similar shares have correlated residuals. Separately, we add the export shares to the same regression to address the concern in Borusyak et al. (2018) that shares x_{ck} do not add up to one. Last, we add to column (6) the weighted average of the suppliers' export shock. These shocks have a positive effect on supplier wages (as we would predict), but they do not affect the coefficient of interest on the buyer's wage.

connections relative to those of workers, suppliers and customers that left the firm between 2010 and 2015. Appendix Table A8 associates the export shock to the share of newly hired workers after the shock, who received higher monthly wages than the firm's average worker before the shock. Thus, Table 4 is not driven by a few outliers among new connections.

Table 5: Firm Sales and Network Connections

Number of	Customers			Suppliers		
	(1)	(2)	(3)	(4)	(5)	(6)
$\log Sales_f$	0.440	0.462	0.459	0.577	0.593	0.590
	(0.016)	(0.013)	(0.013)	(0.011)	(0.009)	(0.009)
$\log Wage_f$			0.278			0.208
			(0.211)			(0.175)
R^2	0.328	0.472	0.472	0.609	0.645	0.645
N	77,418	77,418	77,418	77,418	77,418	77,418
Fixed effects		Ind	Ind		Ind	Ind

Notes: The wage is defined as the average value of monthly payments per worker. All variables are in logarithms. *Ind* refers to 4-digit NACE industries. Robust standard errors are clustered at the industry level.

2.4 Other Characteristics of the Network

Three other features of the data govern our modeling choices. First, sales is the most important indicator of the number of suppliers and customers of a firm. Table 5 reports the endogenous elasticity of the number of customers and suppliers with respect to sales. Firm sales explain about a third of the variation in the number of buyers and 60 percent of the variation in the number of suppliers (R-squared in columns (1) and (4)). Columns (2) and (5) add industry fixed effects, and columns (3) and (6) also add wages. The coefficients on wages are insignificant and do not change the coefficients on sales or the R-squared.

Second, service firms, mostly wholesalers and retailers, account for almost half of the domestic sales and material purchases of manufacturing firms. We do not, however, observe the skill intensity of the materials purchased through these service intermediaries. Thus, we introduce to the model a service sector that aggregates manufacturing inputs into a homogeneous good. The service good is used as an input into manufacturing and as a final good.

Third, imports account for only 4 percent of spending on material inputs by a typical manufacturing firm in our data, compared to a 10 percent share of exports in its total sales. Accordingly, in the open economy model in Section 4, we model manufacturing firms' decisions to export, but for simplicity, only service firms import.[17]

We conclude with a brief point on quality measures. Quality in our model is a latent variable that changes the firm's production function, increasing the relative marginal

[17] We replicate the moments in Section 2.3 for import shocks in place of export shocks and find mostly insignificant effects. This null finding possibly arises because only a small share of the manufacturing firms in our data import their inputs directly.

product of skilled workers and of skill-intensive inputs. Kremer (1993) refers to this variable interchangeably as quality or complexity. But our emphasis, like his, is the complementarity between skilled workers in production. Even if we observed unit values in our data, it is not clear that standard measures of quality would be superior to wages in capturing this complementarity. Since we cannot answer this question with our data, we leave it for future work. Nevertheless, we do observe unit values for a small subset of the data: the foreign sales of exporting firms. For this subset, Appendix A.3 confirms the positive relation between wages and the quality measure of Khandelwal et al. (2013), which uses information on unit values and quantities by destination.[18]

3 The Closed-Economy Model

To highlight the novel features of the model, we first present the closed economy case. There are two sectors: services and manufacturing. The service sector is perfectly competitive. It produces a homogeneous good with constant returns to scale using manufacturing inputs. The manufacturing sector has heterogeneous firms.

Each manufacturing firm chooses its quality q from a line segment $Q \subset \mathbb{R}_+$. This choice determines the firm's production function. All tasks performed in a firm of quality $q \in Q$ are also indexed by q, whether the worker is in production or posting ads. Earnings per worker and the marginal product of higher-q inputs may be higher in the production of higher-q output. Firms post ads to find suppliers and customers. The matching of ads forms the firm-to-firm network. As in Lim (2018), each firm is matched with a continuum of suppliers and customers, and it charges the monopolistic-competition markup. More productive firms endogenously post more ads and have more customers and suppliers. Firms imperfectly direct their ads toward other firms with similar quality levels.

Differences in input intensity in the production function allow skill-intensive firms to spend more on each others' inputs–the intensive margin of assortative matching. Directed search increases the probability that skill-intensive firms match with each other—the extensive margin.

We present the manufacturing sector in Section 3.1. Section 3.1.1 sets up the firm's problem, and Section 3.1.2 aggregates firm choices to form the network. The service sector is in Section 3.2, and the equilibrium is in Section 3.3. Section 3.4 presents key properties of the model. The less-technical reader may skip to Section 3.4. Whenever convenient, we assume functions are continuous, differentiable, and integrable. Parametric assumptions

[18] In our estimation, we use moments based on quintiles of firm wages, and the appendix documents significant overlap between firms grouped on the basis of this quality measure and on the basis of wages.

in the estimation ensure these conditions.

3.1 Manufacturing

3.1.1 The Firm's Problem

The revenue of a firm with quality q, price p and a mass v of ads to find customers (v stands for visibility) is:

$$p^{1-\sigma} v D(q) \tag{7}$$

where $\sigma > 1$ is the elasticity of substitution between manufacturing varieties and $D(q)$ is an endogenous demand shifter.

The cost of a bundle of inputs to produce quality q when the firm posts a measure m of ads to find manufacturing suppliers is:

$$C(m, q) = w(q)^{1-\alpha_m-\alpha_s} P_s^{\alpha_s} [m^{1/(1-\sigma)} c(q)]^{\alpha_m} \tag{8}$$

where $(\alpha_m, \alpha_s) \gg 0$ are Cobb-Douglas weights with $(\alpha_m + \alpha_s) \in (0, 1)$, P_s is the price of the service good, $w(q)$ is the wage rate per efficiency unit of task q, and $c(q)$ is the cost of a bundle of manufacturing inputs when the firm posts a measure one of ads to find suppliers. The marginal cost of the firm is $C(m, q)/z$, where z is its productivity.

The costs of posting v ads to find customers and m ads to find suppliers are, respectively:

$$\begin{aligned} & w(q) f_v \frac{v^{\beta_v}}{\beta_v} \\ & w(q) f_m \frac{m^{\beta_m}}{\beta_m} \end{aligned} \tag{9}$$

where f_m, f_v, β_m, and β_v are positive parameters with $\beta_m > \alpha_m$ and $\beta_v > \beta_m/(\beta_m - \alpha_m)$.

From (7), the firm charges markup $\sigma/(\sigma - 1)$ over marginal cost. Given q and z, she chooses v and m to maximize profit:

$$\max_{v,m} \frac{v m^{\alpha_m}}{\sigma} \left[\frac{\sigma}{\sigma - 1} \frac{C(1, q)}{z} \right]^{1-\sigma} D(q) - w(q) f_v \frac{v^{\beta_v}}{\beta_v} - w(q) f_m \frac{m^{\beta_m}}{\beta_m}. \tag{10}$$

Rearranging the first-order conditions, the firm's revenue x, mass of ads to find customers

v and to find suppliers m, and price p are functions of productivity z and quality q:

$$\begin{aligned}
x(z,q) &= \Pi(q) z^{\gamma(\sigma-1)} \\
v(z,q) &= \left(\frac{x(z,q)}{\sigma f_v w(q)}\right)^{1/\beta_v} \\
m(z,q) &= \left(\frac{\alpha_m x(z,q)}{\sigma f_m w(q)}\right)^{1/\beta_m} \\
p(z,q) &= \frac{\sigma}{\sigma-1}\frac{C(m(z,q),q)}{z}
\end{aligned} \tag{11}$$

where

$$\begin{aligned}
\Pi(q) =& [\sigma w(q)]^{1-\gamma}\left[D(q)\left(\frac{\sigma}{\sigma-1}C(1,q)\right)^{1-\sigma}\left(\frac{f_m}{\alpha_m}\right)^{-\alpha_m/\beta_m} f_v^{-1/\beta_v}\right]^{\gamma} \\
\gamma =& \frac{\beta_v\beta_m}{\beta_v(\beta_m-\alpha_m)-\beta_m} > 1.
\end{aligned} \tag{12}$$

A firm is characterized by a vector $\omega = (\omega_0, \omega_1) \in \mathbb{R}^2$ that determines its productivity for each quality level:

$$z(q,\omega) = \exp\left\{\omega_0 + \omega_1 \log(q) + \overline{\omega}_2[\log(q)]^2\right\} \tag{13}$$

where $\overline{\omega}_2$ is a parameter common to all firms. Parameter ω_0 captures the firm's absolute advantage in production, and ω_1 captures her comparative advantage in producing higher quality. These two dimensions of heterogeneity capture the joint distribution of sales and wages in the estimation. Since profit (10) is a share $1/(\gamma\sigma)$ of revenue, firm ω chooses q to maximize revenue:

$$q(\omega) = \arg\max_{q\in Q}\left\{x(z(q,\omega),q)\right\} = \arg\max_{q\in Q}\left\{z(q,\omega)^{\gamma}\Pi(q)\right\}. \tag{14}$$

If wage $w(q)$ is continuous in q, then function $\Pi(q)$ (below) is continuous in q, and (14) is the maximization of a continuous function in a compact set Q. Firms' quality choices are interconnected through the endogenous terms in $\Pi(q)$. Manufacturing firm-to-firm trade determines the input cost $c(q)$ and the component of demand $D(q)$ that comes from other firms.

3.1.2 Manufacturing Firm-to-Firm Trade

Production Function The quantity produced by firm ω producing quality q is:

$$z(q,\omega)l^{1-\alpha_m-\alpha_s}y_s^{\alpha_s}Y(q)^{\alpha_m}$$

where l is efficiency units of labor, y_s is units of the service good, and $Y(q)$ is an aggregate of manufacturing inputs. This production function yields unit costs in (8). Following Fieler et al. (2018), we assume:

$$Y(q) = \left[\int_{\omega'\in\Omega} y(\omega')^{(\sigma-1)/\sigma}\phi_y(q,q(\omega'))^{1/\sigma}d\omega'\right]^{\sigma/(\sigma-1)} \tag{15}$$

where $y(\omega)$ is the quantity of input ω and function $\phi_y(q,q')$ governs the productivity of an input of quality q' in the production of output of quality q. The ratio of the firm's demand for any two inputs 1 and 2 with prices $p(1)$ and $p(2)$ and qualities $q(1) > q(2)$,

$$\frac{y(1)}{y(2)} = \left(\frac{p(1)}{p(2)}\right)^{-\sigma}\frac{\phi_y(q,q(1))}{\phi_y(q,q(2))}, \tag{16}$$

is strictly increasing in the producing firm's quality q if ϕ_y is log-supermodular.

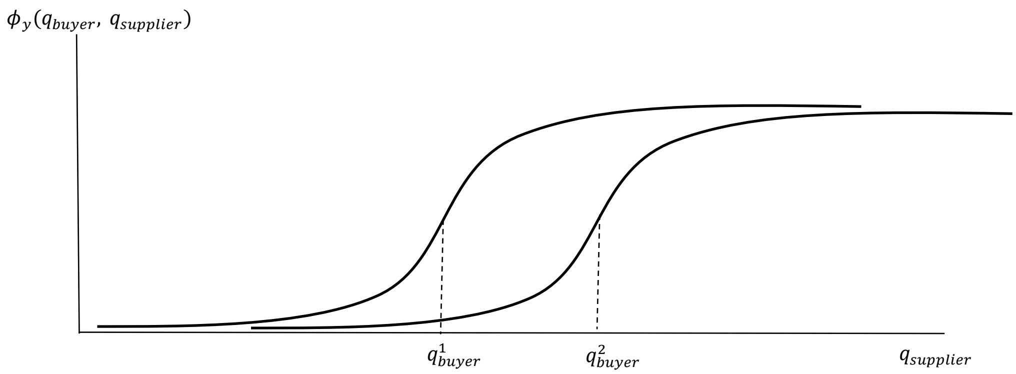
We parameterize:

$$\phi_y(q,q') = \frac{\exp(q'-\nu_y q)}{1+\exp(q'-\nu_y q)}. \tag{17}$$

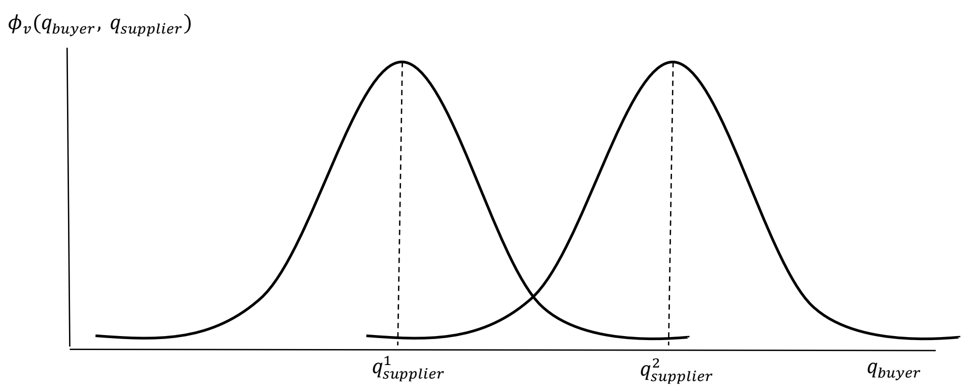
It is increasing in input quality, and if $\nu_y > 0$, it is also log-supermodular and decreasing in output quality. Figure 3A illustrates ϕ_y as a function of supplier quality for two producing firms (buyers). One can see how, given the same prices and matches, the buyer with higher quality q^2_{buyer} spends relatively more on high-quality input suppliers than the buyer with quality q^1_{buyer}.

Directed Search Buyers can only see sales ads that target their own quality level. The ads posted by a seller with quality q' are distributed across buyer qualities $q \in Q$ according to function $\phi_v(q,q')$. We parameterize $\phi_v(q,q')$ as the density of a normal distribution with variance parameter ν_v and mean q', the quality of the seller posting the ads. Figure 3B illustrates the distribution of ads across buyers for two sellers (suppliers). Clearly, the ads posted by the higher-quality supplier q^2_{supplier} are disproportionately targeted toward higher-quality buyers. Here, the direction of ads is exogenous for simplicity. In Appendix I, we modify the model to allow firms to choose the direction of their search (the mean of ϕ_v), and we obtain similar estimation and counterfactual results.

Figure 3: Assortative Matching on Quality in the Model



A. Intensive margin of assortative matching: The marginal product of input supplier is $\phi_y(q_{buyer}, q_{supplier})^{1/\sigma}$ and spending given prices is proportional to $\phi_y(q_{buyer}, q_{supplier})$. The figure plots function $\phi_y(q_{buyer}, q_{supplier})$ for two buyers with output qualities q^1_{buyer} and q^2_{buyer}.



B. Extensive margin of assortative matching: The distribution of ads posted by two sellers with output qualities $q^1_{supplier}$ and $q^2_{supplier}$ are targeted at buyers according to function $\phi_v(q_{buyer}, q_{supplier})$.

Aggregation There is a fixed set of firms Ω. Firm choices in (14) give rise to the measure:

$$J(z,q) = \left|\{\omega \in \Omega : z(q(\omega),\omega) \leq z \quad \text{and} \quad q(\omega) \leq q\}\right|. \tag{18}$$

Assume J has a density denoted with $j(z,q)$. Directed search implies that there is a continuum of matching submarkets, one for each buyer quality. In the submarket of buyers with quality $q \in Q$, the measures of ads posted by buyers and sellers are, respectively:

$$M(q) = \int_Z m(z,q)j(z,q)dz \tag{19}$$

$$V(q) = \int_Q \phi_v(q,q')\overline{V}(q')dq' \tag{20}$$

where $\overline{V}(q)$ is the measure of ads posted by sellers of quality q:

$$\overline{V}(q) = \int_Z v(z,q)j(z,q)dz.$$

A standard matching function determines the measure of matches with buyers of quality q:[19]

$$\tilde{M}(q) = V(q)\left[1 - \exp(-\kappa M(q)/V(q))\right] \tag{21}$$

where parameter $\kappa > 0$ captures efficiency in the matching market. The success rate of ads is $\theta_v(q) = \tilde{M}(q)/V(q)$ for sellers and $\theta_m(q) = \tilde{M}(q)/M(q)$ for buyers.

Using (20), for each ad posted by a buyer of quality q, the probability of finding a supplier with productivity-quality (z',q') is:

$$\theta_m(q)\frac{\phi_v(q,q')v(z',q')j(z',q')}{V(q)}. \tag{22}$$

Combining with the CES price associated with (15), a bundle of manufacturing inputs used by a firm of quality q with a measure one of buying ads costs:

$$c(q) = \left[\frac{\theta_m(q)}{V(q)}\int_Q \phi_y(q,q')\phi_v(q,q')P(q')^{1-\sigma}dq'\right]^{1/(1-\sigma)} \tag{23}$$

where

$$P(q) = \left[\int_Z p(z,q)^{1-\sigma}v(z,q)j(z,q)dz\right]^{1/(1-\sigma)} \tag{24}$$

takes into account the greater visibility of firms that post more sales ads $v(z,q)$.

[19] See Petrongolo and Pissarides (2001) for a survey on matching functions and their properties.

We now turn to demand. A firm with quality q posts price p and a measure v of sales ads. From (19), the measure of buyers with (z', q') matched to the firm is:

$$v\theta_v(q')\phi_v(q', q)\frac{m(z', q')j(z', q')}{M(q')}.$$

Conditional on the match, the firm's sales to a buyer with (z', q') are:

$$\phi_y(q', q)\left(\frac{p}{c(q')}\right)^{1-\sigma}\frac{\alpha_m(\sigma-1)}{\sigma}\frac{x(z', q')}{m(z', q')}.$$

Multiplying these last two expressions and summing over buyers (z', q'), the sales of the firm to other manufacturing firms are[20]

$$p^{1-\sigma}vD_m(q)$$

where

$$D_m(q) = \frac{\alpha_m(\sigma-1)}{\sigma}\int_Q \frac{\theta_v(q')}{M(q')}\phi_y(q', q)\phi_v(q', q)c(q')^{\sigma-1}X(q')dq', \tag{25}$$
$$X(q) = \int_Z x(z, q)j(z, q)dz.$$

3.2 Service Sector and Final Demand

Service firms aggregate manufacturing inputs into a homogeneous good sold in a perfectly competitive market. Their production function is given by $Y(0)$ in (15). There is a fixed set of service firms, each endowed with a fixed measure $\overline{m}$ of manufacturing suppliers. The probability that any of these suppliers has productivity-quality (z, q) is:

$$\frac{v(z, q)j(z, q)}{V_T}$$

[20] We may also derive $D_m(q)$ from buyer connections. Using (23), the share of spending on materials by buyers of quality q' allocated to a supplier with price p, quality q, and v ads is:

$$\theta_m(q')\frac{\phi_y(q', q)\phi_v(q', q)vp^{1-\sigma}}{V(q)c(q')^{1-\sigma}}$$

Multiplying by domestic spending on materials $[\alpha_m(\sigma-1)/\sigma]X(q')$ and integrating over buyers q', demand is:

$$vp^{1-\sigma}\frac{\alpha_m(\sigma-1)}{\sigma}\int_Q \frac{\theta_m(q')}{V(q')}\phi_y(q', q)\phi_v(q', q)c(q')^{\sigma-1}X(q')dq'$$

which is the expression above since $\theta_m(q)/V(q) = \theta_v(q)/M(q)$.

where

$$V_T = \int_Q \overline{V}(q)dq.$$

Then, the price index of the service good is:

$$P_s = \left[\frac{\overline{m}}{V_T}\int_Q \phi_y(0,q)P(q)^{1-\sigma}dq\right]^{1/(1-\sigma)}. \tag{26}$$

Total sales to the service sector by a manufacturing firm with price p, quality q, posting v ads in the home country to find customers are:

$$\frac{v}{V_T}\left(\frac{p}{P_s}\right)^{1-\sigma}\overline{m}\phi_y(0,q)X_s$$

where X_s is the total absorption of services. Using (26), these sales are:

$$p^{1-\sigma}vD_s(q) \tag{27}$$

$$\text{where}\quad D_s(q) = \phi_y(0,q)\left[\int_Q \phi_y(0,q')P(q')^{1-\sigma}dq'\right]^{-1}X_s.$$

They do not depend on $\overline{m}$.

Take total manufacturing absorption to be the numeraire. Households consume only the service good. Then, service absorption X_s is the share of service and labor inputs and profits in manufacturing absorption:

$$X_s = 1 - \frac{(\sigma-1)}{\sigma}\alpha_m.$$

3.3 Equilibrium

The demand shifter faced by a manufacturing firm in (7) is the sum of demand from other manufacturing firms (25) and from services (27):

$$D(q) = D_m(q) + D_s(q). \tag{28}$$

We take the supply of efficiency units of labor to produce task q as an exogenous function $L(q,w)$, where w is the whole wage schedule $w(q)$ for all $q \in Q$. Labor markets clear if for all q:

$$L(q,w) = \frac{1}{w(q)\sigma}\left[(1-\alpha_m-\alpha_s)(\sigma-1)+1-\frac{1}{\gamma}\right]X(q) \tag{29}$$

where the constant is the labor share in manufacturing production in (10).

We have derived aggregate variables as functions of equilibrium wages $w(q)$ and firm outcomes. Measure $J(z,q)$ is in (18). The success rates of ads are $\theta_m(q) = \tilde{M}(q)/M(q)$ and $\theta_v(q) = \tilde{M}(q)/V(q)$, where $M(q)$, $V(q)$ and $\tilde{M}(q)$ are in (19), (20) and (21). Costs $c(q)$ and $C(m,q)$ are in (8) and (23), and demand $D(q)$ is in (28). Firms maximize profits in (10) given wages $w(q)$ and other firms' actions summarized in $c(q)$ and $D(q)$. Denote with Θ a set of firm outcomes, specifying for each $\omega \in \Omega$ its quality, productivity, sales, measures of upstream and downstream ads and price.

An **equilibrium** is a set of wages w and of firm outcomes Θ such that the functions $D(q)$ and $C(1,q)$ exist and that the following conditions are satisfied:

1. The labor market clears (29).

2. Firms maximize profits. Firm ω chooses $q(\omega)$ in (14) and has productivity $z^*(\omega) = z(q(\omega),\omega)$ at the optimal. Its sales, measure of ads, and prices are $x(z^*(\omega),q(\omega))$, $m(z^*(\omega),q(\omega))$, $v(z^*(\omega),q(\omega))$, and $p(z^*(\omega),q(\omega))$ in (11).

3.4 Properties of the Model

The model has two novel features: The use of log-supermodular functions to capture assortative matching and the search-and-matching setup of network formation. We explain these features in Sections 3.4.1 and 3.4.2, respectively.

3.4.1 Assortative Matching

In the estimation below, we assume that the wage per worker is increasing in firm quality. Then, assortative matching in wage per worker in the network arises through buyers' and sellers' quality levels.

For a firm with quality q, the measure of its suppliers that have quality q_1 relative to quality q_2 is (integrating (22)):

$$\frac{\phi_v(q,q_1)}{\phi_v(q,q_2)} \frac{\overline{V}(q_1)}{\overline{V}(q_2)}. \tag{30}$$

The firm's average spending on its suppliers of quality q_1 relative to its suppliers of quality q_2 is (integrating (16)):

$$\frac{\phi_y(q,q_1)}{\phi_y(q,q_2)} \left(\frac{P(q_1)}{P(q_2)}\right)^{1-\sigma} \frac{\overline{V}(q_2)}{\overline{V}(q_1)}. \tag{31}$$

Multiplying these expressions (or using equation (23)), the ratio of the firm's total spending on the two qualities is:

$$\frac{\phi_v(q,q_1)}{\phi_v(q,q_2)}\frac{\phi_y(q,q_1)}{\phi_y(q,q_2)}\left(\frac{P(q_1)}{P(q_2)}\right)^{1-\sigma}. \tag{32}$$

These expressions summarize the extensive margin (30), intensive margin (31) and total (32) assortative matching in the network. Since the terms $\overline{V}(q)$ and $P(q)$ are common to all buyers, functions ϕ_y and ϕ_v alone govern assortative matching. By definition, a function ϕ is log-supermodular if $\phi(q,q_1)/\phi(q,q_2)$ is increasing in q whenever $q_1 > q_2$ or equivalently $\partial^2 \log(\phi(q,q'))/\partial q \partial q' > 0$. Function $\phi_v(q,q')$ governs the distribution of sales ads posted by suppliers with quality q' across buyers of quality q. We parameterize ϕ_v as the density of a normal random variable with variance ν_v. Its derivative $\partial^2 \log(\phi_v(q,q'))/\partial q \partial q' = 1/\nu_v$ is positive. Then, higher-quality firms have relatively more higher-quality suppliers in (30). Function $\phi_y(q,q')$ governs the marginal product of an input of quality q' in the production of output quality q. It is log-supermodular if $\nu_y > 0$ in (17). Then, higher-quality firms spend relatively more on their higher-quality suppliers in (31).

3.4.2 Search and Matching

We consider a special case of the model to highlight its search and matching setup.[21] Assume that there is only one quality and $\beta_v = \beta_m \equiv \beta$. Set $\phi_v = \phi_y = 1$ without loss of generality. Take wages as the numeraire, and drop the quality arguments from functions. We refer to a firm by its productivity z instead of ω. The mass of firms N and the distribution of z are exogenous. Appendix D has the complete, closed-form solution to this special case and analyzes its efficiency properties.[22]

With $\beta_v = \beta_m$, the ratio of ads to find suppliers and customers in (11) is $m(z)/v(z) = (\alpha_m f_v/f_m)^{1/\beta}$, independent of firm productivity. Then, the success rates of ads θ_m and θ_v are exogenous functions of parameters. The number of customers and the number of suppliers of firm z are the same:

$$\theta_v\left(\frac{x(z)}{\sigma f_v}\right)^{1/\beta} = \theta_m\left(\frac{\alpha_m x(z)}{\sigma f_m}\right)^{1/\beta}.$$

[21] This special case relates to the setup of Miyauchi (2019), who incorporates matching frictions in firm-to-firm trade in a version of the multi-location multi-sector Melitz (2003) model.

[22] There are two externalities for each ad in the decentralized equilibrium. A positive externality is that ads increase the total mass of matches $\tilde{M}$. A negative externality is that ads decrease the probability of matching for firms in the same of side of the market as the ads (sellers for v ads and buyers for m ads). The negative externality is always greater than the positive externality, so the planner posts fewer ads than in the market equilibrium. There is no inefficiency from the allocation of ads across heterogeneous firms. The allocation of labor for production is also efficient. Markups are constant in manufacturing, and the service sector has no labor.

They increase log-linearly with firm sales, as in Table 5.

The probability that a firm with productivity z is the buyer or the seller in a match is:

$$\frac{m(z)}{M} = \frac{v(z)}{V} = \frac{z^{\gamma(\sigma-1)/\beta}}{N\mathbb{E}\left(z^{\gamma(\sigma-1)/\beta}\right)}.$$

It does not depend on the other firm in the match. Thus, there is no assortative matching in the network: All firms are more likely to match with more productive firms.[23]

The market share of a firm with productivity z in total manufacturing sales is:

$$x(z) = \frac{z^{\gamma(\sigma-1)}}{N\mathbb{E}\big(z^{\gamma(\sigma-1)}\big)}.$$

The expression is the same as Melitz (2003) except for the added parameter $\gamma > 1$. The effect of productivity on sales is augmented because more productive firms post more ads to find suppliers and customers. Thus, the model needs a smaller dispersion in firms' fundamental productivity z to generate the same distribution of sales as Melitz (2003).

4 Open Economy

We embed the model above into a small open economy setup. The prices of foreign varieties and foreign demand for domestic goods are exogenous. Manufacturing firms may export by paying a fixed cost and posting ads abroad. Service firms combine domestic and foreign varieties with a constant elasticity of substitution σ. We focus here only on the differences from the closed economy case. Appendix E presents the full model.

The manufacturing firm ω has productivity $z(q, \omega)$ in (13). The firm chooses $q \in Q$ and then draws a random fixed export cost of f_E units of the service good from a common distribution. She then decides her export status and posts ads to search for domestic suppliers, for domestic customers, and if exporting, for foreign customers. We introduce randomness in the fixed cost of exporting because firms in the data with similar size and wages have different export statuses. The timing simplifies aggregation in the estimation.

The revenue from foreign sales of an exporter with quality q, price p and v sales ads to find foreign customers is:

$$p^{1-\sigma} v e^{\sigma} D_F(q) \tag{33}$$

where $D_F(q)$ is an exogenous demand function and e is the real exchange rate. The cost of

[23] Bernard et al. (2019b), Lim (2018), and Huneeus (2018) generate an increasing relation between a firm's sales and number of network connections by imposing a fixed cost for firms to trade. Their setting generates strong negative assortative matching because only more productive firms pay a fixed cost to trade with less productive firms.

posting v ads in Foreign is the same as the domestic cost in (9), $w(q)f_v v^{\beta_v}/\beta_v$. Assuming the same curvature β_v is important to maintain the log-linearity in the firm's problem. The cost parameter f_v is the same as that for domestic ads to simplify the notation only, since we do not observe foreign trading partners.

By backward induction, we start with the problem of the firm after it has chosen its quality and export status. A firm with quality q, productivity z and export status $E \in \{0,1\}$ chooses a mass of ads to find suppliers m, a mass of ads to find customers v and the share $r_v \in [0,1]$ of the sales ads that are posted domestically:

$$\max_{m,v,r_v} \frac{v m^{\alpha_m}}{\sigma}\left[\frac{\sigma}{\sigma-1}\frac{C(1,q)}{z}\right]^{1-\sigma}[r_v D_H(q) + (1-r_v)Ee^{\sigma}D_F(q)] \\ - w(q)f_v[r_v^{\beta} + (1-r_v)^{\beta}]\frac{v^{\beta_v}}{\beta_v} - w(q)f_m\frac{m^{\beta_m}}{\beta_m} \tag{34}$$

where $C(1,q)$ is the input cost in (8) and $D_H(q)$ is the endogenous domestic demand shifter, denoted with $D(q)$ in equation (7). The optimal share of ads r_v is a function of quality q and export status E:

$$\frac{1-r_v(q,E)}{r_v(q,E)} = \left(\frac{Ee^{\sigma}D_F(q)}{D_H(q)}\right)^{1/(\beta_v-1)}. \tag{35}$$

Given the optimal r_v, problem (34) differs from the closed economy case (10) only in the level of demand and of the cost of posting v sales ads. Then, the relationship between sales, ads and prices takes the form of (11). Total sales are:

$$x(z,q,E) = \Pi(q,E)z^{\gamma(\sigma-1)} \tag{36}$$

where

$$\Pi(q,E) = [\sigma w(q)]^{1-\gamma}\left[D(q,E)\left(\frac{\sigma}{\sigma-1}C(1,q)\right)^{1-\sigma}\left(\frac{f_m}{\alpha_m}\right)^{-\alpha_m/\beta_m} f_v^{-1/\beta_v}\right]^{\gamma} \tag{37}$$

$$D(q,E) = \left[D_H(q)^{\beta_v/(\beta_v-1)} + E(e^{\sigma}D_F(q))^{\beta_v/(\beta_v-1)}\right]^{(\beta_v-1)/\beta_v}. \tag{38}$$

Exporting increases the firm's profit by more than the sum of the profits from operating separately in each market. The firm uses the same input suppliers to produce all its goods, regardless of destination. Thus, exporting increases the firm's incentives to search for suppliers, which lowers price and increases the firm's incentives to search for customers in both markets. The exponent in the CES term $D(q,E)$ and γ capture these magnification effects.[24]

The firm exports if its fixed exporting cost parameter $f_E \leq \overline{f}_E(z, q)$, where

$$\overline{f}_E(z, q) = \frac{z^{\gamma(\sigma-1)}}{\gamma\sigma P_s} \left[\Pi(q, 1) - \Pi(q, 0)\right]. \tag{39}$$

Denote with Φ the cumulative distribution function of f_E. After observing its productivity $z(q, \omega)$ but before observing f_E, the firm chooses its quality:

$$q(\omega) = \arg\max_{q \in Q} \Bigg\{ \frac{z(q, \omega)^{\gamma(\sigma-1)}}{\gamma\sigma} \Bigg[\Pi(q, 1)\Phi\left(\overline{f}_E(z(q, \omega), q)\right) + \Pi(q, 0)\Big[1 - \Phi\left(\overline{f}_E(z(q, \omega), q)\right)\Big]\Bigg] \\ - P_s \mathbb{E}(f_E | f_E \leq \overline{f}_E(z(q, \omega), q)) \Bigg\}. \tag{40}$$

Appendix E makes exactly the same assumptions on production and network formation as in the closed economy case. The only difference is that because sales, ads and prices depend on export status, aggregation in the open economy model is over two measure functions:

$$\begin{aligned} \tilde{J}(z, q, 1) &= J(z, q)\Phi\left(\overline{f}_E(z, q)\right) \\ \tilde{J}(z, q, 0) &= J(z, q)\left[1 - \Phi\left(\overline{f}_E(z, q)\right)\right] \end{aligned} \tag{41}$$

where $J(z, q)$ is defined in (18). The equilibrium is also similarly defined with the exchange rate e as an additional equilibrium variable and a trade equilibrium condition, in which we allow an exogenous trade imbalance.

5 Estimation and Identification

The key estimation assumption is that the wage per worker ($w(q) \times$ labor endowment per worker) is strictly increasing in q. Using a Roy (1951) model, Teulings (1995) provides a micro foundation for the labor supply function $L(q, w)$ and for this estimation assumption.[25] We also prove that we can construct a set of labor endowments that exactly matches the distribution of wage per worker across firms in the data. See Appendix C for details.

We calibrate some parameters and estimate others using the method of simulated mo-

[24] The interconnection between a firm's decisions on sales, prices and purchases in the domestic market and its participation in other markets (export or not) does not appear in standard models of exporting à la Melitz (2003) but does appears in models of importing such as Antràs et al. (2017).

[25] See Costinot and Vogel (2010) for an application of Teulings (1995) to international trade.

ments. A closed economy is defined by parameters $\{\alpha_m, \alpha_s, \sigma, f_m, f_v, \beta_m, \beta_v, \overline{m}, \kappa, \nu_y, \nu_v, \overline{\omega}_2\}$, the labor supply $L(q, w)$, and the set of firms Ω, itself specified by a mass N and a distribution of firm productivity parameters (ω_0, ω_1). In addition, the open economy has the price of the bundle of imported goods P_F, foreign demand $D_F(q)$, and the distribution of fixed costs of exporting f_E.

5.1 Calibrated Parameters and Normalizations

We calibrate production parameters $\{\alpha_m, \alpha_s, \sigma, \beta_v, \beta_m\}$. We set $\alpha_m = 0.33$ and $\alpha_s = 0.38$ in (8) to the cost shares of manufacturing and services in the Turkish manufacturing sector. The elasticity of substitution $\sigma = 5$ is from Broda and Weinstein (2006). We set $\beta_m = 1/0.59$ and $\beta_v = 1/0.46$ to match the endogenous elasticity of the number of suppliers and customers with respect to firm sales in Table 5.

We also normalize the mass of firms to $N = 1$. We set $f_m = f_v = 1$. Since search efforts are not observable, we cannot separately identify the cost of one ad, f_m and f_v, from the matching efficiency κ in (21). Similarly, parameter $\overline{m}$ is not identified because it governs the theoretical price index P_s in (26) but not the observable sales of manufactures to services in (27). We pick $\overline{m}$ so that $P_s = 1$.

We set equilibrium efficiency wages $w(q) = 1$ for all q and real exchange rates $e = 1$. While these variables endogenously respond to counterfactuals, they may be normalized in the cross-section. We observe the wage per worker in the data, but we can always normalize the endowment of efficiency units of labor per worker so that the efficiency wage $w(q) = 1$. Similarly, we can set e and adjust the foreign demand $D_F(q)$ and price P_F accordingly.

5.2 Parameterization

Assume (ω_0, ω_1) are distributed according to a bivariate normal with standard deviations σ_{ω_0} and σ_{ω_1} and correlation ρ. The fixed export costs f_E are log-normally distributed with mean μ_E and standard deviation σ_E. We parameterize:

$$D_F(q) = b_1 q^{b_2}$$

where b_1 and b_2 are parameters.

5.3 Moments and Identification

We use 39 moments to estimate the remaining 11 parameters: $\{\kappa, \nu_y, \nu_v, \bar{\omega}_2, \sigma_{\omega_0}, \sigma_{\omega_1}, \rho, \mu_E, \sigma_E, b_1, b_2\}$. To exploit information on the joint distribution of firm wages, sales, number of network links, and export activities as well as the novel sorting patterns, we summarize most moments conditional on the 5 quintiles of firm wage per worker:

1. The mean number of suppliers (5 moments) and mean number of customers (5 moments)
2. The share in total network sales (5 moments) and the standard deviation of sales (5 moments)
3. The share of firms exporting (5 moments) and the average export intensity for exporting firms (5 moments)
4. The average log-wage of suppliers, unweighted (4 moments) and weighted by spending shares (4 moments)[26]
5. The shift-share regression coefficient of the wage response to an idiosyncratic export demand shock (1 moment)

Although all parameters are estimated jointly, some parameters are associated with some moments more closely. The average number of trading partners per firm identifies κ, the efficiency in transforming ads into matches in (21). Total sales and the standard deviation by quintile of wages identify the parameters σ_{ω_0}, σ_{ω_1}, and ρ. Parameter μ_E governs the share of firms exporting, and σ_E governs how this share changes across quintiles of firm wages. If σ_E is large, then the share of firms exporting does not vary much across quintiles because it depends more on firm draws of f_E than on quality choices (wages). Parameter b_1 governs the level of export intensity, while b_2 governs how export intensity changes across quintiles of firm wages. If b_2 is large, $D_F(q)/D_H(q)$ is increasing in q, and export intensity increases with the wage quintile.

The moments on suppliers' wages summarize the total and extensive margins of assortative matching in the network. As per Section 3.4, parameter ν_y governs the intensive margin in (31), and parameter ν_v governs the extensive margin (30).

Finally, the shift-share coefficient of Table 3, column (2), identifies $\overline{\omega}_2$. Consider a shock that increases a single firm's export demand $D_F(q)$ by 5 percent. If $D_F(q)/D_H(q)$ is increasing in quality as in our estimated model, the firm increases $q(\omega)$. This increase

[26] This set includes only four moments (and not one per quintile) because we normalize the wages in the lowest quintile to 0 and match the log difference from the lowest quintile in the data and model.

is associated with an increase in the wage per worker since each quality in the estimated model is associated with an average wage per worker in the data (the ranking is the same). The parameter $\overline{\omega}_2$ governs the concavity of $z(q, \omega)$ in (13). If $\overline{\omega}_2$ is large and negative, then $z(q, \omega)$ is very concave, and the firm does not respond much to the export demand shock. If $\overline{\omega}_2$ is small, the response is large.[27]

5.4 Model Computation

We solve the equilibrium of the model for each guess of parameters. We discretize the quality space into a grid of 100 equally spaced choices in [0, 8]. Given a guess of $\sigma_{\omega 0}, \sigma_{\omega 1}, \rho$, we sample 50,000 firms from the bivariate distribution of $\omega = (\omega_0, \omega_1)$ and calculate each firm's productivity at each quality, $z(q, \omega)$ in (13).

The solution algorithm, detailed in Appendix G, is composed of two blocks. The inner block takes the equilibrium distribution of productivity-quality $J(z, q)$ as given. It solves the equilibrium in the matching and product markets given $J(z, q)$ and the optimal export status, search and production decisions for each (z, q). From this inner block, we obtain the aggregate functions $\Pi(q, 0)$ and $\Pi(q, 1)$ that govern each firm's export cutoff $\overline{f}_E(z, q)$ in (39) and quality choice in (40). The outer block solves the optimal quality choice for each firm ω and updates $J(z, q)$ used in the inner block. We iterate over these two blocks until firms do not change their quality choices.[28]

6 Estimation Results

The targeted moments are in Table 7. The estimated parameters in Table 6 are split into three sets. The first set $\{\nu_v, \nu_y, \kappa\}$ governs network formation. Parameter ν_v is the standard deviation of the distribution of ads ϕ_v in Figure 3B. The estimated value $\nu_v = 3.09$ implies, for example, that 65 percent of the ads posted by sellers in the top

[27] In Appendix F, we prove that we can non-parametrically identify the joint distribution of (ω_0, ω_1) using the joint distribution of sales and wages and that $\overline{\omega}_2$ is not identified in the cross-section. We also show its identification through idiosyncratic firm-specific shocks.

To construct the model's response, we sample firms and estimate the expected effect from the idiosyncratic demand shocks as the average change in wages per worker weighted by firms' export probabilities.

[28] The estimated function $\Pi(q, E)$ is concave in q because all buyers' (service and manufacturing firms') valuation of quality, ϕ_y in (15), is concave. Then, the quadratic form of $z(q, \omega)$ in (13), together with $\bar{\omega}_2 < 0$, implies that all firms' problem of choosing quality (14) is concave and that quality choices are bounded even for firms that have a comparative advantage in producing higher quality, $\omega_1 > 0$.

Although we cannot guarantee the uniqueness of the equilibrium, we conduct 500 Monte Carlo simulations, each with random starting choices of firm quality. In all simulations, the algorithm converges to the same equilibrium. We conduct these simulations for the parameter estimates and the baseline counterfactuals.

Table 6: Parameter Estimates

	Parameter	Estimate	Standard error
Matching friction	κ	0.00087	(0.00003)
Directed search	ν_v	3.09	(0.06)
Complementarity	ν_y	0.35	(0.03)
Sd of quality capability	σ_{ω_1}	0.116	(0.001)
Sd of efficiency capability	σ_{ω_0}	0.110	(0.000)
Correlation	ρ	0.137	(0.002)
Efficiency cost of quality	$\overline{\omega}_2$	-0.103	(0.001)
Mean of log export cost	μ_E	-3.95	(0.02)
Sd of log export cost	σ_E	1.52	(0.04)
Foreign demand shifter	b_1	93.16	(2.49)
Foreign demand curvature	b_2	0.49	(0.01)

Notes: We calculate the standard errors using the bootstrapped variance-covariance matrix of the moments.

quintile of quality go to buyers also in the top quintile and 8 percent go to buyers in the lowest quintile. Parameter $\nu_y = 0.35$ governs the complementarity in production, i.e., the log-supermodularity of function ϕ_y in Figure 3A. Take two suppliers, one in the highest quintile of quality and one in the lowest quintile. The marginal product of the first input is 46 percent higher when output is in the top quintile of quality and 10 percent higher when output is in the bottom quintile.[29] Parameter $\kappa = 8.7 \times 10^{-4}$ implies a low probability of finding a trading partner per ad. This is not surprising given that the number of partners per firm in the data is a tiny fraction of all manufacturing firms. The average number of suppliers and customers per quintile of wages ranges from 5.6 to 25.8 in Table 7. The model fits these averages well. With only two parameters, ν_v and ν_y, to govern assortative matching, it also fits the increasing relation between buyers' and sellers' wages, weighted and unweighted, reasonably well.

The second set of parameters $\{\sigma_{\omega_0}, \sigma_{\omega_1}, \rho\}$ are those of the joint distribution of (ω_0, ω_1), where ω_0 determines a firm's productivity level and ω_1 its comparative advantage in higher quality. This distribution governs the joint distribution of wages and sales. There is a large dispersion of sales across quintiles of wages in Table 7. Firms in the highest quintile account for 78 percent of network sales in the data and in the model.

The third set $\{\mu_E, \sigma_E, b_1, b_2\}$ governs export patterns. The log of the export cost has mean $\mu_E = -3.95$ and standard deviation $\sigma_E = 1.52$. The share of firms exporting is

[29] We use the median of each quintile to calculate these numbers.

Table 7: Model Fit: Targeted Moments

	Quintiles of average wage per worker				
	1	2	3	4	5 (largest)
Mean number of suppliers					
Data	5.8	6.7	5.8	11.4	25.8
Model	4.7	4.7	6.0	9.1	29.4
Mean number of customers					
Data	5.6	7.0	6.7	11.7	25.1
Model	5.4	5.9	7.6	10.9	23.8
Standard deviation of log sales					
Data	1.37	1.34	1.37	1.52	1.79
Model	1.20	1.18	1.20	1.24	1.55
Share of total network sales					
Data	0.03	0.04	0.04	0.10	0.78
Model	0.04	0.03	0.05	0.11	0.78
Fraction of exporters					
Data	0.08	0.18	0.16	0.34	0.57
Model	0.11	0.13	0.18	0.29	0.60
Export intensity of exporters					
Data	0.24	0.21	0.23	0.23	0.26
Model	0.18	0.21	0.22	0.23	0.25
Unweighted average log wage of suppliers					
Data	-	0.01	0.01	0.04	0.14
Model	-	0.02	0.04	0.07	0.12
Weighted average log wage of suppliers					
Data	-	0.02	0.02	0.07	0.23
Model	-	0.04	0.07	0.11	0.17
Shift-share IV coefficient (5% export shock)					
Data	0.21%				
Model	0.21%				

higher among high-wage firms, but still about 10 percent of low-wage firms export in the data and in the model. Parameters $b_1 = 93$ and $b_2 = 0.49$ govern export intensity by wage quintile. Conditional on exporting, export intensity is increasing in firm wages in the data. The model captures this pattern with an estimate of $D_F(q)/D_H(q)$ that is increasing in q.

This increasing ratio $D_F(q)/D_H(q)$ matters because a firm-specific shock that increases $D_F(q)$ leads the firm to upgrade its quality and thereby increase its wage per worker. This prediction is consistent with the shift-share regressions in Table 3. In the data, a 5 percent export shock on average increases the wage per worker by 0.21 percent for exporting firms, and we pick $\bar{\omega}_2 = -0.103$ to exactly match this response. In column (6) of Table 3, a 1 percent increase in the firm's wage in response to the shock increases its suppliers' wages by 0.434 percent (with a standard error of 0.185 percent). Out-of-sample, this number is 0.219 percent in the model.

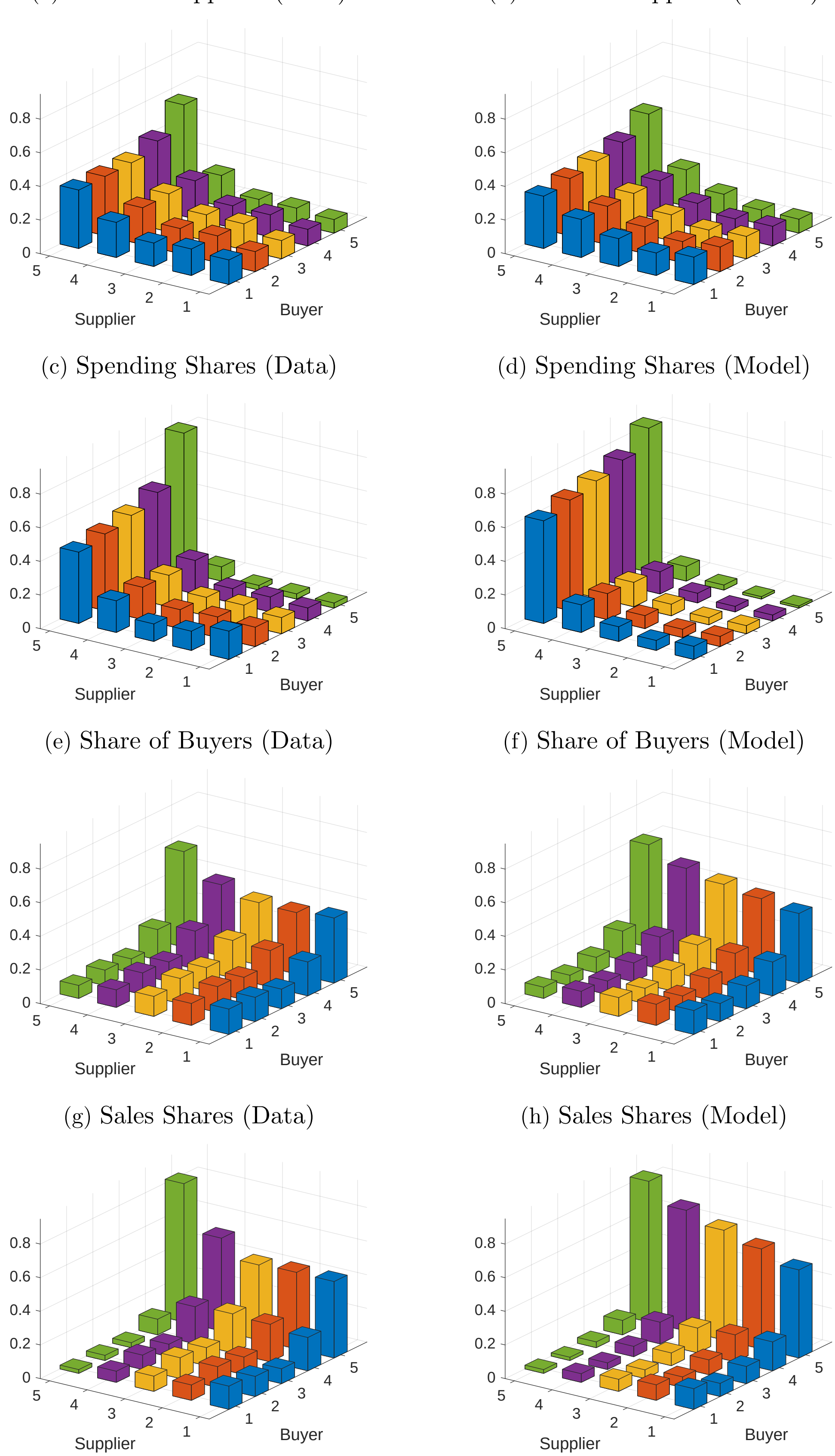
Overall, the moments of the model and the data are similar in Table 7. As further validation, Figure 4 illustrates the predictions of the model for the non-parametric patterns of assortative matching of Figure 2 above. These figures are related to targeted moments, but they were not directly targeted. The model matches well the extent to which firms with similar wages disproportionately transact with each other, upstream and downstream, on the intensive and extensive margins.

Equipped with these estimates, we investigate how a counterfactual increase in export demand affects firm quality. Such a shock potentially has a large indirect effect through the production network, because in the data and the model, exporters are large and skill-intensive and have many domestic trading partners.

7 Counterfactual Analysis

Starting with the equilibrium of the estimated model, our baseline counterfactual increases export demand $D_F(q)$ by 5 percent. It maintains the efficiency wages $w(q) = 1$ for all q, the real exchange rate $e = 1$ and the price of services $P_s = 1$. We allow gross manufacturing output and the trade balance to increase with the shock. We choose this as the baseline because it captures the effect of the shock on manufacturing but shuts down the interaction between manufacturing and the rest of the economy by assuming that (i) labor supply in and out of manufacturing is perfectly elastic ($w(q) = 1$), (ii) the export expansion does not lead to a real exchange rate appreciation ($e = 1$), and (iii) the price of the inputs that

Figure 4: Firm-to-Firm Trade Links and Values by Quintile



(a) Share of Suppliers (Data)

(b) Share of Suppliers (Model)

(c) Spending Shares (Data)

(d) Spending Shares (Model)

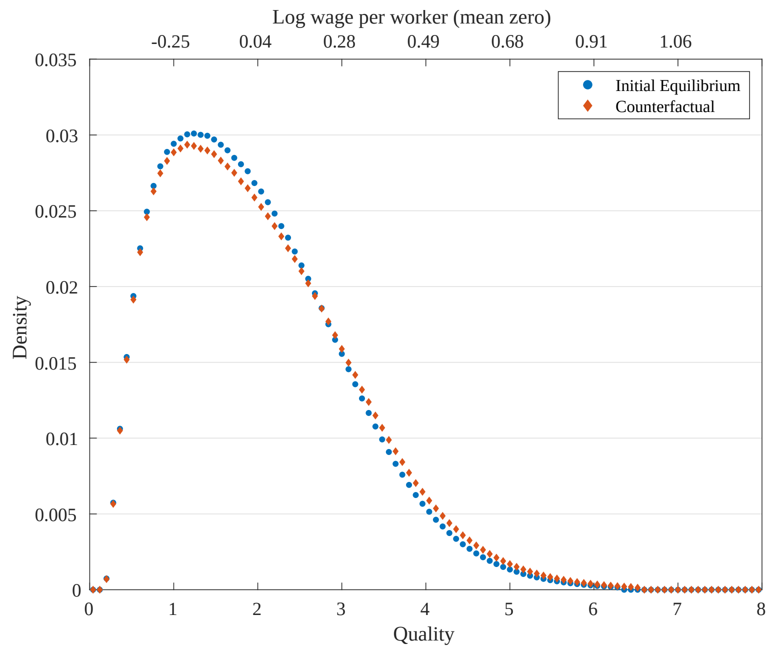
(e) Share of Buyers (Data)

(f) Share of Buyers (Model)

(g) Sales Shares (Data)

(h) Sales Shares (Model)

Figure 5: Distribution of Quality Choices



manufacturing firms use from distributors does not change ($P_s = 1$).[30] Relaxing each of these assumptions, as we do in Section 8, requires out-of-sample assumptions.

Figure 5 plots the density of quality choices. The counterfactual first order stochastically dominates the initial equilibrium. By assumption, the rankings of quality and average wage per worker (efficiency wage $w(q)\times$ labor endowment per worker) are kept constant in the counterfactual, and the model exactly matches the distribution of wage per worker across firms in the data. Thus, the wage per worker in the top x-axis of Figure 5 lends an economic interpretation to quality. Since $w(q) = 1$ in the counterfactual, changes in firm wages reflect only quality upgrading (shifting to higher-quality tasks).

Table 8 reports the changes in wages, sales and number of trading partners for exporters and non-exporters by *ex ante* quintile of the quality distribution. The wage per worker increases in all groups of firms, especially among the *ex ante* high-quality firms. For example, wages in non-exporting, high-quality firms increase by 2.5 log points.

The network propagates the shock from exporting to non-exporting firms. Profit shifter $\Pi(q, 0)$ summarizes the benefit of upgrading quality for non-exporters. As per equations (57) and (8), $\Pi(q, 0)$ is proportional to a demand component $D(q, 0)^\gamma$ and a cost component $c(q)^{\gamma\alpha_m(1-\sigma)}$. Figure 6(a) plots the counterfactual changes relative to the initial equilibrium of $\Pi(q, 0)$ and each of these components. First, take the demand

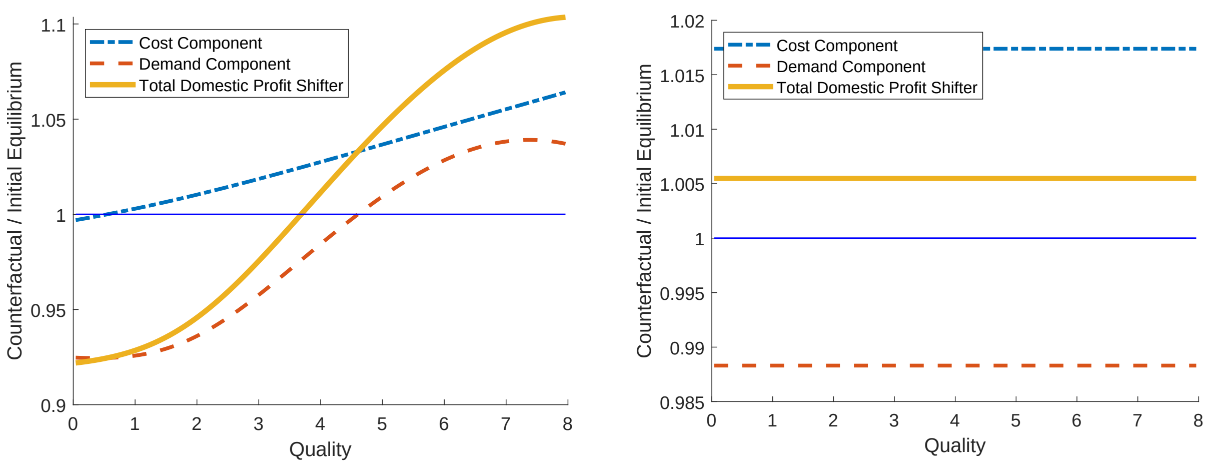
[30]The price stays at $P_s = 1$ in a limiting case in which domestic manufacturing is a small share of inputs into services. Other inputs may be imports or other (not modeled) domestic goods or factors.

Table 8: Counterfactual Changes by Quintile of Quality

	Ex-ante quintiles of quality				
	1	2	3	4	5 (largest)
log*(Wage per worker)*$\times 10^{-2}$, *counterfactual – initial equilibrium*					
Exporters	0.31	0.52	0.92	1.66	2.90
Non-exporters	0.23	0.48	0.89	1.61	2.53
All Firms	0.24	0.48	0.90	1.63	2.76
log*(Sales)*$\times 10^{-2}$, *counterfactual – initial equilibrium*					
Exporters	-1.25	0.50	1.48	3.05	6.58
Non-exporters	-7.69	-7.03	-6.03	-4.25	-1.23
All Firms	-6.93	-5.98	-4.58	-2.01	3.60
log*(Number of Suppliers)*$\times 10^{-2}$, *counterfactual – initial equilibrium*					
Exporters	-0.74	0.29	0.88	1.81	3.90
Non-exporters	-4.56	-4.17	-3.58	-2.52	-0.73
All Firms	-4.11	-3.55	-2.71	-1.19	2.14
log*(Number of Customers)*$\times 10^{-2}$, *counterfactual – initial equilibrium*					
Exporters	-2.47	-1.28	-0.12	1.47	3.82
Non-exporters	-3.55	-2.58	-1.43	0.16	2.14
All Firms	-3.42	-2.40	-1.18	0.56	3.18

Figure 6: Decomposition of Changes in Domestic Profit Shifter

(a) Baseline Counterfactual (b) No Complementarity



Notes: The figure displays the counterfactual changes in the domestic profit shifter. This shifter $\Pi(q,0)$ is proportional to $D(q,0)^{\gamma} \cdot c(q)^{\alpha_m(1-\sigma)\gamma}$, and we separately plot these demand and cost components. The baseline counterfactual is in the left panel, and the special case with no complementarity ($\nu_y = 0$, $\nu_v = \infty$) is in the right panel.

component $D(q,0)^\gamma$ on the red dotted curve. Exporters upgrade quality and increase their posting of ads. Then, the probability of matching increases for high-quality suppliers who direct their ads toward high-quality market segments. At the intensive margin, conditional on the match, exporters increase their spending on high- relative to low-quality domestic suppliers. Second, take the cost component $c(q)^{\gamma\alpha_m(1-\sigma)}$ on the blue dotted curve. The increased search effort and quality upgrading among exporters decrease the cost of manufacturing inputs for all firms. This decrease accrues disproportionately to high-quality firms whose production is intensive in high-quality inputs (estimated $\nu_y > 0$). The more firms respond to these shifts by upgrading their qualities, the more they augment the effect of the shock. Overall, the profitability for non-exporters increases by 7 percent in the high-quality segment ($q \approx 6$), and it decreases by about 7 percent in the low-quality segment ($q \approx 1$). Both $c(q)$ and $D(q,0)$ significantly contribute to these changes.

Exporters (not in the figure) experience similar indirect effects. Their profit shifter $\Pi(q,1)$ is proportional to the same cost component $c(q)^{\gamma\alpha_m(1-\sigma)}$, and their demand component $D(q,1)^\gamma$ is a CES aggregate of domestic demand $D(q,0)$ and foreign demand $D_F(q)$ in equation (38). In all, the average wage increases by 1.0 percent for non-exporters, 1.92 percent for exporters and 1.22 percent for all firms. This increase in exporters' wages is an order of magnitude larger than the increase of 0.21 percent induced by the idiosyncratic export demand shocks of the same magnitude.

The effect of the counterfactual on sales and network connections is more heterogeneous. The domestic market for inputs becomes more competitive ($c(q)$ decreases), and the appeal of low-quality inputs decreases because their marginal product is low in the production of high quality. As a result, lower-quality, non-exporting firms decrease their sales and search efforts. In Table 8, the number of suppliers and customers decreases by 4 log points, and sales decrease by 7.7 log points for these firms. In spite of the positive cross-sectional correlations, the counterfactual simultaneously predicts reductions in sales and network connections and increases in quality for non-exporting firms.

To further probe these mechanisms, we study a special case of the model without the complementarity in matching ϕ_v and in production ϕ_y. The value of high- and low-quality inputs in production is independent of the output quality ($\nu_y = 0$), and all firms' ads are uniformly distributed across the quality set Q ($\nu_v \to \infty$). We re-estimate the model with these parameter restrictions in Appendix H. By assumption, the special case cannot match the increasing relation between buyer and supplier wage. For all other moments, the fit is similar to the general model. Importantly, the ratio $D_F(q)/D_H(q)$ is increasing in quality so that exporters upgrade quality when $D_F(q)$ increases.

We experiment with the same 5 percent counterfactual increase in export demand

$D_F(q)$ in this special case. The average wage increase for exporters is 0.23 percent, very close to the average firm response to an idiosyncratic export demand shock of 0.21 percent. Figure 6 panel (b) plots the change of $\Pi(q,0)$ and of its cost $c(q)^{\gamma\alpha_m(1-\sigma)}$ and demand $D(q,0)^\gamma$ components. The shock decreases the price index $P(q)$ in the domestic market. Competition tightens decreasing demand and costs. However, these changes are independent of quality. Profit shifter $\Pi(q,0)$ increases by 0.5 percent for all non-exporters. In the model, firms choose quality before observing their exporting cost. The flattened $\Pi(q,0)$ mutes the quality response of all firms in this special case, especially those with a low probability of exporting.

Manufacturing output increases by 6.03 percent in the general model and 5.78 percent in the special case. These effects are larger than the classical Hulten (1978) prediction because the positive shock increases exporters' search efforts and leads other firms to tilt their input purchases toward exporters, whose prices decrease. Baqaee and Farhi (2019a) highlight this role of an elasticity of substitution greater than one ($\sigma = 5$ in the estimation). Despite similar predictions on output, the dramatic differences in quality upgrading between the general model and the special case indicate that economies of scale are not sufficient to explain the effect of international trade on developing countries.

8 Alternative Counterfactuals and Discussion

In all of the counterfactuals below, foreign demand $D_F(q)$ increases 5 percent, as in the baseline counterfactual. We experiment with four specifications:

1. We allow the real exchange rate e (wages in Foreign relative to Home) to move to balance trade. In Appendix E, trade balances if

$$\left(\frac{eP_F}{P_s}\right)^{1-\sigma} X_s = \int_{q\in Q} \left(\frac{e^\sigma D_F(q)}{D_H(q)}\right)^{\beta_v/(\beta_v-1)} \left[\int_z x(z,q,1)j(z,q,1)dz\right] dq.$$

   Clearly, imports on the left-hand side decrease with e, and exports on the right-hand side increase. In the baseline, we kept $e = 1$ and allowed the trade surplus to increase with the export shock.

2. We incorporate free entry. In the baseline, average profits increase with the foreign demand shock. Here, we allow the mass of firms to increase, which tightens competition and maintains the average profit as in the initial equilibrium.

3. We allow the wages of skilled workers to increase. In the baseline, we maintain the wage schedule $w(q) = 1$ for all q, assuming that the labor supply is perfectly elastic.

There, labor demand for high-quality tasks increases. Here, we allow the wage $w(q)$ to rise relative to the initial equilibrium and to rise proportionately with quality. In particular, we assume $w(q)$ is linear, set $w(0) = 1$ and pick $w(q^{max})$ so that the average counterfactual quality change across firms is zero.

4. We increase the productivity of the highest-quality firms under the assumption that the agglomeration of skilled workers in manufacturing increases firm productivity, as estimated in Diamond (2016). In the baseline, the stock of labor in the *ex ante* top quintile of quality rises by 0.846 percent. Diamond (2016) estimates that an increase in college graduates of one percent in a location increases their productivity by 0.854 percent. Using these numbers, we increase the productivity $z(q, \omega)$ of firms in the *ex ante* top quintile of quality by 0.72 percent ($= 0.854 \times 0.00846$).[31]

Table 9 summarizes the results. With free entry (2), the number of firms increases by 1.13 percent, but the remaining results are close to the baseline. With balanced trade (1), the real exchange rate appreciates by 1.15 percent, and in counterfactual (3), wages at the top quality $w(q^{max})$ increase by 0.79 percent. Both of these price changes decrease the incentives for firms to upgrade quality. Although they are small, they have a significant effect, because both the positive and negative effects are magnified in general equilibrium. The average wage per worker increases by 0.20 percent with balanced trade (1) and by 0.16 with the increase in the skill premium (3) in comparison to 1.22 percent in the baseline.

Agglomeration effects increase the productivity of high-quality firms in specification (4). By the same general equilibrium effects, even a small increase, of 0.72 percent, has a large effect: The average wage per worker increases by 3.17 percent and output by 13.7 percent, roughly double the baseline numbers.[32]

Shifter $D_F(q)$ summarizes the foreign market size, price index, and frictions in matching with foreign customers. Thus, the counterfactual increase in $D_F(q)$ may be interpreted as a foreign shock or as a policy to promote exports through decreases in search frictions, e.g., export fairs and conferences.[33] Counterfactuals (1), (3) and (4) highlight critical factors in the effectiveness of these export-promotion policies. In counterfactual (3), the

[31] In Diamond's (2016) model, the inverse demand function for college graduates is

$$\log w_H = \gamma \log L_H - (1/\sigma) \log L_H$$

where L_H is the supply of college graduates in a location, σ is the elasticity of substitution between skilled and unskilled workers and γ is the external scale parameter. She estimates $\sigma = 1.6$ and $\gamma - 1/\sigma = 0.229$, yielding $\gamma = 0.854$.

[32] This exercise is akin to Jones (2011), who emphasizes the roles of complementarity and economies of scale in economic growth.

[33] Rauch (2001) surveys case studies of this type of export-promotion policies.

Table 9: Summary of Counterfactuals

	Counterfactual Specifications				
	Baseline	Balanced Trade	Free Entry	Δ Skill Premium	Agglomeration
Percentage changes in		(1)	(2)	(3)	(4)
Output (X)	6.03	0.00	6.39	3.07	13.72
Exchange rate (e)	-	-1.15	-	-	-
Mass of firms (N)	-	-	1.13	-	-
Efficiency wage at $w(q^{max})$	-	-	-	0.79	-
Average wage per worker (All)	1.22	0.20	1.30	0.16	3.17
Average wage per worker (Exporters)	1.92	0.32	2.06	0.13	5.01
Average Quality (All)	2.06	0.34	2.19	0.00	5.24

rise in the skill premium dampens the incentives for firms to upgrade to skill-intensive qualities. This points to the importance of ensuring an elastic supply of skilled workers into manufacturing, perhaps through education and training. In counterfactual (1), the effects of export promotion in quality upgrading are dampened when a real exchange rate appreciation prevents the country from running a trade surplus. In addition, in counterfactual (4), output grows when the agglomeration of skilled workers in manufacturing increases firm productivity. These counterfactuals together rationalize the concomitant increases in the trade surplus and in manufacturing production and upgrades commonly observed in fast-growing emerging markets, notably in East Asia.

9 Conclusion

We document novel facts about firm-to-firm trade using data from Turkey. High-wage firms are more likely to match with each other in the network, and the value of transactions is larger when the trading partners' wages are both high. Over time, a firm-specific demand shock from a rich export destination is associated with an increase in the firm's wage and in the average wage of its suppliers.

We rationalize these findings in a model where firms' choices of quality and skill intensity are interconnected through the production network. Higher-quality production is intensive in skilled labor and in higher-quality inputs, and higher-quality firms direct their search toward other higher-quality firms. Counterfactuals show that even a small export shock leads to large and widespread quality upgrades in manufacturing firms because of the complementarity in their quality choices.

These findings are broadly consistent with those of Goldberg and Reed (2020), who show that exporting even a small amount of output to developed countries is associated with economic growth in developing countries. Alternative counterfactual scenarios in Section 8 point to other economic factors that interact with the effects of international trade on manufacturing firms: education, trade imbalances, and agglomeration effects.

References

ABOWD, J. M., F. KRAMARZ, AND D. N. MARGOLIS (1999): "High Wage Workers and High Wage Firms," *Econometrica*, 67, 251–333.

ACEMOGLU, D., V. M. CARVALHO, A. OZDAGLAR, AND A. TAHBAZ-SALEHI (2012): "The Network Origins of Aggregate Fluctuations," *Econometrica*, 80, 1977–2016.

ADÃO, R., M. KOLESÁR, AND E. MORALES (2019): "Shift-Share Designs: Theory and Inference," *Quarterly Journal of Economics*, 134, 1949–2010.

ALFARO-URENA, A., I. MANELICI, AND J. P. VASQUEZ (2019): "The Effects of Multinationals on Workers: Evidence from Costa Rica," .

ANTRÀS, P., T. C. FORT, AND F. TINTELNOT (2017): "The Margins of Global Sourcing: Theory and Evidence from US Firms," *American Economic Review*, 107, 2514–64.

BAQAEE, D. R. AND E. FARHI (2019a): "The Macroeconomic Impact of Microeconomic Shocks: Beyond Hulten's Theorem," *Econometrica*, 87, 1155–1203.

——— (2019b): "Productivity and Misallocation in General Equilibrium," *Quarterly Journal of Economics*, 135, 105–163.

BARTIK, T. J. (1991): *Who Benefits from State and Local Economic Development Policies?*, WE Upjohn Institute for Employment Research.

BASTOS, P., J. SILVA, AND E. VERHOOGEN (2018): "Export Destinations and Input Prices," *American Economic Review*, 108, 353–92.

BECKER, G. S. (1973): "A Theory of Marriage: Part I," *Journal of Political Economy*, 81, 813–846.

BERNARD, A. B., E. DHYNE, G. MAGERMAN, K. MANOVA, AND A. MOXNES (2019a): "The Origins of Firm Heterogeneity: A Production Network Approach," *National Bureau of Economic Research Working Paper Series*, No. 25441.

BERNARD, A. B., A. MOXNES, AND Y. U. SAITO (2019b): "Production Networks, Geography, and Firm Performance," *Journal of Political Economy*, 127, 639–688.

BIGIO, S. AND J. LA'O (2020): "Distortions in Production Networks," *Quarterly Journal of Economics*.

Bombardini, M., G. Orefice, and M. D. Tito (2019): "Does Exporting Improve Matching? Evidence from French Employer-Employee Data," *Journal of International Economics*, 117, 229–241.

Borusyak, K., P. Hull, and X. Jaravel (2018): "Quasi-Experimental Shift-Share Research Designs," *National Bureau of Economic Research Working Paper Series*, No. 24997.

Brambilla, I., D. Lederman, and G. Porto (2012): "Exports, Export Destinations, and Skills," *American Economic Review*, 102, 3406–38.

Broda, C. and D. E. Weinstein (2006): "Globalization and the Gains from Variety," *Quarterly Journal of Economics*, 121, 541–585.

Carvalho, V. M. and N. Voigtländer (2014): "Input Diffusion and the Evolution of Production Networks," *National Bureau of Economic Research Working Paper Series*, No. 20025.

Costinot, A. (2009): "An Elementary Theory of Comparative Advantage," *Econometrica*, 77, 1165–1192.

Costinot, A. and J. Vogel (2010): "Matching and Inequality in the World Economy," *Journal of Political Economy*, 118, 747–786.

De Loecker, J. (2007): "Do Exports Generate Higher Productivity? Evidence from Slovenia," *Journal of International Economics*, 73, 69–98.

Dhyne, E., A. K. Kikkawa, M. Mogstad, and F. Tintelnot (2018): "Trade and Domestic Production Networks," *National Bureau of Economic Research Working Paper Series*, No. 25120.

Diamond, R. (2016): "The Determinants and Welfare Implications of US Workers' Diverging Location Choices by Skill: 1980-2000," *American Economic Review*, 106, 479–524.

Dingel, J. I. (2017): "The Determinants of Quality Specialization," *Review of Economic Studies*, 84, 1551–1582.

Dupuy, A. and A. Galichon (2015): "Canonical Correlation and Assortative Matching: A Remark," *Annals of Economics and Statistics*, 375–383.

Eaton, J., S. Kortum, and F. Kramarz (2018): "Firm-to-Firm Trade: Imports, Exports, and the Labor Market," .

Feenstra, R. C. and J. Romalis (2014): "International Prices and Endogenous Quality," *Quarterly Journal of Economics*, 129, 477–527.

Fieler, A. C., M. Eslava, and D. Y. Xu (2018): "Trade, Quality Upgrading, and Input Linkages: Theory and Evidence from Colombia," *American Economic Review*, 108, 109–46.

Goldberg, P. K. and T. Reed (2020): "Income Distribution, International Integration, and Sustained Poverty Reduction," *National Bureau of Economic Research Working Paper Series*, No. 27286.

Goldsmith-Pinkham, P., I. Sorkin, and H. Swift (2020): "Bartik Instruments: What, When, Why, and How," *American Economic Review*, 110, 2586–2624.

Hallak, J. C. (2006): "Product Quality and the Direction of Trade," *Journal of International Economics*, 68, 238–265.

Hulten, C. R. (1978): "Growth Accounting with Intermediate Inputs," *Review of Economic Studies*, 45, 511–518.

Hummels, D., R. Jørgensen, J. Munch, and C. Xiang (2014): "The Wage Effects of Offshoring: Evidence from Danish Matched Worker-Firm Data," *American Economic Review*, 104, 1597–1629.

Huneeus, F. (2018): "Production Network Dynamics and the Propagation of Shocks," .

Johnson, R. A. and D. W. Wichern (1988): *Applied Multivariate Statistical Analysis*, vol. 5, Prentice-Hall, Inc.

Jones, C. I. (2011): "Intermediate Goods and Weak Links in the Theory of Economic Development," *American Economic Journal: Macroeconomics*, 3, 1–28.

Khandelwal, A. (2010): "The Long and Short (of) Quality Ladders," *Review of Economic Studies*, 77, 1450–1476.

Khandelwal, A. K., P. K. Schott, and S.-J. Wei (2013): "Trade Liberalization and Embedded Institutional Reform: Evidence from Chinese Exporters," *American Economic Review*, 103, 2169–95.

Kremer, M. (1993): "The O-Ring Theory of Economic Development," *Quarterly Journal of Economics*, 108, 551–575.

Kugler, M. and E. Verhoogen (2011): "Prices, Plant Size, and Product Quality," *Review of Economic Studies*, 79, 307–339.

Lenoir, C., J. Martin, and I. Mejean (2019): "Search Frictions in International Good Markets," CEPR Discussion Papers 13442, C.E.P.R. Discussion Papers.

Lim, K. (2018): "Endogenous Production Networks and the Business Cycle," .

Liu, E. (2019): "Industrial Policies in Production Networks," *Quarterly Journal of Economics*, 134, 1883–1948.

Manova, K. and Z. Zhang (2012): "Export Prices Across Firms and Destinations," *Quarterly Journal of Economics*, 127, 379–436.

Melitz, M. J. (2003): "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity," *Econometrica*, 71, 1695–1725.

Milgrom, P. and J. Roberts (1990): "The Economics of Modern Manufacturing: Technology, Strategy, and Organization," *American Economic Review*, 80, 511–528.

Miyauchi, Y. (2019): "Matching and Agglomeration: Theory and Evidence from Japanese Firm-to-Firm Trade," .

Mortensen, D. T. (1986): "Job Search and Labor Market Analysis," *Handbook of Labor Economics*, 2, 849–919.

Petrongolo, B. and C. A. Pissarides (2001): "Looking into the Black Box: A Survey of the Matching Function," *Journal of Economic Literature*, 39, 390–431.

Rauch, J. E. (2001): "Business and Social Networks in International Trade," *Journal of Economic Literature*, 39, 1177–1203.

Rogerson, R., R. Shimer, and R. Wright (2005): "Search-Theoretic Models of the Labor Market: A Survey," *Journal of Economic Literature*, 43, 959–988.

Roy, A. D. (1951): "Some Thoughts on the Distribution of Earnings," *Oxford Economic Papers*, 3, 135–146.

Schott, P. K. (2004): "Across-Product Versus Within-Product Specialization in International Trade," *Quarterly Journal of Economics*, 119, 647–678.

Teulings, C. N. (1995): "The Wage Distribution in a Model of the Assignment of Skills to Jobs," *Journal of Political Economy*, 103, 280–315.

Verhoogen, E. A. (2008): "Trade, Quality Upgrading, and Wage Inequality in the Mexican Manufacturing Sector," *Quarterly Journal of Economics*, 123, 489–530.

Voigtländer, N. (2014): "Skill Bias Magnified: Intersectoral Linkages and White-Collar Labor Demand in U.S. Manufacturing," *Review of Economics and Statistics*, 96, 495–513.